JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

/.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

VOL. XXXIX.

LONDON:
CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.
PARIS: 8, RUE LAMARTINE, 8.
BERLIN: CARLSTRASSE 11. MELBOURNE: McCARRON, BIRD & CO.
NEW YORK: THE SPECTATOR COMPANY.

1905.

[The Council of the Institute of Actuaries wish it to be understood that while they consider it their duty to give, from time to time, publicity to certain of the papers presented to the Institute, and to abstracts of the discussions at the Sessional Meetings, they are not responsible for the opinions put forward therein.]

LONDON:
PRINTED BY CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.

CONTENTS OF VOL. XXXIX.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Opening Address by the President, HENRY COCKBURN, Esq.

[Delivered 28 November 1904.]

TO have been elected by you to fill the highest position in the Institute is an honour of which any one of us may indeed be proud; and I tender you my sincere acknowledgment of the very great compliment which has been paid me, and which I feel might well have fallen to the lot of a member more worthy of it. With a great satisfaction at having, by your courtesy, attained so honourable a place, there mingles thought of the duties and responsibilities which it must entail; and I can but say that it will be my endeavour, with your friendly support, to uphold the status and usefulness of the Institute, from association with which, during many years past, I have derived great instruction and advantage.

Since the last Presidential Address was delivered before you, the extensive mortality investigation undertaken jointly by the Institute and the Faculty of Actuaries in Scotland has been brought to a conclusion, and the results of the enquiry are of the greatest professional interest to us; and, further, looking to the vast sums to which the accumulated funds of our British Life Offices, ordinary and industrial, now amount (some £300,000,000), it is evident that the bases of our various calculations form a matter of increasing responsibility for our profession, and of very considerable public importance.

It is generally agreed that to the passing of the Life Assurance Companies Act, 1870, in connection with which eminent actuaries were consulted, and to the diffusion by the Institute of sound principles and information, including especially the compilation of the H^M Tables, published in 1872, may very largely be attributed the satisfactory position to-day of our life offices as a whole, contrasting favourably with earlier times, when no official returns were required, and no restriction existed upon the formation and continuance of weak companies.

Such statistical work is in its nature recurrent; and after the time which had elapsed since the compilation of the H^M Table, a generation back, it was right to test afresh the basis upon which rests so great a superstructure. If the general results of the work last undertaken do not indicate the need for any marked changes in life assurance practice, still the investigation is none the less valuable, for it is, of course, in periodically reviewing our position by the light of experience, and by following out fresh lines of enquiry which may suggest themselves, that we make certain of maintaining a status and practice in harmony with the facts with which we are dealing.

Our interest in the new "British Offices" Tables will, in one respect at least, be shared by the community at large, who are interested to know whether or not improvement has taken place in the value of life, according to assurance office experience. As regards male lives, we find, as you know, that on the whole, as compared with the older H^M Table, the new O^M Table shows a superior vitality. A similar result is seen by comparing the old and new Female Experience (H^F and O^F); and, whatever be the reason or reasons, the improvement in the value of assured life is, on the whole, greater among females than among males—a fact noticeable also in the new experience of Annuitant life.

After paying due regard to the methods employed in collection and tabulation, and to the skilful graduation of the unadjusted data, our professional interest centres largely round the very practical question of Reserves; and it is, I should say, satisfactory to find that in the matter of Reserve the new Table closely corroborates the H^M, so that our finances suffer no disturbance, and no difficulty arises in passing from the older to the newer basis, as some offices have already done, although in doing so the Reserve may in some cases be fractionally greater.

Then we have some useful results arising from the separate investigation of the rate of mortality in different classes of

assurance, a matter not much thought about, or at all events little investigated, since Mr. Meikle, in 1869, exhibited the separate rates of mortality among lives assured in Scottish Offices by policies participating and non-participating. In the preparation of the H^M Table little analysis was undertaken. Distinction was made between healthy and under-average, and between male and female lives; but in the new tables a wider range of enquiry has been entered upon; and while healthy lives only have been brought into account, we are able in a degree not before possible to observe the selection which persons seeking insurance exercise in their choice of table or class of policy, notably under whole-life Policies, participating and non-participating respectively, and, after the question of Reserves, this is perhaps one of the most interesting results of the recent investigation.

We have here two forms of selection at work, that exercised by the Office in its acceptance of risks, and the selection exercised by the proposer in his choice of table. This last selection is not to be deprecated or combated, for our aim is to encourage providence in every phase; but life policies are not all effected for provident purposes, and we can now look to the construction, and watch the working, of our various office tables, in view of better knowledge as to the mortality likely to be experienced in the several classes of policy. The selection exercised by the proposer is a force likely, I should say, to increase rather than diminish, as a choice of tables wider than formerly continues to be offered by competing offices, and as the public possibly become, in insurance matters, still more discriminating or, at all events, more prone to adopt variants of the ordinary policy which may in form suit individual tastes or requirements, although these variants cannot, in essence, offer greater ultimate benefit.

The new tables exhibit also a continued improvement in the value of annuitant life, especially (as before mentioned) among females, which must be attributed in some degree to the "self-selection" exercised by the purchasers. The margin for profit on this business is at best very narrow, and it is conceivable that through a further fall in interest, or the action of competition, that margin might disappear entirely. The mortality experience among Government Annuitants, upon which Life Offices have of late years relied in their annuity business, related, as you know, to the period 1808-1875, and, looking to the continued improvement in mortality as shown by the latest investigation, it would certainly be of interest

and value to have the more recent mortality experience of the Government Annuitants made available to us. Since the publication of the British Offices Annuity Tables, some offices have reconsidered their terms. Government, however, is a powerful competitor in the business. We should be glad of the additional light which the suggested enquiry would afford.

As regards the larger question of a general improvement in the value of human life in this country, and apart from the experience in any special class, there is no doubt that advances in matters relating to sanitary science, the care of disease, more healthful ways of living, have diminished at most ages the chances of death, and, indeed, the remark is not unusual that "people live longer than formerly." This improvement in vitality takes the form of, at each age, a greater average number of years lived by those born into the world, constituting, in actuarial parlance, a greater "expectation of life." A larger number than formerly reach adult age and usefulness, though without materially increasing the small proportion who will as heretofore attain to the highest ages; and one distinguishes between this position and longevity in its usual meaning—the prolongation of life to advanced age. In that matter it is difficult to say or to determine that among the population as a whole there is any appreciable increase. Yet, in the main, life is being rendered more valuable, and, we may hope, more agreeable.

In the collection and arrangement of the experience of British assured and annuitant life, the Institute, in its corporate capacity, has, for the present, fulfilled its part. Yet the eight volumes relating to the new British Offices experience tables constitute, as a whole, a repository of information which it may take some time to explore thoroughly, and from which the brains and energy of individual members may yet discover to us much of interest, particularly with reference to the power of selection, of one kind or another—a matter touching, singly and in combination, the questions of rates of premium, reserve, and division of surplus.

One volume of tables, based on Select experience, has just been published by individual enterprise; and we may hope that others, based on the O^M experience and adapted for use in every-day actuarial work, will also make their appearance.

The business of life assurance, which is our principal care, will doubtless continue to present fresh features and questions calling for the employment of business foresight, statistical

method, and mathematical skill. For one thing, the question of investment becomes more absorbing. The vast and increasing funds of our Life Offices are now further augmented by the larger investment element in the policies effected. It is worthy of notice that the endowment assurance contracts in force in "ordinary" companies now outnumber the whole-life ones, although the sums assured by the latter are still considerably the greater. The rate of interest has fallen,—although the fall seems, for the time being, arrested; the total funds are held by fewer offices; and altogether, as compared with the circumstances a generation back, the question of investment demands wider views and consideration from those responsible, especially if the profit from interest, at anything like former rates, is to be maintained. One might allow this profit to, so to speak, take care of itself, and regard solely the paramount matter of the safety of the capital; but it is likely that rather broader views will prevail, and that investments will be selected in a larger field.

In the past thirty years the funds of British Life Offices have trebled in amount, and this increase has been accompanied by changes in the classes of securities most resorted to. Thirty years ago, for instance, between 45 and 50 per-cent of the Funds of British Offices were invested upon Mortgages of property. Not more than 25 per-cent or so is now so placed; and relatively less, also, is invested in British Government and similar "gilt-edged" securities, which do not now yield a sufficient rate of interest, nor have they proved free from disturbing fluctuations in capital value to which all Stock Exchange securities have in late years been subject. Mortgage securities of the kinds most favoured a generation ago have not, I take it, increased in the same ratio as have the Funds for investment. Trust and other monies for which these securities are in demand have accumulated with the prosperity of the country, while the value of land, houses, and such like, has not increased in the same proportion; and now-a-days debentures and debenture stocks, shares and stocks, ground-rents, Indian and Colonial Government securities, foreign Government securities, figure to a greater extent than formerly in the balance sheets of our Life Offices. A marked widening has taken place in the range of their investments. We cannot forecast the course of events in the political and financial world, upon which depends to a great extent the rate of interest obtainable. I hesitate to offer an opinion upon so large and difficult a question as the future rate of interest, but I should say, from words and

writings of my professional brethren, and from the gradual reduction in valuation rates of interest, the view most generally held among us is that we should be prepared, as time goes on, for some further, if gradual, diminution, continuing the desirability of opening up fresh channels of investment. One quarter per-cent in the valuation rate of interest may be a much more important question than the particular table of mortality to be used. However capable and resourceful the Boards of Directors, it is of high importance that the principal officers of the companies should also possess sound and definite ideas in regard to the varied category of investments offering or to be sought for; and if, as is possible, new fields or forms of investment have to be considered, in the endeavour to maintain our rate of interest, a faculty for cautious adaptation to new circumstances must continue to be aimed at by us. Whether the chief officer of a Life Office be the actuary, or the manager, or fills the combined office,—in favour of which last arrangement sound reasons can in my opinion be adduced,—the question of investment is to him of great responsibility, the ultimate judgment of his directors frequently depending upon his own examination of the securities under consideration.

Another fact of modern assurance practice is, I should say, an increased confidence in the acceptance of lives. Cases which in past years might, on account of family history, or personal history or condition, have been declined or heavily surcharged are now admitted to somewhat more favourable treatment. Various disabilities are better understood by the medical profession; and also there is, I think, more co-operation between the physician and the actuary, furnishing the key-note, in my view, to the best assessment of risks, so long as their respective functions are kept from overlapping. But we still lack material for guidance in determining what constitutes, and how to deal with, under-average lives, a class which was not brought under review in the recent mortality investigation; and although I do not forget the work of one or two of our members in the matter, a wide field of enquiry is still open in this connection. Possibly some system might be devised, not too elaborate, to extract and tabulate the experience of offices in the more typical cases in which it is customary, or would be proper, to require additional premium.

In all work there is a tendency as time goes on to specialize, and with actuaries to fix our attention, perhaps, on Life Assurance.

The problems connected with the various descriptions of provident institutions are already recognized as our own; but beyond these lie numerous questions bearing also on the social well-being of the people,—questions to be solved through statistical enquiry. The Institute,—I quote from our charter,—has among its objects: "The extension and improvement of the data and methods of the "science which has its origin in the application of the doctrine "of probabilities to the affairs of life"; and as the determination and application of probability must, as a rule, rest upon statistical investigation, the functions of the actuary may range throughout the whole field of general statistics.

Statistical work—in which I include both the collection and tabulation of facts and the science of correctly reasoning from them—has a two-fold value, for it inculcates the need of a clear conception of the subject for investigation, of due discernment in the selection of factors, of accuracy in compilation, of discrimination to secure a true deduction, and, at the same time, it provides us with working material. In statistical research there is, we know, room for greater care and study than might at first be thought necessary. In our own special course of study the acquisition of statistical method, embracing the more modern systems of procedure, has its place. The attention of our younger members is mainly directed, no doubt, to enquiries connected with the business of life assurance, sickness, and such like. It is suitable, however, that actuaries who have engaged in intricate questions in these branches of research should extend their powers to enquiries in a wider sphere, and I look upon statistical method as an important feature in our educational programme.

Statistics of population, of marriage and birth, are no new fields of work. One matter, for instance, may at present claim attention: the fact of a general decline in the birth-rate. Are men and women less able or less willing to incur the responsibilities of marriage, and if so, for what reasons? Replenishment of the race vies in importance with improvement in vitality. Then statistics of occupation, migration, health, sickness, invalidity: these all furnish matter in which the Actuary may especially work, and in which, also, he may collaborate with the statesman, the physician, the political economist, the social reformer. Beyond these subjects lies a whole region of general statistics, commercial, financial, political, which appear to produce an ever-growing mass of statistical literature. In connection with certain enquiries, such as the Fiscal question now under public discussion, the

importance of some training in statistical method and in the use of arithmetical illustrations has sometimes seemed to assert itself, arguments having been put forward on one side or the other seemingly deficient in cautious preparation.

I do not presume to suggest that so many-sided a matter might be referred for examination to a committee of actuaries,— though such an arrangement might help to elucidate, away from the arena of politics, questions which are of public interest and not necessarily of a political aspect. There can, however, be no doubt that actuarial training is of advantage in relation to enquiries of all kinds depending for their value upon statistical investigation. Highly as we all prize the purely theoretical side of the Institute's work, it does not seem to me desirable that the Institute should ever seek to dissociate that from the practical side of affairs. On the contrary, our strength will be best utilized if directed, so far as our constitution permits, into channels where theory and practice intermingle. The cause of life assurance, our principal concern, is surely deserving of all the fostering care which we can bestow upon it as a whole. And if any widening in our sphere of work is to take effect, it must be largely in effort of a practical kind. I therefore think that the Institute does well when it concerns itself with general questions affecting the business or interests which its members control; and that to this end we should be ready as occasion arises to join with other bodies or persons if by so doing, and while always maintaining the independence of the Institute, we help forward some result which we all desire.

Mr. Higham, four years ago, gave us his views on, at all events, one important phase of this question. My immediate predecessor, Mr. Hughes, urged that the Institute shall be a complete school of insurance, and that while the discussion of mathematical theory must necessarily form a large part of our proceedings, the discussion of practical subjects, though in a scientific spirit, claims its proper share of consideration.

These views I fully endorse. There is of course a well-defined limit beyond which our discussion or cognizance of practical affairs could not in any case extend; but within that limit it should be our object to secure as far as possible that sound and successful practice should flow from our work in the realms of theory.

The further opening-up of statistical work for our profession has been dwelt upon by former Presidents of the Institute. Mr. Samuel Brown, indeed, instituted the prize which bears his

name "for the purpose of engaging the attention and abilities of "the members in discussing some new questions to which the "doctrine of probabilities may be applied beyond that of the "mere practice of insurance, in which we are more especially "engaged",—and he exemplified various directions in which they may proceed. (*J.I.A.*, xv, 459.)

Problems relating to health and sickness have special claim upon us. Thrift and health, indeed, are closely connected. Papers which have appeared in our *Journal* on vaccination; the increase or otherwise of cancer; total abstinence and life assurance; are, let us hope, forerunners of other investigations connected with the health of the people. We are concerned with provident institutions not merely that they should be established and conducted on sound principles, but also, as good citizens, because the practice of thrift is an important factor in combating poverty, disease, and crime which result from its non-observance. The amelioration or abatement of these afford scope for continued research. Sickness assurance, long practised among the wage-earning population, shows sign of spreading in some degree to the professional classes, whose incomes, however, afford more margin for contingencies, and to whom the provision of a sum at death is in most cases the greater necessity. Still, this provision depends on ability to continue payment of premiums. Whether a modern practice of effecting policies against specified diseases, or against all diseases except certain specified ones, is an economical arrangement for the public is open to question. There are companies, no doubt, which grant sickness assurance of more general scope. Assurance restricted as to range, or time, is of but moderate advantage. We know, however, as a crucial fact in sickness assurance, that special precautions of one kind and another (not always easy of application) are necessary to secure the society from ill-founded claims. As regards the professional and mercantile classes, sickness statistics are still scanty. Difficult, for these reasons, as the task may be, it will be to the credit of the profession if this protective insurance could be extended, on a scientific basis, with proper safeguards in management, and at the same time in a contract running not merely for one year, or for a few years, but during the whole working period of life.

Various statistical enquiries falling within our province are followed by means of Census Returns. The desirability of a more frequent Census-taking is a matter which has engaged the

attention of the Council for some time past; and "The Case for Census Reform" was presented by Mr. G. H. Ryan in an able and instructive paper read before the Institute in 1901. It is to be hoped, therefore, that the sympathetic attitude of the President of the Local Government Board towards an important deputation which waited upon him in May last to urge the desirability of a Quinquennial Census may bear fruit. The desirability of this, and of the institution of a permanent Census department, tending to greater method in enumeration and in deduction of results, have now been fully demonstrated. The Institute was represented, very ably, on the deputation in question, and I think that no one reading Mr. Ryan's paper or the evidence given on the occasion referred to can fail to recognize how closely statistical enquiry on sound methods is bound up with our social condition, and in how many directions, as principal or as collaborator, the actuary may take part in questions affecting the health and wealth of the country. I notice that at the last Actuarial Congress, Mr. North, Director of the United States Census Bureau (which is now a permanent Department), expressed a desire to place the Department in close association with our professional brethren in that country. If we in Great Britain should be called upon in a similar manner by those in authority, we shall not fail to respond to the call.

In the early part of this year a movement of some interest in relation to our professional status was initiated in the introduction of a Bill in Parliament, styled the Chartered Societies Bill, designed to afford protection to the members of chartered societies in the use of the designations and distinctive letters determined by their respective constitutions.

The suggestion of such legislation is, as regards the Institute, no new one, for you will recollect that in the first volume of the *Journal of the Institute* (or the *Assurance Magazine*, as it was then called), published in 1855, there is given a draft "Bill for "regulating the profession and practice of Actuaries throughout "Great Britain and Ireland", but little more seems to have been heard of it. If such a Bill was thought desirable when the Institute was in its infancy, its subsequent progress and expansion, recognized in the grant of its Royal Charter in 1884, and the extent to which the public now depend upon competent actuarial guidance, render very evident the reasonableness of the movement.

The Council of the Institute joined readily with the other Chartered Societies in promoting the Bill, the object of which,

to quote from a memorandum prefixed to it, was "to provide a "simple procedure enabling societies and institutes, incorporated "by Royal Charter for the purpose of improving the status and "training of the professions which they represent, to protect the "public and their members against frauds, occasioned by their "inability at law to restrain unauthorized persons from using the "professional designations and distinctive initials which they are "entitled by virtue of their charters to use and which are "recognised by the public as denoting membership. The Bill "does not seek to create any monopoly, and the offence which it "constitutes is carefully limited to one 'knowingly' committed."

A deputation, upon which the Institute was represented, waited upon the President of the Board of Trade, in May last, to explain the objects of the Bill, and were informed in reply, that although the Government could not undertake to introduce legislation on the subject that Session, the Chartered Societies seemed to have made out a good case for the protection suggested. The Bill did not reach the stage for discussion; but a similar Bill may, we hope, pass ere long into law, affording due protection to the public in relation to the vast and increasing interests represented by our provident institutions and in the other spheres of our work, and emphasizing the professional status of members of the Institute of Actuaries and of the Faculty of Actuaries in Scotland.

May we hope, further, that some day the Life Assurance Companies Act, 1870, with other Acts requiring the certification of the official returns of provident institutions may be amended or passed in such form as to require, *inter alia,* that all valuation statements be certified by a Chartered Actuary. This, again, is no new suggestion, for at the time when the Bill to amend the law relating to Life Assurance Companies was introduced into the House of Commons in 1870, notice was given of an amendment to enact that the actuaries responsible for the periodical investigation of Life Assurance Societies should be members of this Institute. And it is of interest further to note that notice of this amendment was, as stated by the Council at the time, entirely spontaneous on the part of the member giving it, and was not due, directly or indirectly, to any member of the Institute.

The question of the Income Tax as affecting Life Offices is another subject which has specially engaged our attention during the past year, and we have recently had an interesting paper upon it from Mr. J. E. Faulks, followed by a useful discussion. We

were in hopes, at one time, that a Parliamentary Committee on Income Tax, appointed last Spring, to "enquire and report "whether it is desirable to effect any alteration in the system of "the Income Tax as at present prescribed and administered", under certain stated heads, might permit of evidence being received by the Commissioners on the question of the taxation of Life Offices. That, however, proved to be outside the scope of the reference, and could not be brought forward.

Evidence was accepted, however, on behalf of the Institute in regard to terminable annuities and securities generally of a wasting nature, the point to be established being, of course, that annuities and terminable charges of any description consist, in their nature, of the two things, interest on capital, and provision for its replacement, and that the former element should alone be taxed. I am afraid, however, that the Committee regarded this point as, in relation to taxation, somewhat academic, unless in the ordinary case of terminable charges where interest and capital are set out separately at the outset; and, at all events, they did not indicate that general effect could at present be given to it. The point, however, has been carried recently in the case of the Great Indian Peninsula Railway Company; and as there are certainly other cases in which taxation of the whole income constitutes a hardship, we will hope for a wider application in future of the principle contended for.

Last year witnessed another International Congress of Actuaries, this time in New York, and those who were present have told of the cordial reception accorded to its members by representatives of the Government of the United States, and by our actuarial brethren in that country, who extended to all a warm welcome, and a profuse and graceful hospitality. As in Brussels and in Paris, so in New York, the Congress was recognized by those in highest authority as of public importance and interest, and was welcomed accordingly. Let us hope that when a Congress is again held in this country, similar official encouragement may be given to it. The Hon. George B. Cortelyou, LL.D., Secretary of the Department of Commerce and Labour in the United States Government, accepted the position of Honorary President of the Congress, and delivered an opening Address, in which, after words of greeting, he referred to the development of insurance, and to the rising-up of the actuarial profession. Mr. Cortelyou gave expression, moreover, to a wide

and liberal conception of the functions of the actuary, as did also Mr. Israel C. Pierson, President of the Congress; and their inspiring words will find ready acceptance by those who recognize that our sphere of usefulness is capable of extension, however gradual.

The invitation of the Organizing Committee led to some seventy papers being prepared on, as may be imagined, a considerable number of subjects. These papers, with the discussion upon each, we shall soon, I hope, be able to study in the volume of *Transactions* now in preparation; and while certain subjects may appeal to one school or nationality more than to another, there is a permanent value in such a volume, and there can be little doubt that its perusal will suggest matter for fresh enquiry, inspire new effort, and promote co-operation. In the six days devoted to a Congress, it has been difficult to adequately consider or discuss the whole programme. To this end a wider knowledge of foreign tongues would greatly contribute; and even if this is not, with most of us, a daily need, still it leads doubtless to wider sympathies and better understandings, in more ways than one. Possibly, without limiting the scope of any Congress, some alteration of arrangement may be devised to afford more time for discussion.

Not least among the advantages of these meetings is the opportunity of intercourse and friendship with our professional brethren from all parts of the globe, men of like mind and purpose with ourselves. Our brethren across the sea have ever been ready to recognize the Institute as a parent Society. We, on our part, have seen with satisfaction the birth and progress of similar bodies in other lands, labouring to help forward improvements in social economy, rendering the conditions of life more stable and more pleasant.

A gradual increase in our membership is noticeable; and there seem to be reasons for expecting this to continue rather than to diminish. For one thing, the difficulty experienced in this country by many young men in obtaining a suitable opening in life is probably not decreasing. Then the opportunities now obtainable of University teaching in general actuarial and insurance subjects, while in no way competing in range or importance with the special teaching of the Institute, may attract more attention to the career furnished by our profession, and lead some to join the Institute. The Insurance Institutes, too, which are now established and doing good work in important

provincial centres, give encouragement to a better study of various branches of insurance, and some of the members may join our own body. The establishment of these Institutes, the co-operation which exists between them, evidence the sense of responsibility which animates the members in their daily work, and their desire to improve and extend the practice of, in its various forms, the great insurance principle. And while in the past, many have become students of this Institute through chancing to obtain an insurance office opening, let us hope that in future young men of liberal education will, as in say, Law or Medicine, join the profession through more independent appreciation of its aims, scope and opportunities. While maintaining our position as the only school through which, in England, the actuarial degree can be obtained, we desire to observe friendly relations with any other bodies whose teaching may embrace the more elementary insurance subjects, and from which progressive students may come to swell our own ranks.

A large increase has taken place in our Colonial Members. The step taken by the Council in 1892, when it determined to hold examinations in the Colonies, has been appreciated; and in these younger countries, which form so important a part of the Empire, qualified men, all, I am sure, actuated by a feeling of loyalty to and interest in the Institute as their Alma Mater, have risen and are rising up to carry on and extend the work of our profession. We have had good papers from Colonial Members, and we hope for more, whether on subjects specially connected with Colonial work or questions, or upon matters of general interest to the profession at large.

While our numbers have been increasing, and may still further increase, the modern tendency to the concentration and consolidation of undertakings has exemplified itself in the business of insurance, and through absorptions and amalgamations the number of our Life Offices has somewhat diminished. Apart from a feeling of regret which many may experience at the disappearance of offices of old standing and high character, such arrangements, honestly conceived and carried out for the sake of efficiency and economy, can hardly, I imagine, be objected to on public grounds, so long as the number of offices in existence continues sufficient to maintain a healthy competition, and to afford to the public the variety in constitution and operation which are desirable in the general interest. Nor has the business of life insurance fallen away in volume or degree under the

tendency referred to. On the contrary, it has increased; and while our field of usefulness may gradually expand, it is nevertheless likely that the business of life assurance will continue to form our chief consideration and occupation. We desire to see the principal appointments filled by Fellows of the Institute—in other words, that the chief officer of every office should be manager as well as actuary. There is certainly no reason why the successful manager and the capable actuary should not be combined in one person. We wish to attract to the Institute as students young men of liberal education who are willing to study both theory and practice, and of whom the more capable and practical should secure the better appointments. But in connection with the question of our numbers it is the case that rather fewer chief appointments than formerly may in future be open to our members, though the conduct of so vast a business should always ensure the employment in important positions of a large number of skilled men, and is likely to call, as time goes on, for a higher standard of general efficiency among the staffs of offices. If we can enlarge our field of operations, it will be of advantage, in more ways than one, to do so.

In alluding to subjects which are, to most of you, familiar, my object has been to indicate that the Institute continues, both in its corporate capacity and in the efforts of its members individually, to exhibit a healthful activity. As a corporation, we continue our educational functions on, as regards the number of our students, a somewhat increasing scale; we have taken some part in relation to practical questions affecting the great cause of life assurance; the furtherance of statistical science; the status of our profession. The Council has rendered further assistance to the Royal Patriotic Fund Corporation. Individual thought and work, however, are no less important—and in, perhaps, a wider sphere than heretofore. I should say, indeed, that upon the general results of individual effort will mainly depend the place held by the profession as a whole. My old official chief, Mr. William Thomas Thomson,—to whom was due a suggestion which led, in 1848, to the formation of the Institute,—a man of wide and liberal conceptions,—said, twenty-five years later, "the "profession of actuary is one of the learned professions. It has "worked its way by degrees to an important standing which is "more and more acknowledged every day, as the public become "acquainted with the onerous duties which the members of our "profession are called upon and are qualified to perform."

Much has happened since these words were spoken. More work, better methods, have opened up to us; and we must strive, each in his own way, to bear our share of the trust which has descended to us.

In conclusion, I will venture to address a few words specially to our younger members.

Those who have attained to the degree of F.I.A. will have realized how, in the complex business of life assurance, while principles remain fixed their application alters and extends; and there is still scope for fresh thought and effort. They may have been chiefly in touch, so far, with the purely educational side of the Institute's work. Important as that is, it constitutes but one phase of our aims and operations; and there are many respects in which the evolution of professional competence must be continued individually, in practical opportunity and experience. And in and beyond official and professional work there is the cultivation of character, and of general business ability. Many years ago, Dr. Sprague expressed the view that the prosperity of a Life Office depends principally on the character of its chief officer; and according to that view you will recognize that our aims and acquirements should embrace qualities not necessarily implied in the Institute's degree, but to be gained in that sphere or spheres where we may acquire the virtue represented by the term *savoir faire*. It is chiefly you who, ere so very long, will be conducting the affairs of the Institute; and you will be aided in doing so by keeping abreast, as far as possible, of what is doing year by year, and so have in your minds the Institute's work and history.

To those who are still Associates or Students the immediate interest, no doubt, is to pass our examinations. These form a subject to which the Council, in their desire to apply fair and even tests, while maintaining our standards, pay continuous attention. In a Society such as our own, where examinations are not competitive, and which places no limit on the number of times a candidate may occupy the attention of the Examiners, there may be temptation to do so on a doubtful amount of preparation. When one is young, a possible year of waiting seems a long time. Whatever be the reason, the percentage of passes in late years has not been satisfactory; and I would suggest that in the long run more thorough preparation would involve no real loss of time in attaining proficiency. Any want of thoroughness in the earlier studies will make itself felt later on, and necessitate a retracing of one's steps, or leave some permanent weakness of grasp. To cope with fresh problems in the wider

fields alluded to demands a thorough grounding in principles and methods. It is somewhat hard, no doubt, in a busy official life, to find time for private study, and also for attendance at our meetings; to which our younger members, I fancy, do not come regularly. I would, however, urge them to do so; for, although the subject of the paper may be strange or difficult, it will provide a change of thought, and you will almost certainly carry away some new information or idea from it or from the discussion. You will have the opportunity also of making acquaintance with your fellow-members.

Let us hope that all our younger members will strive to become not only experts in the theory of our special work, but also men of affairs, fitted in mind and manner to carry on and expand the work of a profession of high purpose and great possibilities, largely of a beneficent nature, in our social system.

On the Retrospective Method of Valuation. *By* FREDERICK BELL, F.I.A., *of the Alliance Assurance Company, Limited; Actuary for the Imperial Life Assurance Fund.*

[Read before the Institute, 19 December 1904.]

THE method of valuation generally adopted in practice is that known as the Prospective Method; by it, the value of a policy is obtained by a consideration of the future, and is the discounted value of the payment to be made, less the discounted value of the payments (if any) to be received; discounted, that is, in each case, for both mortality and interest.

It is probable that the general acceptance of the method arose in the first place mainly for two reasons—

(1) Because the object of making reserves was to provide for liabilities payable in the future—in itself a strong inducement to estimate that reserve by consideration of the future; and

(2) Because the method regarded the policy as a sort of debtor and creditor account, the items of which might be pitted against each other, and a balance struck at each valuation, and it is conceivable that this aspect of the matter appealed not only to methodical actuarial minds, but also to the many business men, who, though not actuaries, had done much to raise the business of Life Assurance to high repute.

In a sense, too, the Prospective Method has received the approval of the Legislature, for—drafted, no doubt, under actuarial advice—the Life Assurance Companies Act, 1870,

requires the statement, separately, of the values of Sums Assured, and the values of premiums, in the following form—

(Form referred to under heading No. 7, in Fifth Schedule.)

Summary and Valuation of the Policies of the   as at   18

Description of Transactions.	Particulars of the Policies for Valuation				Valuation — Value by the Table, Interest per-cent			
	Number of Policies	Sums Assured and Bonuses	Office Yearly Premiums	Net Yearly Premiums, if ascertained	Sums Assured and Bonuses	Office Yearly Premiums	Net Yearly Premiums if computed	Net Liability
ASSURANCES.								
I. *With participation in profits.*								
For whole term of life . . .								
Other classes (to be specified) . .								
Extra premiums payable . .								
Total Assurances with profits .								
II. *Without participation in profits.*								
For whole term of life . . .								
Other classes (to be specified) . .								
Extra premiums payable . .								
Total Assurances without profits.								
Total Assurances . .								
Deduct re-assurances . .								
Net amount of assurances . .								
Adjustments, if any . .								
ANNUITIES.								
Immediate								
Other classes (to be specified) . .								
Total of the results . .								

The term " Extra Premium " in this Act shall be taken to mean the charge for any risk not provided for in the minimum contract premium. If policies are issued in or for any country at rates of premium deduced from tables other than the

Under these circumstances, it is not surprising that the Retrospective Method, though well known, has received (as far as I am able to trace) but little consideration from Actuaries.

That storehouse of information, the Institute *Text-Book*, speaks of the two Methods as being of "great and equal importance"; a considerable part of Chapter XVIII is devoted to the explanation of the Retrospective Method, and its use is exemplified in valuing various classes of policies, but I do not remember to have come across it frequently in practical work;* it is, of course, equally sound, actuarially, and I have endeavoured, in the following notes, to show that it has advantages which its more popular rival lacks, and can be conveniently and usefully employed for the valuation of most of the classes of policies which are commonly found on the books of a Life Office.

Quoting again from the Institute *Text-Book* (Part II, 1887 ed., p. 315): "The value of a policy consists of the " portion assignable to that particular contract of the difference " between the accumulated premiums and the accumulated claims " in respect of all the entrants. That is

$$\text{“} \qquad {}_n\mathrm{V}_x = \frac{\mathrm{P}_x(\mathrm{N}_{x-1}-\mathrm{N}_{x+n-1})-(\mathrm{M}_x-\mathrm{M}_{x+n})}{\mathrm{D}_{x+n}}. \text{”}$$

Although the "policy" here spoken of is "an ordinary " whole-life policy of 1, effected exactly n years ago on a life then " aged x, at an annual premium of P_x", the formula is not expressly restricted to whole-life policies, and was possibly not intended to be so restricted; it is, in fact, a general formula, and applies equally to all classes of policies effected at level annual premiums, where only one life is at risk, the net premium P_x, varying, of course, according to the class of Assurance: thus

For Whole-Life, Limited premiums, P_x becomes ${}_t\mathrm{P}_x$;
" Term Assurances, P_x " $\mathrm{P}^1_{x\overline{t|}}$;
" Endowment Assurances, P_x " $\mathrm{P}_{x\overline{t|}}$;

and so on.

For reasons which will appear later, I prefer to write this general formula

$${}_n\mathrm{V}_x = \frac{\mathrm{P}_x(\mathrm{N}_{x-1}-\mathrm{N}_{x+n-1})-(\mathrm{M}_x-\mathrm{M}_{x+n})}{\mathrm{D}_{x+n}}$$

in the alternative form

$${}_n\mathrm{V}_x = \mathrm{A}_{x+n} - \mathrm{P}_x(1+a_{x+n}) + \frac{(\mathrm{P}_x-\pi_x)\mathrm{N}_x}{\mathrm{D}_{x+n}} \quad . \quad . \quad (\mathrm{A})$$

where π is the ordinary whole-life premium, and P the special

* See note on p. 63.

premium; and it may be interesting—especially to students—if I here show the identity of this formula (A) with the formulæ generally employed. Thus—

WHOLE-LIFE ASSURANCE.

In the case of Whole-Life Policies it is clear that the final term disappears, and we have left, the usual formula—

$$_{n}\mathrm{V}_{x}=\mathrm{A}_{x+n}-\mathrm{P}_{x}(1+a_{x+n}).$$

WHOLE-LIFE ASSURANCE, LIMITED PREMIUMS.

$$\mathrm{V}=\mathrm{A}_{x+n}-{}_{t}\mathrm{P}_{x}(1+a_{x+n})+\frac{({}_{t}\mathrm{P}_{x}-\pi_{x})\mathbb{N}_{x}}{\mathrm{D}_{x+n}}$$

$$=\mathrm{A}_{x+n}+\frac{\mathrm{M}_{x}}{\mathbb{N}_{x}-\mathbb{N}_{x+t}}\cdot\frac{\mathbb{N}_{x}-\mathbb{N}_{x+n}}{\mathrm{D}_{x+n}}-\frac{\mathrm{M}_{x}}{\mathrm{D}_{x+n}}$$

$$=\mathrm{A}_{x+n}+\frac{\mathrm{M}_{x}(\mathbb{N}_{x}-\mathbb{N}_{x+n}-\mathbb{N}_{x}+\mathbb{N}_{x+t})}{(\mathbb{N}_{x}-\mathbb{N}_{x+t})\mathrm{D}_{x+n}}$$

$$=\mathrm{A}_{x+n}-{}_{t}\mathrm{P}_{x}(1+a_{x+n\cdot\overline{t-n-1}}).$$

TERM ASSURANCE.

$$\mathrm{V}=\mathrm{A}_{x+n}-\mathrm{P}^{1}_{x\overline{t}}(1+a_{x+n})+\frac{(\mathrm{P}^{1}_{xt}-\pi_{x})\mathbb{N}_{x}}{\mathrm{D}_{x+n}}$$

$$=\mathrm{A}_{x+n}-\left(\frac{\mathrm{M}_{x}-\mathrm{M}_{x+t}}{\mathbb{N}_{x}-\mathbb{N}_{x+t}}\right)\left(\frac{\mathbb{N}_{x+n}-\mathbb{N}_{x}}{\mathrm{D}_{x+n}}\right)-\frac{\mathrm{M}_{x}}{\mathrm{D}_{x+n}}$$

$$=\mathrm{A}_{x+n}-\frac{\mathrm{M}_{x}(\mathbb{N}_{x+n}-\mathbb{N}_{x+t})-\mathrm{M}_{x+t}(\mathbb{N}_{x+n}-\mathbb{N}_{x})}{(\mathbb{N}_{x}-\mathbb{N}_{x+t})\mathrm{D}_{x+n}}$$

$$=\frac{\mathrm{M}_{x+n}}{\mathrm{D}_{x+n}}-\frac{(\mathrm{M}_{x}-\mathrm{M}_{x+t})(\mathbb{N}_{x+n}-\mathbb{N}_{x+t})+\mathrm{M}_{x+t}(\mathbb{N}_{x}-\mathbb{N}_{x+t})}{(\mathbb{N}_{x}-\mathbb{N}_{x+t})\mathrm{D}_{x+n}}$$

$$=\frac{\mathrm{M}_{x+n}-\mathrm{M}_{x+t}}{\mathrm{D}_{x+n}}-\mathrm{P}^{1}_{x\overline{t}}\,\frac{\mathbb{N}_{x+n}-\mathbb{N}_{x+t}}{\mathrm{D}_{x+n}}$$

$$=\mathrm{A}^{1}_{x+n\cdot\overline{t-n}|}-\mathrm{P}^{1}_{x\overline{t}}(1+a_{x+n\cdot\overline{t-n-1}}).$$

ENDOWMENT ASSURANCE.

Premiums payable throughout duration—

$$\mathrm{V}=\mathrm{A}_{x+n}-\mathrm{P}_{\overline{t}}(1+a_{x+n})+\frac{(\mathrm{P}_{x\overline{t}|}-\pi_{x})\mathbb{N}_{x}}{\mathrm{D}_{x+n}}$$

$$=\frac{\mathrm{M}_{x+n}}{\mathrm{D}_{x+n}}-\left(\frac{\mathrm{M}_x-\mathrm{M}_{x+t}+\mathrm{D}_{x+t}}{\mathbb{N}_x-\mathbb{N}_{x+t}}\right)\left(\frac{\mathbb{N}_{x+n}-\mathbb{N}_x}{\mathrm{D}_{x+n}}\right)-\frac{\mathrm{M}_x}{\mathrm{D}_{x+n}}$$

$$=\frac{(\mathrm{M}_{x+n}-\mathrm{M}_x)(\mathbb{N}_x-\mathbb{N}_{x+t})-(\mathrm{M}_x-\mathrm{M}_{x+t}+\mathrm{D}_{x+t})(\mathbb{N}_{x+n}-\mathbb{N}_x)}{\mathrm{D}_{x+n}(\mathbb{N}_x-\mathbb{N}_{x+t})}$$

$$=\frac{\mathbb{N}_x(\mathrm{M}_{x+n}-\mathrm{M}_x+\mathrm{M}_x-\mathrm{M}_{x+t}+\mathrm{D}_{x+t})-\mathbb{N}_{x+t}(\mathrm{M}_{x+n}-\mathrm{M}_x)-\mathbb{N}_{x+n}(\mathrm{M}_x-\mathrm{M}_{x+t}+\mathrm{D}_{x+t})}{\mathrm{D}_{x+n}(\mathbb{N}_x-\mathbb{N}_{x+t})}$$

$$=\frac{\mathbb{N}_x(\mathrm{M}_{x+n}-\mathrm{M}_{x+t}+\mathrm{D}_{x+t})-\mathbb{N}_{x+t}(\mathrm{M}_{x+n}-\mathrm{M}_{x+t}+\mathrm{D}_{x+t})-(\mathbb{N}_{x+n}-\mathbb{N}_{x+t})(\mathrm{M}_x-\mathrm{M}_{x+t}+\mathrm{D}_{x+t})}{\mathrm{D}_{x+n}(\mathbb{N}_x-\mathbb{N}_{x+t})}$$

$$=\frac{\mathrm{M}_{x+n}-\mathrm{M}_{x+t}+\mathrm{D}_{x+t}}{\mathrm{D}_{x+n}}-\left(\frac{\mathrm{M}_x-\mathrm{M}_{x+t}+\mathrm{D}_{x+t}}{\mathbb{N}_x-\mathbb{N}_{x+t}}\right)\left(\frac{\mathbb{N}_{x+n}-\mathbb{N}_{x+t}}{\mathrm{D}_{x+n}}\right)$$

$$=\mathrm{A}_{x+n\cdot\overline{t-n}|}-\mathrm{P}_{x\overline{t}|}\,(1+a_{x+n\cdot\overline{t-n-1}|}).$$

Endowment Assurance, Limited Premiums.

Premiums payable for s years only, n being less than s—

$$\mathrm{V}=\mathrm{A}_{x+n}-{}_s\mathrm{P}_{x\overline{t}|}(1+a_{x+n})+\frac{({}_s\mathrm{P}_{x\overline{t}|}-\pi_x)\mathbb{N}_x}{\mathrm{D}_{x+n}}$$

$$=\frac{(\mathrm{M}_{x+n}-\mathrm{M}_x)(\mathbb{N}_x-\mathbb{N}_{x+s})-(\mathrm{M}_x-\mathrm{M}_{x+t}+\mathrm{D}_{x+t})(\mathbb{N}_{x+n}-\mathbb{N}_x)}{\mathrm{D}_{x+n}(\mathbb{N}_x-\mathbb{N}_{x+s})}$$

$$=\frac{\mathbb{N}_x(\mathrm{M}_{x+n}-\mathrm{M}_{x+t}+\mathrm{D}_{x+t})-\mathbb{N}_{x+s}(\mathrm{M}_{x+n}-\mathrm{M}_x)-\mathbb{N}_{x+n}(\mathrm{M}_x-\mathrm{M}_{x+t}+\mathrm{D}_{x+t})}{\mathrm{D}_{x+n}(\mathbb{N}_x-\mathbb{N}_{x+s})}$$

$$=\frac{(\mathbb{N}_x-\mathbb{N}_{x+s})(\mathrm{M}_{x+n}-\mathrm{M}_{x+t}+\mathrm{D}_{x+t})-(\mathbb{N}_{x+n}-\mathbb{N}_{x+s})(\mathrm{M}_x-\mathrm{M}_{x+t}+\mathrm{D}_{x+t})}{\mathrm{D}_{x+n}(\mathbb{N}_x-\mathbb{N}_{x+s})}$$

$$=\mathrm{A}_{x+n\cdot\overline{t-n}}-{}_s\mathrm{P}_{x\overline{t}|}(1+a_{x+n\cdot\overline{s-n-1}|}).$$

It is thus clear that, if for entry age x, P_x be the net annual premium for the policy to be valued, and π_x the annual whole-life premium, the use of the formula

$$\mathrm{V}=\mathrm{A}_{x+n}-\mathrm{P}_x(1+a_{x+n})+\frac{(\mathrm{P}_x-\pi_x)\mathbb{N}_x}{\mathrm{D}_{x+n}}$$

will give the correct value of the policy after n years' duration—the $(n+1)$th premium being due and unpaid—whether the policy

be for Assurance for the whole of life, with premiums payable throughout life, or for the whole of life by limited premiums, or for a term of years, or for an Endowment Assurance.

It will be observed that the only expressions made use of are the whole-life single premium at the advanced age, the net premium (for the risk) at the entry age multiplied by the whole-life annuity at the advanced age, and a correction—

$$\frac{(P_x - \pi_x)\mathbb{N}_x}{D_{x+n}},$$

of which the numerator is constant throughout the duration of the policy, and the denominator is the commutation D for the advanced age. Except in as far as it enters into P, the number of premiums yet to be paid under a limited premium policy: or the number of years yet to run under a term policy: or the number of years yet to run, or the age at maturity, or the number of premiums yet to be paid under an Endowment Assurance Policy, does not enter into the calculation of the value of the policy. Hence it follows that, if this formula be made use of, all the foregoing classes of policies may be grouped together for valuation purposes, one grouping, namely, by the current age, being sufficient to include all endowment assurances, the only condition being that a level premium is to be payable in respect of them.

For convenience, I propose to refer to the numerator of the correction $(P_x - \pi_x)\mathbb{N}_x$ by the single letter k so that formula (A) may be written—

$$V = A_{x+n} - P_x(1 + a_{x+n}) + \frac{k_x}{D_{x+n}} \quad . \quad . \quad . \quad . \quad (A)$$

In order to make use of this method, it is necessary to note on the valuation cards the value of k_x; and additional columns are required in our valuation sheets for k_x and $\frac{k_x}{D_{x+n}}$ respectively; but apart from this, the valuation of these somewhat troublesome groups of policies may be proceeded with, with almost the same facility as if they were whole-life cases; in fact, except that it may be desired to keep them apart for the purpose of Board of Trade Returns, there is no reason why they should not be grouped with whole-life cases (for which $k_x = 0$), and all valued together.

Calculation of k.

In order that this paper may be of practical value, I have appended tables of k_x (Numerators of Corrections) by O^{M}, and $O^{[M]}$ Mortality at $2\frac{1}{2}$, $2\frac{3}{4}$, 3, $3\frac{1}{2}$ and 4 per-cent for Whole-Life Assurance by Limited Premiums payable for 10, 15, 20, 25 and 30 years, and for Endowment Assurance maturing at ages 45, 50, 55, 60 and 65.

The readiest means of calculating these was found to be by the use of the formulæ

$$_tk_x = {_tP_x}\mathrm{N}_{x+t};$$

and

$$k_{x\overline{t|}} = \frac{\mathrm{N}_{x+t}}{1+a_{x.\overline{t-1|}}},$$

and I desire here to acknowledge the kind courtesy of Messrs. H. J. Baker and A. H. Raisin, who, by giving me access to their calculations of $_tP_{[x]}$ and $a_{[x]\overline{t|}}$ prior to publication, have very greatly assisted and expedited the work.

The rationale of the above two equations is evident, and it will be interesting to show how they may be derived from the general formula

$$k_x = (P_x - \pi_x)\mathrm{N}_x.$$

Thus, for Whole-Life, Limited payments—

$$\begin{aligned} _tk_x &= ({_tP_x} - \pi_x)\mathrm{N}_x \\ &= {_tP_x}\mathrm{N}_x - \mathrm{M}_x \\ &= {_tP_x}\mathrm{N}_x - {_tP_x}(\mathrm{N}_x - \mathrm{N}_{x+t}) \\ &= {_tP_x}\mathrm{N}_{x+t} \end{aligned}$$

and, for Endowment Assurance—

$$\begin{aligned} k_{x\overline{t|}} &= (P_{x\overline{t|}} - \pi_x)\mathrm{N}_x \\ &= \left(\frac{1}{1+a_{x.\overline{t-1|}}} - \frac{1}{1+a_x}\right)\mathrm{N}_x \\ &= \frac{\mathrm{D}_x(\mathrm{N}_x - \mathrm{N}_x + \mathrm{N}_{x+t})}{\mathrm{N}_x - \mathrm{N}_{x+t}} \\ &= \frac{\mathrm{N}_{x+t}}{1+a_{x.\overline{t-1|}}}. \end{aligned}$$

Tables of correction-numerators for Simple Endowments and for Term Assurances could readily be formed from the formulæ—

$$k_{x\,\overset{1}{\overline{t}|}}=\frac{\mathrm{D}_{x+t}(1+a_x)}{1+a_{x.\overline{t-1}|}}\,;$$

and $$k^{1}_{x\,\overline{t}|}=k_{x\overline{t}|}-k_{x\,\overset{1}{\overline{t}|}}$$

which are obtained as follows:

For Simple Endowments—

$$k_{x\,\overset{1}{\overline{t}|}}=\mathrm{P}_{x\,\overset{1}{\overline{t}|}}\mathbb{N}_x,$$

since in this case there is no mortality risk. Hence, by substitution—

$$k_{x\,\overset{1}{\overline{t}|}}=\frac{\mathrm{D}_{x+t}(1+a_x)}{1+a_{x.\overline{t-1}|}}\,.$$

For Term Assurances—

$$\begin{aligned}
k^{1}_{x\overline{t}|}&=(\mathrm{P}^{1}_{x\overline{t}|}-\pi_x)\mathbb{N}_x\\
&=\frac{(\mathrm{M}_x-\mathrm{M}_{x+t})\mathbb{N}_x}{\mathbb{N}_x-\mathbb{N}_{x+t}}-\mathrm{M}_x\\
&=\frac{(\mathrm{M}_x-\mathrm{M}_{x+t})\mathbb{N}_x-\mathrm{M}_x(\mathbb{N}_x-\mathbb{N}_{x+t})}{\mathbb{N}_x-\mathbb{N}_{x+t}}\\
&=\frac{(d\mathbb{N}_{x+t}-\mathrm{D}_{x+t})\mathbb{N}_x-(d\mathbb{N}_x-\mathrm{D}_x)\mathbb{N}_{x+t}}{\mathbb{N}_x-\mathbb{N}_{x+t}}\\
&=\frac{\mathrm{D}_x\mathbb{N}_{x+t}-\mathrm{D}_{x+t}\mathbb{N}_x}{\mathbb{N}_x-\mathbb{N}_{x+t}}\\
&=k_{x\overline{t}|}-k_{x\,\overset{1}{\overline{t}|}}\,.
\end{aligned}$$

It is self-evident that, since $\mathrm{P}^{1}_{x\overline{t}|}+\mathrm{P}_{x\,\overset{1}{\overline{t}|}}$ is equal to $\mathrm{P}_{x\overline{t}|}$, $k^{1}_{x\overline{t}|}+k_{x\,\overset{1}{\overline{t}|}}$ will be equal to $k_{x\overline{t}|}$, but the above deduction from the general formula for correction-numerators will not be without interest as tending to confirm the correctness of the formula.

VALUATION OF BONUSES.

No method of valuation of special classes of policies is of practical use unless it enables us readily to value bonus additions to the sums assured, and in this respect the Retrospective Method presents no difficulty.

If A'_x be the single premium for the benefit at entry age x

$$_nV_x = \frac{A'_x D_x}{D_{x+n}} - \frac{M_x - M_{x+n}}{D_{x+n}},$$

or in the alternative form

$$_nV_x = A_{x+n} + \frac{(A'_x - A_x)D_x}{D_{x+n}}. \qquad \text{(B)}$$

It wants but little consideration of this formula (B), for its identity, with the formula usually adopted, to be apparent, but for the sake of completeness and for the benefit of students, I append the proof; thus—

WHOLE-LIFE ASSURANCE.

$$_nV_x = A_{x+n} + \frac{(A_x - A_x)D_x}{D_{x+n}}$$

here, obviously the second expression $=0$, and we have—

$$_nV_x = A_{x+n}.$$

TERM ASSURANCE.

$$V = A_{x+n} + \frac{(A^1_{x\overline{t}|} - A_x)D_x}{D_{x+n}}$$

$$= \frac{M_{x+n} + M_x - M_{x+t} - M_x}{D_{x+n}}$$

$$= \frac{M_{x+n} - M_{x+t}}{D_{x+n}}$$

$$= A^1_{x+n.\overline{t-n}|}.$$

ENDOWMENT ASSURANCE.

$$V = A_{x+n} + \frac{(A_{x\overline{t}|} - A_x)D_x}{D_{x+n}}$$

$$= \frac{M_{x+n} + M_x - M_{x+t} + D_{x+t} - M_x}{D_{x+n}}$$

$$= \frac{M_{x+n} - M_{x+t} + D_{x+t}}{D_{x+n}}$$

$$= A_{x+n.\overline{t-n}|}.$$

The only expressions made use of in formula (B) are the whole-life single premium and a correction, $\frac{(A'_x - A_x)D_x}{D_{x+n}}$, of which the numerator is constant throughout the duration of the policy, and the denominator is the commutation D for the advanced age. For convenience I propose to refer to this numerator by the letter K, so that this formula (B) may be written

$$V = A_{x+n} + \frac{K}{D_{x+n}}$$

It will be observed that the expression K is in the same general form as k, for

$$K \text{ (for single premium cases) } = (A'_x - A_x)D_x$$

$$k \text{ (for annual premium cases) } = (P_x - \pi_x)N_x.$$

When made use of for valuing bonuses the same quantity K may be employed at successive valuations, throughout the duration of the policy, irrespective of the date of the declaration of the bonus and the age of the life assured at the time of such declaration, since in the cases under consideration, $(A'_x - A_x)D_x$ is constant for all ages. This is a very great convenience, since K being once entered on the valuation card its multiplication by the last declared bonus and the addition of the result to the existing entries (of k multiplied by the original sum assured, and K multiplied by previous—existing—bonus additions) is but routine work, and can readily be performed by an ordinary clerical staff. On our valuation sheets k and K are, of course, dealt with together, in one column, the common multiplier being the reciprocal of D_{x+n}.

The formula (B) is obviously not restricted to the valuation of bonus additions to the sum assured, but applies also to paid up policies—which bonus additions in effect are—but in valuing paid up Policies it is usual to make a special reserve, in addition to the value of the sum assured, of an annual sum, for expenses and profit during the remainder of the duration of the assurance.

The amount of this special loading being f per annum, and its value at the commencement of the assurance, being $f(1 + a_{x.\overline{t-1}})$,—*i.e.*, an annuity-due for the term of t years during

which the assurance is to run,—the amount in hand at the end of the nth year will be

$$V = f\left\{\frac{(1+a_{x.\overline{t-1}})D_x}{D_{x+n}} - \frac{N_x - N_{x+n}}{D_{x+n}}\right\}$$

$$= f(1+a_{x+n}) + f\frac{(a_{x.\overline{t-1}} - a_x)D_x}{D_{x+n}}$$

$$= f(1+a_{x+n}) - \frac{fN_{x+t}}{D_{x+n}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (C)$$

The first of these expressions, $f(1+a_{x+n})$, is of the same character as the premiums to be valued under formula (A), but it must be noted that it is a *positive* quantity and must therefore be dealt with separately as a deduction from $P(1+a_{x+n})$ in the valuation sheets; the second expression $\frac{fN_{x+t}}{D_{x+n}}$ is of the same character as the corrections in formulæ (A) and (B), and can be dealt with in the same manner; it is a *negative* quantity.

In the case of Whole-Life Policies, $(x+t)$ is, of course, the limiting age of the table, and $fN_{x+t}=0$.

It will be noted that the special reserve is identical with that produced by the Prospective Method, for if the term of years for which it was originally made be t, the reserve at the end of the nth year—n being less than t—will be

$$f\frac{N_{x+n} - N_{x+t}}{D_{x+n}} = f(1+a_{x+n.\overline{t-n-1}}).$$

Reductions of premium, by the application of bonus, or otherwise, are similar in every respect to the "annual sum" referred to above, and will be valued by formula (C); the quantity fN_{x+t} is constant throughout the duration of the assurance, and independent of the age of the assured at the time of the commencement of the reduction, hence N_{x+t} being once entered on the valuation card, its multiplication by the amount of the last reduction and the addition of the result to the previously recorded values of k and K multiplied by their proper coefficients (and fN_{x+t} if any) are easily effected.

The amounts of reduction per annum which a Cash Bonus, or payment, will purchase, may be obtained by the use of formula (C), from the relation

$$f = \frac{\text{Cash Bonus}}{1 + a_{x+n} - \frac{N_{x+t}}{D_{x+n}}}$$

this is, however, only likely to be of service where N_{x+t} is already entered on the cards, and a table of temporary annuities is not readily available.

It will be convenient to say here that the corrections k and K will be positive or negative according to the nature of the assurances to be valued; for instance, for Endowment Assurances they will be positive, for Term Assurances, negative. Therefore it will be necessary to adopt a clear and simple method of distinguishing on the valuation cards whether a positive or negative correction is to be included in the valuation sheets; the adoption of a special coloured ink—red, for example—should prove sufficient to avoid all possibility of error in this particular. In the illustrative cards included in this paper the negative corrections are printed in old style type. It will be convenient to have two columns on the valuation policy-lists, for positive and negative corrections respectively; the difference only of the two columns will require to be valued, but, as this may be positive or negative, two columns of k (or K) will be needed on the valuation sheets.

To revert to the subject of special loadings, it not infrequently happens that we desire to accumulate during a fixed term of years a special loading to provide for expenses and profit after the fixed term has expired; that is, generally, after the payment of premiums has ceased.

Assuming that the loadings (g) are to be accumulated during t years, the value of the loadings in hand at the end of n years, n being less than t, will be

$$\mathrm{V}=g\frac{\mathrm{N}_x-\mathrm{N}_{x+n}}{\mathrm{D}_{x+n}}$$

$$=-g(1+a_{x+n})+g\frac{\mathrm{N}_x}{\mathrm{D}_{x+n}} \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{D})$$

It is clear that we can readily incorporate this special loading with the premium payable; so that, in this case our formula (B) becomes

$$\mathrm{V}=\mathrm{A}_{x+n}-(\mathrm{P}_x+g)(1+a_{x+n})+\frac{(\mathrm{P}_x+g-\pi_x)\mathrm{N}_x}{\mathrm{D}_{x+n}}.$$

At the end of the tth year the amount of accumulation of special loadings in hand, will be $g\dfrac{\mathrm{N}_x-\mathrm{N}_{x+t}}{\mathrm{D}_{x+t}}$, and if this be the

amount required to provide f per annum during the remainder of the duration of the assurance (s years, say), then

$$g\frac{N_x - N_{x+t}}{D_{x+t}} = f(1 + a_{y.\overline{s-1}|})$$

which latter quantity enters into formula (C) given above, y being here written in place of $x+n$ so as to show clearly the identity of the expression with that in formula (C).

In all the foregoing it has been assumed that a net premium valuation is desired, but the use of the Retrospective Method is not limited to net premium valuations.

When $(P_x + \phi_x)$ is the premium to be valued, the formula for the value of the policy at the end of the nth year will be—

$${}_nV_x = A_{x+n} - (P_x + \phi_x)(1 + a_{x+n}) + \frac{(P_x - \pi_x)N_x}{D_{x+n}} + \frac{\phi N_{x+t}}{D_{x+n}} \quad . \quad (E)$$

In the case of a whole-life policy, with premiums payable throughout life, $(x+t)$ is the limiting age of the table and therefore $N_{x+t} = 0$.

This formula is a combination of the formulæ (A) and (C), and although somewhat formidable in appearance, when once ϕ_x and the correction for ϕ_x are entered on our valuation cards, it does not at all add to the work or trouble of valuation. It will be observed that in this case the expressions in formula (C) take the reverse sign, since ϕ_x is not a loading to be accumulated by the Office as a provision for a future liability, but it is, rather, a contribution by the assured to the discharge of a liability (namely, expenses) incurred by the Office on his behalf.

I am aware that this formula does not agree with that given on p. 324 of the *Text-Book* (1887 ed.), which states :

" If the premium P_x payable under the policy be loaded to " the extent of ϕ_x by the Prospective Method we should have

$$\text{"}\quad {}_nV'_x = A_{x+n} - (P_x + \phi_x)(1 + a_{x+n})$$

$$\text{"}\quad = {}_nV_x - \phi_x(1 + a_{x+n}),$$

" and by the Retrospective Method we should have

$$\text{"}\quad {}_nV''_x = \frac{(P_x + \phi_x)(N_{x-1} - N_{x+n-1}) - (M_x - M_{x+n})}{D_{x+n}}$$

$$\text{"}\quad = {}_nV_x + \frac{\phi_x(N_{x-1} - N_{x+n-1})}{D_{x+n}}.$$

"It thus appears that in the case of a loaded premium, the value "of a policy by the Prospective Method is smaller than the "net premium value, while by the Retrospective Method it "is larger"; but I submit that this formula (for the value of a whole-life policy) omits to take account of one of the primary conditions assumed in the use of $(P_x+\phi_x)$ namely, that the Policy at the outset is weighted with a charge which requires the annual payment of ϕ_x to liquidate it. It is an actuarial axiom that at the commencement of the assurance the Benefit and Payment sides of the contract are equal. Assuming then that the whole of the annual payments of $(P_x+\phi_x)$ is required to provide A_x we have

$$\text{Benefit}\;;\;\; A_x=$$

$$\text{Payment}\;;\;\; (P_x+\phi_x)(1+a_x),$$

which, in the language of Euclid, is absurd. It is clear that the Benefit is $A_x+\phi_x(1+a_x)$, that is, that the assurance is loaded at the commencement with a charge of $\phi_x(1+a_x)$.

Proceeding on this basis, we have as the value of the policy by the Retrospective Method

$$\begin{aligned}{}_nV''_x &= \frac{(P_x+\phi_x)(N_x-N_{x+n})-(M_x-M_{x+n})-\phi_x(1+a_x)D_x}{D_{x+n}}\\ &=A_{x+n}-(P_x+\phi_x)(1+a_{x+n})\\ &={}_nV_x-\phi_x(1+a_{x+n}),\end{aligned}$$

so that the values by the Prospective and Retrospective Methods are equal, as they undoubtedly should be.

This proof of the equality of the results by the two methods need not be limited to whole-life policies with premiums payable throughout their duration, for, if we assume that the assurance is to be secured by, say, t premiums our formula becomes

$$V=A_{x+n}-({}_tP_x+\phi_x)(1+a_{x+n})+\frac{({}_tP_x-\pi_x)N_x}{D_{x+n}}+\frac{\phi_xN_{x+t}}{D_{x+n}}.$$

Proceeding from this to the prospective formula, we have

$$\begin{aligned}V&=A_{x+n}-{}_tP_x(1+a_{x+n})+\frac{({}_tP_x-\pi_x)N_x}{D_{x+n}}-\frac{\phi_x(N_{x+n}-N_{x+t})}{D_{x+n}}\\ &=A_{x+n}-({}_tP_x+\phi_x)(1+a_{x+n\,.\,\overline{t-n-1}|})\end{aligned}$$

In the same way the values by the Prospective and Retrospective Methods may be proved to be equal for endowment assurances, and other classes of assurances, provided the initial charge on the policy be taken into account.

Endowments Simple.

Although Simple Endowments are not assurances in which lives are at risk, and therefore are not policies for the valuation of which formulæ (A) and (B) are suitable; and although their valuation by the formulæ usually employed is very readily effected, it will be desirable to consider in what way their valuation can be included in a valuation by the Retrospective Method, because bonus additions to sums assured are sometimes made contingent upon the survival of the life assured to an agreed age.

In this latter case the value of the bonus is obviously $\dfrac{D_{x+t}}{D_{x+n}}$, where $(x+t)$ is the endowment age; and D_{x+t} (multiplied by the amount of contingent bonus) is the quantity to be added to k (or K) to increase the reserve by the value of the contingent bonus. It is hardly necessary to add that contingent bonuses must be entered in a separate column on our valuation policy-lists, as, if included with other bonuses, they would be multiplied by A_{x+n}, and the result included in our valuation summary.

Simple endowments secured by annual premiums can, if desired, be included with other classes of policies in our valuation policy-lists, and by the use of formula (D), valued together with them, thus—

$$V = -P_{\overset{1}{x\,t}}(1+a_{x+n}) + \frac{P_{\overset{1}{x\,t}} N_x}{D_{x+n}};$$

this is obviously identical with the formula of the Prospective Method, for

$$V = \left(\frac{D_{x+t}}{N_x - N_{x+t}}\right)\left(\frac{N_x - N_{x+n}}{D_{x+n}}\right)$$

$$= \frac{D_{x+t}(N_x - N_{x+t} - N_{x+n} + N_{x+t})}{(N_x - N_{x+t})D_{x+n}}$$

$$= \frac{D_{x+t}}{D_{x+n}} - P_{\overset{1}{x\,t}} \frac{N_{x+n} - N_{x+t}}{D_{x+n}}$$

$$= A_{x+n\,.\,\overset{1}{\overline{t-n}|}} - P_{\overset{1}{x\,t}}(1 + a_{x+n\,.\,\overline{t-n-1}|})$$

In this case, again, if the Retrospective Method be used, care must be taken that the amounts assured be not included in the

cast of the Sums Assured column in our valuation policy-lists—this could be guarded against by using either a special card, or a different coloured ink, as suggested above—for, if so included, the result of their multiplication by A_{x+n} would be thus included in our valuation summary and result in an over-statement of our liability.

Recapitulation of Formulæ.

I trust I have shown that the values of policies may be expressed in formulæ, derived by the Retrospective Method, which are mathematically accurate, and which, I suggest, are more convenient than those of the Prospective Method, in the practical work of valuation; not perhaps in the valuation of isolated cases, but in the valuation of large numbers of policies of different descriptions, such as we have periodically to undertake.

Before proceeding to give examples of their use, it will be well to recapitulate the principal of these formulæ; they are as follows:

For net premium valuation—

(1) For a policy subject to level annual premiums

$$V = A_{x+n} - P_x(1 + a_{x+n}) + \frac{k_x}{D_{x+n}} \quad . \quad . \quad . \quad . \quad . \quad (A)$$

(2) For a paid-up policy (or reversionary bonus)

$$V = A_{x+n} + \frac{K}{D_{x+n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

(3) For a reduction of premium, or special loading of f per annum

$$V = f(1 + a_{x+n}) - \frac{f\mathrm{N}_{x+t}}{D_{x+n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (C)$$

Hence—

(4) for a policy subject to level annual premiums, and entitled to a reduction of premiums of f per annum by bonus or otherwise

$$V = A_{x+n} - (P_x - f)(1 + a_{x+n}) + \frac{k_x - f\mathrm{N}_{x+t}}{D_{x+n}};$$

(5) for a paid-up policy, including a special loading reserve of f per annum

$$V = A_{x+n} + f(1 + a_{x+n}) + \frac{K - f\mathrm{N}_{x+t}}{D_{x+n}};$$

(6) For a simple endowment, subject to level annual premiums of g per annum, or for the accumulation of an amount of g per annum from the commencement of the policy as an additional reserve

$$V = -g(1+a_{x+n}) + g\frac{N_x}{D_{x+n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (D)$$

Hence—

(7) For a policy subject to level premiums, including the accumulation of g per annum

$$V = A_{x+n} - (P_x + g)(1 + a_{x+n}) + \frac{k_x + gN_x}{D_{x+n}}.$$

For valuation using a loaded premium—

(8) For a policy subject to level annual premiums

$$V = A_{x+n} - (P_x + \phi)(1 + a_{x+n}) + \frac{k_x + \phi N_{x+t}}{D_{x+n}} \quad . \quad . \quad . \quad (E)$$

This formula is obtained by combining formulæ (A) and (C), but it may also be derived from formula (D), thus—

$$V = A_{x+n} - (P_x + \phi)(1 + a_{x+n}) + \frac{k_x + \phi N_x}{D_{x+n}} - \frac{\phi(N_x - N_{x+t})}{D_{x+n}}$$

$$= A_{x+n} - (P_x + \phi)(1 + a_{x+n}) + \frac{k_x + \phi N_{x+t}}{D_{x+n}} \quad . \quad . \quad . \quad . \quad (E)$$

Note.—Where the policy is for the whole of life, with level premiums payable throughout its duration, $x+t$ is the limiting age of the table, and we have therefore

$$V = A_{x+n} - (P_x + \phi)(1 + a_{x+n}) + \frac{k_x + \phi N_x}{D_{x+n}} - \frac{\phi N_x}{D_{x+n}},$$

and, since in this case $k_x = 0$, the last two terms disappear and we have the ordinary prospective formula.

Practical Examples.

We are now in a position to test the method by employing the foregoing formulæ in the valuation of typical cases—

1. A Whole-Life Policy effected at age 20, sum assured £100, net annual premium £1·422, bonus additions £24.
2. A Whole-Life Policy effected by single premium at age 22, sum assured £200. A loading of £0·4 per annum for the remainder of life to be reserved for.

3. A Whole-Life Policy effected at age 24, sum assured £300, net annual premium £6·729, payable for 25 years only. The amount of £0·257 per annum to be accumulated during the term of 25 years to provide for future loadings.
4. A Term Assurance Policy effected at age 26, sum assured £400, net annual premium £3·532 payable until age 55, when the assurance ceases.
5. A Term Assurance Policy effected by single premium at age 28, sum assured £500. A loading of £1 per annum until age 60 (when the assurance ceases) to be reserved for.
6. An Endowment Assurance Policy effected at age 30, sum assured £600, net annual premium £25·5, payable until age 50, when the endowment matures. Bonus additions £105.
7. An Endowment Assurance Policy effected at age 32, sum assured £700, net annual premium £21·105, payable for 25 years. Endowment matures at age 65. The amount of £0·546 per annum to be accumulated during the term of 25 years to provide for future loadings.
8. An Endowment Assurance Policy effected by single premium at age 34, sum assured £800. A loading of £1·6 per annum until age 60 (when the endowment matures) to be reserved for.
9. An Endowment Simple Policy effected at age 36, sum assured £900, net annual premium £31·491 payable until age 55, when the endowment matures.
10. An Endowment Simple Policy effected by a single premium at age 38, sum assured £1,000. A loading of £2 per annum until age 50 (when the endowment matures) to be reserved for.
11. A Whole-Life policy effected at age 31, Sum Assured £1,100. Original net annual premium £21·747, reduced by bonus by £2·284 per annum.
12. An Endowment Assurance Policy effected at age 33, sum assured £1,200. Net annual premium £46·824, payable until age 55, when the endowment matures. Contingent Bonus addition of £90, payable on attainment of age 55 only.

At attained age 43, independent valuation of the foregoing policies by O^M $2\frac{1}{2}$ per-cent gives the following reserve values:—

Policy No.	1	Reserve value . .	£41·624
,,	2	,, ,, . .	117·036
,,	3	,, ,, . .	134·327
,,	4	,, ,, . .	18·534
,,	5	,, ,, . .	111·982
,,	6	,, ,, . .	436·152
,,	7	,, ,, . .	218·714
,,	8	,, ,, . .	572·195
,,	9	,, ,, . .	252·998
,,	10	,, ,, . .	784·474
,,	11	,, ,, . .	242·371
,,	12	,, ,, . .	506·076
			£3,436·483

Grouping the policies together we have the following schedule:

CURRENT AGE 43. O^M $2\frac{1}{2}$ PER-CENT.

No. of Policy	Sum Assured and Bonus Additions	Contingent Sum Assured and Bonus Additions	Net Premiums and Special Loading		Correction	
			Positive	Negative	Positive	Negative
1	124	...	1·422	...	...	...
2	200	...	...	·400	...	...
3	300	...	6·986	...	2,852	...
4	400	...	3·532	...	...	3,885
5	500	...	...	1·000	...	5,187
6	705	...	25·500	...	14,994	...
7	700	...	21·651	...	6,782	...
8	800	..	...	1·600	2,988	..
9	...	900	31·491	...	24,020	...
10	...	1,000	...	2·000	21,464	...
11	1,100	...	21·747	2·284	...	...
12	1,200	90	46·824	...	20,554	...
	6,029	1,990	159·153	7·284	93,654	9,072
Deduct:	...	...	7·284	...	9,072	...
	£6,029	1,990	151·869	...	84,582	...

VALUATION—

$$A_{43} \times \quad 6{,}029 = 3{,}304\cdot676$$

$$\frac{1{,}000}{D_{43}} \times \; 84{,}582 = 2{,}945\cdot365$$

$$6{,}250\cdot041$$

$$(1 + a_{43}) \times 151\cdot869 = 2{,}813\cdot525$$

$$\text{Liability } £3{,}436\cdot516$$

It will be observed that although the policies included in this group are of widely different descriptions and for various terms of years, the resulting reserve is practically identical with that produced by independent valuation, the only difference which can arise, in fact, being caused by the adjustment in the final figure in the tables, not only of the correction, but also of A, a, π, and D, which probably affects the independent valuation as much as it does the grouped valuation.

The foregoing examples assume that at the moment of valuation the lives assured have exactly attained the valuation age, and also that the $(n+1)$th premium is due and unpaid.

These are, of course, not the circumstances prevailing at an office valuation, and it is necessary to give effect to both the dates of birth, and the renewal dates being distributed throughout the year.

Assuming that, in each case, these are uniformly distributed, and taking the nearest age as the valuation age, the necessary adjustment is readily effected by the addition of $\frac{1}{2}$P to the reserve, in whole-life cases; for endowment assurances and other cases where the premiums are terminable, the same correction (the addition of $\frac{1}{2}$P) will apply, provided that the life assured be assumed to have entered at exact age x; so that, if a life entering at age 29 and 4 months be accepted as entering at, and required to pay the rate of premium for, age 30, we assume for valuation purposes, that he was born 30 years before entry. It is now usual for endowment assurances to mature on the anniversary of entry, so that, in such cases, the assumption that the date of birth is exactly x years before entry, tallies as nearly with the fact as the assumption that the endowment matures at age $(x+t)$.

In order to show more clearly the effect of this procedure, let us take four examples, all endowment assurances to mature on the anniversary of entry in the 60th year.

Policy (*a*). Assured born in March, 1860, entered in January 1900. His age at entry being 40 next birthday, his policy is for 20 years and matures in January 1920.

Policy (*b*). Assured born in September 1860, entered in January 1900. His age at entry being 40 next birthday, his policy is for 20 years and matures in January 1920.

Policy (*c*). Assured born in March 1860, entered in December 1900. His age at entry being 41 next birthday, his policy is for 19 years and matures in December 1919.

Policy (*d*). Assured born in September 1860, entered in December 1900. His age at entry being 41 next birthday, his policy is for 19 years and matures in December 1919.

If these policies are to be valued (by the Retrospective Method) on the 31 December 1900, the nearest actual age at date of valuation will be for—

Policy	(*a*) 41,	whereas	the entry	age was	40,
,,	(*b*) 40,	,,	,,	,,	40,
,,	(*c*) 41,	,,	,,	,,	41,
,,	(*d*) 40,	,,	,,	,,	41;

therefore, the use of the actual age assumes that at the valuation date one premium has been paid on policy (*a*), no premium on policies (*b*) and (*c*), and *minus* one premium on policy (*d*); the addition of $\frac{1}{2}$P will not rectify this, nor leave a positive value in the case of policy (*d*).

On the other hand, if, for policies (*a*) and (*b*) we assume the date of birth January 1860, and for policies (*c*) and (*d*) we assume the date of birth December 1859, then the nearest assumed age at date of valuation will be for—

Policy	(*a*) 41,	whereas	the entry	age was	40,
,,	(*b*) 41,	,,	,,	,,	40,
,,	(*c*) 41,	,,	,,	,,	41,
,,	(*d*) 41,	,,	,,	,,	41;

the use of the assumed age gives at the date of valuation one premium payment in the case of policies (*a*) and (*b*) and no premium payment in the case of policies (*c*) and (*d*); the addition of $\frac{1}{2}$P produces an average payment of one premium, and when successive years are taken together, the under-estimate of the payments of entrants in the second six months of any year are balanced by the over-estimate of the payments of entrants in the first six months of the next year.

A similar result (that is, the average payment of one premium) is produced if we assume the lives to enter at the nearest age at the date of entry, but the term of the policy must be unaltered, and the premiums to be valued will therefore be, for

Policy	(*a*)	the premium for	entry age	40	to mature at	60,
,,	(*b*)	,,	,,	39	,,	59,
,,	(*c*)	,,	,,	41	,,	60,
,,	(*d*)	,,	,,	40	,,	59.

The effect of this procedure is that credit is taken at the outset for the commuted value of the difference between the net premiums actually payable and those valued, that is, for $\Sigma\left(\frac{A_{x.\bar{n}|}-A_{x-1.\bar{n}|}}{2}\right)$.

I suggest that the same method (of assuming the date of birth to be exactly x years before entry) may very properly be applied to whole-life assurances, since, as the above examples show, the usual method of valuing at the nearest current age, the premium for age next birthday at entry, may result in producing undisclosed negative values.

In practice there is found a marked tendency for lives to enter in the latter half of the financial year, *i.e.*, to defer entry until just before the close of the financial year; the intensity of this tendency differs, no doubt, in different offices, and probably, also, in some degree, in different classes of assurance, and effect should be given to it in deciding the proportion of the year's premium to be added to the value produced by the use of $P(1+a)$ so as to obtain the ultimate reserve value.

VALUATION CARDS AND SCHEDULES.

The following cards were used in valuing the foregoing typical cases, and, as I have endeavoured to include, in these, examples of all cases likely to commonly occur, will probably be found suitable for practical use.

PREMIUM PAYABLE. Born within 6 months of 31st December 18

No.

Description of Policy:

Name

Entered .
Born . .
Age at Entry

Yearly Premium.

Office . . .
Extra (if any) .
Total . . .

Rates of Correction-Numerator for

Sum Assured $\frac{k}{1000}$ = .

Bonus Additions $\frac{K}{1000}$ = .

Prem. Reduction (if for a term) $\frac{N_{x+t}}{1000}$ = .

Cumulative Loadg. (or Simple Endowt) $\frac{N_x}{1000}$ = .

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	.	.		
Sp. Ldg. cumul.)	—	.		
1899	.	.		
1904	.	.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

Remarks

N.B.—Quantities in Red Ink are Negative.

PAID UP. Born within 6 months of 31st December 18

No.

Description of Policy:

Name

Entered .
Born . .
Age at Entry

Rates of Correction-Numerator for

Sum Assured and Bonus Additions $\frac{K}{1000}$ = .

Special Loading (if for a term) $\frac{N_{x+t}}{1000}$ = .

Contingent Assce. (or Contingent Bonus Add.) $\frac{D_{x+t}}{1000}$ = .

	Sum Assured.	Yearly Loading.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	.	—		
Sp. Ldg.	—	.		
1899	.	—		
1904	.	—		
1909	.	—		
1914	.	—		
1919	.	—		

Exit 19 by

Remarks

N.B.—Quantities in Red Ink are Negative.

Provision is made for recording on the cards the rates of correction-numerator; as these are constant throughout the duration of the policy, their entry on the cards facilitates the entry and checking of the numerators, while it also renders it unnecessary to refer to the tables for later entries, such as for second or subsequent bonus additions to the sum assured, or premium reductions. The note on the top right hand corner "Born within six months of 31st December, 18 ", obviously applies only where policies are valued at the nearest age at valuation date; it is of assistance in sorting. The cards as drawn provide for five valuations; this number could of course be increased, but most offices allot bonuses quinquennially, and twenty-five years' wear will probably be thought sufficient for a card; it is, of course, not necessary to print in the dates of bonus, but in my opinion it is an advantage. It is neat in appearance, and tends to keep the information on the card clear and distinct.

The difference between Card A and Card B is so slight that a single form of card could readily be made to serve for the two, but I personally prefer to use a special card for paid-up cases, and I therefore include it; it is rather clearer and simpler than Card A.

In order to exemplify the use of these cards I append copies of those used for the twelve policies valued on pp. 35 and 36. It is hardly necessary for me to comment on them, except to point out that in all these cases, though of widely different descriptions, only the two forms of card are used. It will be noticed that the rate of correction-numerator for contingent bonuses, $\frac{D_{x+t}}{1000}$, is substituted for $\frac{K}{1000}$ on the card for Policy No. 12; this form of bonus being somewhat unusual, I did not think it necessary to provide for it on the typical Card A. It will be remembered that, in the illustrative cards, old style type is used, instead of red ink, to denote negative quantities.

No. 1

Description of Policy: ***Whole-Life, With Profits.***

Name

		Rates of Correction-Numerator for		
Entered .	*1881*	Sum Assured	$\frac{k}{1000}$ =	.
Born . .	*1861*	Bonus Additions	$\frac{K}{1000}$ =	.
Age at Entry	*20*	Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000}$ =	.
Yearly Premium.		Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000}$ =	.
Office . .	.			
Extra (if any)	.			
Total . .	.			

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000
	100·	*1·422*		
Sp. Ldg. (cumul.)	—	.		
1899	*24·*	.		
1904	.	.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

Remarks

N.B.—Quantities in RED INK are NEGATIVE.

No. 2.

Description of Policy: ***Whole-Life, Without Profits.***

Name

		Rates of Correction-Numerator for		
Entered .	*1883*	Sum Assured and Bonus Additions	$\frac{K}{1000}$ =	.
Born . .	*1861*	Special Loading (if for a term)	$\frac{N_{x+t}}{1000}$ =	.
Age at Entry	*22*	Contingent Assce. (or contingent Bonus Add.)	$\frac{D_{x+t}}{1000}$ =	.

	Sum Assured	Yearly Loading.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	200·	—		
Sp. Ldg.	—	·400		
1899	.	—		
1904	.	—		
1909	.	—		
1914		—		
1919		—		

Exit 19 by

Remarks

N.B.—Quantities in RED INK are NEGATIVE.

PREMIUM PAYABLE. Born within 6 months of 31st December *1861*.

No. *3*.

Description of Policy: *Whole-Life, Without Profits. Limited Premiums for 25 years to 1909 inclusive.*

Name

		Rates of Correction-Numerator for		
Entered .	*1885*	Sum Assured	$\frac{k}{1000} =$	*8·3948*
Born . .	*1861*			
Age at Entry	*24*	Bonus Additions	$\frac{K}{1000} =$	·
Yearly Premium.				
Office . .	·	Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000} =$	·
Extra (if any)	·			
Total . .	·	Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000} =$	*1298·991*

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	300·	*6·729*	*2,518*	
Sp. Ldg. (cumul.)	—	*·257*	*334*	
1899	·	·		
1904	·	·		
1909	·	·		
1914	·	·		
1919	·	·		

Exit 19 by

PREMIUM PAYABLE. Born within 6 months of 31st December *1861*.

No. *4*.

Description of Policy: *Term Assurance for 29 years terminating in 1916.*

Name

		Rates of Correction-Numerator for		
Entered .	*1887*	Sum Assured	$\frac{k}{1000} =$	9·712
Born . .	*1861*			
Age at Entry	*26*	Bonus Additions	$\frac{K}{1000} =$	
Yearly Premium.				
Office . .	·	Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000} =$	·
Extra (if any)	·			
Total . .	·	Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000} =$	·

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	400·	*3·532*	3,885	
Sp. Ldg. (cumul.)	—	·		
1899	·	·		
1904	·	·		
1909	·	·		
1914	·	·		
1919	·	·		

Exit 19 by

31st December 1861.

No. 5.

Description of Policy: *Term Assurance for 32 years terminating in 1921.*

Name

Entered .	1889
Born . .	1861
Age at Entry	28

Rates of Correction-Numerator for

Sum Assured and Bonus Additions	$\frac{K}{1000}$ =	10·039
Special Loading (if for a term)	$\frac{H_{x+t}}{1000}$ =	166·827
Contingent Assce. (or contingent Bonus Add.)	$\frac{D_{x+t}}{1000}$ =	

	Sum Assured.	Yearly Loading.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	500·	—	5,020	
Sp. Ldg.	—	1·000	167	
1899	·	—		
1904	·	—		
1909	·	—		
1914	·	—		
1919	·	—		

Exit 19 by

Remarks

N.B.—Quantities in Red Ink are Negative.

31st December 1861.

No. 6.

Description of Policy: *Endowment Assurance, With Profits, maturing in 1911 (age 50).*

Name

Entered .	1891
Born . .	1861
Age at Entry	30

Yearly Premium.

Office . .	·
Extra (if any)	·
Total . .	·

Rates of Correction-Numerator for

Sum Assured	$\frac{k}{1000}$ =	23·491
Bonus Additions	$\frac{K}{1000}$ =	8·5655
Prem. Reduction (if for a term)	$\frac{H_{x+t}}{1000}$ =	·
Cumulative Loadg. (or Simple Endowt)	$\frac{H_x}{1000}$ =	·

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	600·	25·500	14,095	
Sp. Ldg. (cumul.)	—	·		
1899	105·	·	899	
1904	·	·		
1909	·	·		
1914	·	·		
1919	·	·		

Exit 19 by

Remarks

N.B.—Quantities in Red Ink are Negative.

PREMIUM PAYABLE. Born within 6 months of 31st December 1861.

No. 7.

Description of Policy: *Endowment Assurance, Without Profits, maturing in 1926 (age 65). Limited Premiums for 25 years until 1917 inclusive.*

Name

		Rates of Correction-Numerator for		
Entered .	1893	Sum Assured	$\frac{k}{1000}$ =	8·9714
Born . .	1861	Bonus Additions	$\frac{K}{1000}$ =	.
Age at Entry	32	Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000}$ =	.
Yearly Premium.		Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000}$ =	920·141
Office . .	.			
Extra (if any)	.			
Total . .	.			

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	700·	21·105	6,280	
Sp. Ldg. (cumul.)	—	·546	502	
1899	.	.		
1904	.	.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

Remarks

PAID UP. Born within 6 months of 31st December 1861.

No. 8.

Description of Policy: *Endowment Assurance, Without Profits, maturing in 1921 (age 60).*

Name

		Rates of Correction-Numerator for		
Entered .	1895	Sum Assured and Bonus Additions	$\frac{K}{1000}$ =	4·0689
Born . .	1861	Special Loading (if for a term)	$\frac{N_{x+t}}{1000}$ =	166·827
Age at Entry	34	Contingent Assce. (or contingent Bonus Add.)	$\frac{D_{x+t}}{1000}$ =	

	Sum Assured.	Yearly Loading.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	800·	—	3,255	
Sp. Ldg.	—	1·600	267	
1899	.	—		
1904	.	—		
1909	.	—		
1914	.	—		
1919	.	—		

Exit 19 by

Remarks

No. 9.

Description of Policy: *Simple Endowment maturing in 1916 (age 55).*

Name

		Rates of Correction-Numerator for		
Entered .	*1897*	Sum Assured	$\frac{k}{1000}$ =	.
Born . .	*1861*	Bonus Additions	$\frac{K}{1000}$ =	.
Age at Entry	*36*	Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000}$ =	.
Yearly Premium.		Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000}$ =	*762·766*
Office . .	.			
Extra (if any)	.			
Total . .	.			

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	Contingent *900·*	*31·491*	*24,020*	
Sp. Ldg. (cumul.)	—	.		
1899	.	.		
1904	.	.		
1909	.			
1914	.			
1919	.	.		

Exit 19 by

Remarks

N.B.—Quantities in Red Ink are Negative.

No. 10.

Description of Policy: *Simple Endowment maturing in 1911 (age 50).*

Name

		Rates of Correction-Numerator for		
Entered .	*1899*	Sum Assured and Bonus Additions	$\frac{K}{1000}$ =	.
Born . .	*1861*	Special Loading (if for a term)	$\frac{\Pi_{x+t}}{1000}$ =	351·185
Age at Entry	*38*	Contingent Assce. (or contingent Bonus Add.)	$\frac{D_{x+t}}{1000}$ =	*22·166*

	Sum Assured.	Yearly Loading.	Correction-Numerator, divided by 1000,	Cor.-Num. for Bonus Estimate, divided by 1000.
	Contingent *1000·*	—	*22,166*	
Sp. Ldg.	—	2·	702	
1899	.	—		
1904	.	—		
1909	.	—		
1914	.	—		
1919	.	—		

Exit 19 by

Remarks

N.B.—Quantities in Red Ink are Negative.

PREMIUM PAYABLE. Born within 6 months of 31st December 1861.

No. *11.*

Description of Policy : *Whole-Life, With Profits.*

Name

		Rates of Correction-Numerator for		
Entered .	*1892*	Sum Assured	$\frac{k}{1000} =$	.
Born . .	*1861*			
Age at Entry	*31*	Bonus Additions	$\frac{K}{1000} =$	.
Yearly Premium.				
Office . .	.	Prem. Reduction (if for a term)	$\frac{\Pi_{x+t}}{1000} =$	.
Extra (if any)	.			
Total . .	.	Cumulative Loadg. (or Simple Endowt)	$\frac{\Pi_x}{1000} =$	.

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	1100·	*21·747*		
Sp. Ldg. (cumul.)	—	.		
1899	.	2·284		
1904	.	.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

Remarks

PREMIUM PAYABLE. Born within 6 months of 31st December 1861.

No. *12.*

Description of Policy : *Endowment Assurance maturing in 1916 (age 55) CONTINGENT BONUS.*

Name

		Rates of Correction-Numerator for		
Entered .	*1894*	Sum Assured	$\frac{k}{1000} =$	*15·780*
Born . .	*1861*			
Age at Entry	*33*	Contingent Bonus Additions	$\frac{D_{x+t}}{1000} =$	*17·980*
Yearly Premium.				
Office . .	.	Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000} =$	.
Extra (if any)	.			
Total . .	.	Cumulative Loadg. (or Simple Endowt)	$\frac{\Pi_x}{1000} =$	.

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	1200·	*46·824*	*18,936*	
Sp. Ldg. (cumul.)	.	.		
1899	Contingent *90·*	.	*1,618*	
1904		.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

Remarks

The form of schedule for listing policies is mainly different from that ordinarily used, in that it requires a double column for Premium and Loading instead of the usual single column, and an additional double column for Correction-Numerators, the object of doubling these columns, as before explained, being to provide for the negative quantities which arise under certain classes of policies. For with-profit policies a further double column will be required to enable the cost of declaring a reversionary bonus to be calculated. I will deal with the use of this column later.

In case the appearance of this schedule be thought formidable, I should like to mention that the form of list I have been in the habit of using contains nine columns, whereas this list contains eight, if double columns be counted as one, or eleven, if as two columns. The last double column is for Bonus Estimate, and is used only for participating policies. It should be kept in mind that this list is intended for with and without-profit policies for the Whole of Life, Endowment Assurances, Simple Endowments, and Term Assurances, including policies in each class effected subject to limited premiums, or paid-up, and that these can, if desired, be listed together; thus, even if it were slightly more cumbersome than the usual form—which I do not wish to suggest—it is not one ordinary list that it takes the place of, but many.

This form may also be used for summarizing the valuation particulars,

POLICY LIST.

Lives born within 6 months of 31 December 18 .

	No. of Policy	Sum Assured and Bonus Additions	Contingent Sum Assured and Bonus Additions	Office Yearly Premium	Extra Premium	Net Premium and Special Loading		Correction-Numerator		For Bonus Estimate Correction-Numerator	
						Positive	Negative	Positive	Negative	Positive	Negative

the only alteration required being the substitution of the number of policies at each age for the policy numbers, and the addition of a column, preferably on the left hand side, for the valuation ages. It will be observed that a margin has been left for this purpose. The difference between the positive and negative sides of each of the double columns will have been taken, when the policy lists were cast, and the net result in each case will therefore appear on the valuation particulars. Almost without exception, these will be found to be positive, and the negative columns will, I anticipate, be rarely used.

The Valuation Schedule is simple; it requires only five columns.

Valuation.

Valuation Age	Value of					Liability
	Sums Assured and Bonus Additions $\times A_{x+n}$	Net Premium $\times (1+a_{x+n})$		Correction $\times D^{-1}_{x+n}$		
		Positive	Negative	Positive	Negative	

Here, again, double columns are given for Net Premiums and Corrections, but the negative columns could probably be dispensed with. It will be noticed that I do not provide for valuing the office premiums; it is necessary to consider the subject of gross premium valuation with special reference to

Valuations by Mixed Mortality.

I have hitherto presupposed a net premium valuation, and in this I do not include valuation by combined tables of mortality, such as by $H^{M(5)}$ using H^{M} premiums, or $O^{M(5)}$ using O^{M} premiums; these are, of course, gross premium valuations, the amount thrown off for future expenses and profit being usually left unascertained.

It is a condition precedent to the use of my formulæ that the net premium, calculated by the table of mortality employed in the valuation, be valued; but it does not follow from this that the method is not suitable for valuing by combined tables; all that is required is that the premium to be reserved in such cases be regarded as a loaded premium, and that formula (E) be employed. Thus, for valuation by $O^{M(5)}$ using O^{M} premiums, the net premium used in calculating k, and K, will be the

$O^{M(5)}$ premium, and the loading will be the difference between the O^M and $O^{M(5)}$ premiums.

As an example, to show the effect of this procedure, let us take the case of a term assurance effected at age x for the term of t years, and n years in force.

Indicating functions calculated by $O^{M(5)}$ mortality by Roman type, and those by O^M mortality by *italic* type, we have—

(1) The usual formula (prospective method)

$$\mathrm{V}=\mathrm{A}^{\ 1}_{x+n.\overline{t-n}|}-P^1_{x\overline{t}|}(1+a_{x+n.\overline{t-n-1}|}).$$

(2) Formula (E) (retrospective method)

$$\mathrm{V}=\mathrm{A}_{x+n}-P^1_{x\overline{t}|}(1+a_{x+n})+\frac{k^{\,1}_{x\overline{t}|}+\phi\mathrm{N}_{x+t}}{\mathrm{D}_{x+n}}.$$

This is seen to be identical with the prospective formula thus: substituting $(P^1_{x\overline{t}|}-\mathrm{P}^1_{x\overline{t}|})$ for ϕ, we have

$$\mathrm{V}=\mathrm{A}_{x+n}-P^1_{x\overline{t}|}(1+a_{x+n})+\frac{(\mathrm{P}^1_{x\overline{t}|}-\pi_x)\mathrm{N}_x+(P^1_{x\overline{t}|}-\mathrm{P}^1_{x\overline{t}|})\mathrm{N}_{x+t}}{\mathrm{D}_{x+n}}$$

$$=\mathrm{A}_{x+n}-P^1_{x\overline{t}|}(1+a_{x+n})+\frac{\mathrm{P}^1_{x\overline{t}|}(\mathrm{N}_x-\mathrm{N}_{x+t})-\pi_x\mathrm{N}_x+P^1_{x\overline{t}|}\mathrm{N}_{x+t}}{\mathrm{D}_{x+n}}$$

$$=\mathrm{A}_{x+n}-P^1_{x\overline{t}|}(1+a_{x+n})+\frac{\mathrm{M}_x-\mathrm{M}_{x+t}-\mathrm{M}_x+P^1_{x\overline{t}|}\mathrm{N}_{x+t}}{\mathrm{D}_{x+n}}$$

$$=\frac{\mathrm{M}_{x+n}-\mathrm{M}_{x+t}}{\mathrm{D}_{x+n}}-P^1_{x\overline{t}|}\left(\frac{\mathrm{N}_{x+n}-\mathrm{N}_{x+t}}{\mathrm{D}_{x+n}}\right)$$

$$=\mathrm{A}^{\ 1}_{x+n.\overline{t-n}|}-P^1_{x\overline{t}|}(1+a_{x+n.\overline{t-n-1}|}).$$

The above formula (for the retrospective method) serves to point out that the substitution of $P^1_{x\overline{t}|}$ for $\mathrm{P}^1_{x\overline{t}|}$ in the premium column requires a compensating correction of $(P^1_{x\overline{t}|}-\mathrm{P}^1_{x\overline{t}|})\dfrac{\mathrm{N}_{x+t}}{\mathrm{D}_{x+n}}$ in all cases where a terminable premium is payable; in other cases (*i.e.*, where the premium is payable throughout life) no correction is of course required, since $(x+t)=$ the limiting age of the table. The entry of the compensating correction-numerator $(P^1_{x\overline{t}|}-\mathrm{P}^1_{x\overline{t}|})\mathrm{N}_{x+t}$ on the cards is not a difficult matter, and when that is once done, the work of valuation proceeds as easily as a net premium valuation.

In order to exemplify the use of the method, I have re-valued the twelve typical policies by $O^{M(5)}$ mortality, valuing the same premiums as before. The twelve cards will be as follows—

PREMIUM PAYABLE. Born within 6 months of 31st December, 1861.

No. 1.

Description of Policy: *Whole-Life, With Profits.*

Name

Entered .	1881
Born .	1861
Age at Entry	20

Yearly Premium.

Office . .	.
Extra (if any)	.
Total . .	.

Rates of Correction-Numerator for

Sum Assured	$\frac{k}{1000} =$	.
Bonus Additions	$\frac{K}{1000} =$	.
Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000} =$	.
Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000} =$	.

	Sum Assured	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000
	100·	*1·422*		
Sp. Ldg. (cumul.)	—	.		
1899	*24·*	.		
1904	.	.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

PAID UP. Born within 6 months of 31st December 1861.

No. 2.

Description of Policy: *Whole-Life, Without Profits.*

Name

Entered .	1883
Born . .	1861
Age at Entry	22

Rates of Correction-Numerator for

Sum Assured and Bonus Additions	$\frac{K}{1000} =$	.
Special Loading (if for a term)	$\frac{N_{x+t}}{1000} =$	.
Contingent Assce. (or contingent Bonus Add.)	$\frac{D_{x+t}}{1000} =$	.

	Sum Assured.	Yearly Loading.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	200·	—		
Sp. Ldg.	—	*·400*		
1899	.	—		
1904	.	—		
1909	.	—		
1914	.	—		
1919	.	—		

Exit 19 by

Description of Policy: *Whole-Life, Without Profits. Limited Premiums for 25 years to 1909 inclusive.*

Name

		Rates of Correction-Numerator for		
Entered .	1885			
Born . .	1861	Sum Assured	$\frac{k}{1000}$ =	8·8064
Age at Entry	24			
		Bonus Additions	$\frac{K}{1000}$ =	.
Yearly Premium.				
Office . .	.	Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000}$ =	375·602
Extra (if any)	.			
Total . .	.	Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000}$ =	1318·222

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	300·	7·035	2,642	
ΔP		·306	115	
Sp. Ldg. (cumul.)	—	·257	339	
1899	.	.		
1904	.	.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

Remarks

N.B.—Quantities in Red Ink are Negative.

Description of Policy: *Term Assurance for 29 years terminating in 1916.*

Name

		Rates of Correction-Numerator for		
Entered .	1887			
Born . .	1861	Sum Assured	$\frac{k}{1000}$ =	9·547
Age at Entry	26			
		Bonus Additions	$\frac{K}{1000}$ =	.
Yearly Premium.				
Office . .	.	Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000}$ =	249·434
Extra (if any)	.			
Total . .	.	Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000}$ =	.

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	400	3·912	3,819	
ΔP		·380	95	
Sp. Ldg. (cumul.)	—	.		
1899	.	.		
1904	.	.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

Remarks

N.B.—Quantities in Red Ink are Negative.

PAID UP. Born within 6 months of 31st December *1861*.

No. *5*.

Description of Policy: *Term Assurance for 32 years terminating in 1921*.

Name

Entered .	*1889*
Born . .	*1861*
Age at Entry	*28*

Rates of Correction-Numerator for

Sum Assured and Bonus Additions	$\frac{K}{1000} =$	10·078
Special Loading (if for a term)	$\frac{N_{x+t}}{1000} =$	167·044
Contingent Assce. (or contingent Bonus Add.)	$\frac{D_{x+t}}{1000} =$	.

	Sum Assured.	Yearly Loading.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	500·	—	5,039	
Sp. Ldg.	—	1·000	167	
1899	.	—		
1904	.	—		
1909	.	—		
1914	.	—		
1919	.	—		

Exit 19 by

PREMIUM PAYABLE. Born within 6 months of 31st December *1861*.

No. *6*.

Description of Policy: *Endowment Assurance, With Profits, maturing in 1911 (age 50)*.

Name

Entered .	*1891*
Born . .	*1861*
Age at Entry	*30*

Yearly Premium.

Office . .	.
Extra (if any)	.
Total .	.

Rates of Correction-Numerator for

Sum Assured	$\frac{k}{1000} =$	*23·773*
Bonus Additions	$\frac{K}{1000} =$	*8·5946*
Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000} =$	352·384
Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000} =$	.

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	600·	*25·842*	*14,264*	
ΔP		·342	121	
Sp. Ldg. (cumul.)	—	.		
1899	*105·*	.	*902*	
1904	.	.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

Description of Policy: *Endowment Assurance, Without Profits, maturing in 1926 (age 65). Limited Premiums for 25 years until 1917 inclusive.*

Name

Entered .	*1893*
Born .	*1861*
Age at Entry	*32*

Yearly Premium.

Office . .	.
Extra (if any)	.
Total . .	.

Rates of Correction-Numerator for

Sum Assured	$\frac{k}{1000}$ =	*9·0936*
Bonus Additions	$\frac{K}{1000}$ =	.
Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000}$ =	214·104
Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000}$ =	*928·768*

	Sum Assured.	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	700·	*21·462*	*6,366*	
ΔP		·357	76	
Sp. Ldg. (cumul.)	—	*·546*	*507*	
1899	.	.		
1904	.	.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

Remarks

N.B.—Quantities in Red Ink are Negative.

Description of Policy: *Endowment Assurance, Without Profits, maturing in 1921 (age 60).*

Name

Entered .	*1895*
Born . .	*1861*
Age at Entry	*34*

Rates Correction-Numerator for

Sum Assured and Bonus Additions	$\frac{K}{1000}$ =	*4·0742*
Special Loading (if for a term)	$\frac{N_{x+t}}{1000}$ =	167·044
Contingent Assce. (or contingent Bonus Add.)	$\frac{D_{x+t}}{1000}$ =	.

	Sum Assured.	Yearly Loading.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	800·	—	*3,259*	
Sp. Ldg.	—	1·600	267	
1899	.	—		
1904	.	—		
1909	.	—		
1914	.	—		
1919	.	—		

Exit 19 by

Remarks

N.B.—Quantities in Red Ink are Negative.

PREMIUM PAYABLE. Born within 6 months of 31st December *1861*.

No. *9*.

Description of Policy: *Simple Endowment maturing in 1916 (age 55).*

Name

		Rates of Correction-Numerator for		
Entered .	*1897*	Sum Assured	$\frac{k}{1000} =$	.
Born .	*1861*	Bonus Additions	$\frac{K}{1000} =$	.
Age at Entry	*36*	Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000} =$	*249·434*
Yearly Premium.		Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000} =$	*768·549*
Office . .	.			
Extra (if any)	.			
Total .	.			

	Sum Assured. Contingent *900·*	Yearly Net Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
		31·326	*24,076*	
ΔP		*·165*	*41*	
Sp. Ldg. (cumul.)	—	.		
1899	.	.		
1904	.	.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

PAID UP. Born within 6 months of 31st December *1861*.

No. *10*.

Description of Policy: *Simple Endowment maturing in 1911 (age 50).*

Name

		Rates of Correction-Numerator for		
Entered .	*1899*	Sum Assured and Bonus Additions	$\frac{K}{1000} =$	.
Born . .	*1861*	Special Loading (if for a term)	$\frac{11_{x+t}}{1000} =$	352·384
Age at Entry	*38*	Contingent Assce. (or contingent Bonus Add.)	$\frac{D_{x+t}}{1000} =$	*22·320*

	Sum Assured. Contingent *1000·*	Yearly Loading.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
		—	*22,320*	
Sp. Ldg.	—	2·000	705	
1899	.	—		
1904	.	—		
1909	.	—		
1914	.	—		
1919	.	—		

Exit 19 by

No. *11.*

Description of Policy : *Whole-Life, With Profits.*

Name

Entered .	1892	Rates of Correction-Numerator for		
Born . .	1861	Sum Assured	$\frac{k}{1000}$ =	.
Age at Entry	31	Bonus Additions	$\frac{K}{1000}$ =	.
Yearly Premium.				
Office . .	.	Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000}$ =	.
Extra (if any)	.			
Total . .	.	Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000}$ =	.

	Sum Assured.	Yearly <u>Net</u> Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	1100·	*21·747*		
Sp. Ldg. (cumul.)	—	.		
1899	.	2·284		
1904	.	.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

Remarks

N.B.—Quantities in Red Ink are Negative.

No. *12.*

Description of Policy : *Endowment Assurance maturing In 1916 (age 55) CONTINGENT BONUS.*

Name

Entered .	1894	Rates of Correction-Numerator for		
Born . .	1861	Sum Assured	$\frac{k}{1000}$ =	*5·926*
Age at Entry	33	Contingent Bonus Additions	$\frac{D_{x+t}}{1000}$ =	*18·069*
Yearly Premium.				
Office .	.	Prem. Reduction (if for a term)	$\frac{N_{x+t}}{1000}$ =	249·434
Extra (if any)	.			
Total . .	.	Cumulative Loadg. (or Simple Endowt)	$\frac{N_x}{1000}$ =	.

	Sum Assured.	Yearly <u>Net</u> Premium.	Correction-Numerator, divided by 1000.	Cor.-Num. for Bonus Estimate, divided by 1000.
	1200·	*47·352*	*19,111*	
ΔP		·528	132	
Sp. Ldg. (cumul.)	—	.		
1899	Contingent *90·*	.	*1,626*	
1904	.	.		
1909	.	.		
1914	.	.		
1919	.	.		

Exit 19 by

Remarks

N.B.—Quantities in Red Ink are Negative.

Transferring the particulars to our schedule, we have—

CURRENT AGE 43. O^{M} AND $O^{M(5)}$ $2\frac{1}{2}$ PER-CENT.

No. of Policy	Sum Assured and Bonus Additions	Contingent Sum Assured and Bonus Additions	Net Premium and Special Loading: Positive	Net Premium and Special Loading: Negative	Correction: Positive	Correction: Negative
1	124	...	1·422	...	...	...
2	200	...	...	·400	...	...
3	300	..	7·292	·306	2,981	115
4	400	..	3·912	·380	...	3,914
5	500	...	...	1·000	...	5,206
6	705	...	25·842	·342	15,166	121
7	700	...	22·008	·357	6,873	76
8	800	...	...	1·600	3,259	267
9	...	900	31·491	...	24,117	...
10	...	1,000	...	2·000	22,320	705
11	1,100	...	21·747	2·284	...	...
12	1,200	90	47·352	·528	20,737	132
	6,029	1,990	161·066	9·197	95,453	10,536
Deduct:	...	...	9·197	...	10,536	...
	£6,029	1,990	151·869	...	84,917	...

VALUATION—

$$A_{43}\times\ 6{,}029=3{,}318\cdot060$$

$$\frac{1{,}000}{D_{43}}\times\ 84{,}917=2{,}927\cdot163$$

$$6{,}245\cdot223$$

$$(1+a_{43})\times 151\cdot869=2{,}799\cdot857$$

Liability £3,445·366

Independent valuation by prospective formulæ gives the following values—

Policy No.	1	Reserve value	£42·027
,,	2	,, ,,	117·444
,,	3	,, ,,	135·116
,,	4	,, ,,	20·132
,,	5	,, ,,	114·160
,,	6	,, ,,	436·491
,,	7	,, ,,	220·346
,,	8	,, ,,	572·915
,,	9	,, ,,	250·761
,,	10	,, ,,	781·968
,,	11	,, ,,	246·565
,,	12	,, ,,	507·470
			£3,445·395

As before, the results are practically identical.

Objections and Advantages.

There will probably at once occur to the minds of those who have borne with me thus far, the objections

(1) that this method does not provide for the separate calculation of the value of the sum assured required under the 5th Schedule of the "Life Assurance Companies Act, 1870"; and that, the value of the sum assured not being ascertained, there is no means of estimating the cost of allotting a reversionary bonus;

(2) that it does not provide for the calculation of the present value of office premiums, also required under the Act, and that unless the present values of both office and net premiums, be calculated, the present value of future loadings cannot be ascertained; and

(3) that single life assurances of all kinds being listed together, separate lists would have to be prepared in order to comply with the requirements of the Act under Schedule VI, Q 2, 3, 4, 5 and 6.

These three objections are alike in showing that the Act of 1870 did not contemplate a retrospective valuation; but I submit that it does not follow from that, that the Retrospective Method is either unsuitable or undesirable; on the contrary, I am inclined to regard it as simple, quick, and convenient.

The Schedules to the Act are not like the laws of the Medes and Persians that may not be changed, for Section 9 of the Act reads as follows—

"The Board of Trade, upon the application or with the "consent of a company, may alter the forms contained in "the Schedules to this Act, for the purpose of adapting them "to the circumstances of such company, or of better carrying "into effect the objects of this Act", and I think it unlikely that the Board would object to adapt the Schedules to suit the Retrospective Method, when once they were satisfied that the operation of that method was reliable, and the resulting liability identical with that produced by the use of the Prospective Method.

But even if the Board of Trade were to insist on having a statement of the value of the sums assured and bonuses, and of the office and net yearly premiums, it is by no means impossible to produce them. It will have been noticed that on the sample cards, and on the sample policy-list, space is left for "Correction-Numerator for Bonus Estimate." In order to estimate the cost of allotting a reversionary bonus, it is necessary to know the present value of the sums assured and bonus additions of participating policies; the Retrospective Method does not give this in the cases of policies subject to premium payments, and to obtain it, it is necessary to value these policies as if they were paid-up, *i.e.*, to schedule the value of SK (the sum assured multiplied by the correction-numerator for paid-up assurance); as, however, K is constant throughout the duration of a policy, this work is very readily performed. The values of the sums assured and bonus additions having been thus ascertained, the deductions from them of the liability will give the value of the premiums, including any special loadings, reductions and additions; in other words, the value of those premiums which are applicable to the risks. The value of the office premiums can of course be ascertained by the use of formula (C), the loading f being the difference between the office premium and the theoretical, or original net premium.

Provision is made in the Schedule as required by the Act, for listing the office premiums, and therefore the total yearly office premiums and the total yearly net premiums can be compared, and the *yearly* loading ascertained. The calculation of the value of the office yearly premiums does not seem to me to serve any useful purpose, and I very much doubt if the Board of Trade would insist upon it. The amount available for expenses is the loading reserved, and as long as the amount of this loading could be readily seen, or ascertained, and compared with the actual expenses, the purpose for which this item was included in the Fifth Schedule to the Act would, I think, be served, and the Board of Trade would, I anticipate, be satisfied. I know that many Companies mention, in their valuation reports, the value of future loadings; it is a big figure and looks imposing, but I am afraid it conveys very little real information to the public, for whose edification it is no doubt intended; the actual annual expenditure compared with the amount available seems to me far more convincing, and I think it even preferable to the percentages

of these amounts, which many Companies quote; the public would, I have no doubt, find it more intelligible.

The Act of 1870 followed soon after the disastrous failures of the *Albert* and *European* Offices, and the statements required under the Sixth Schedule were no doubt the outcome of those failures and intended to enable an independent valuation to be made of a Company's liabilities. In 1870, life assurance business consisted almost entirely of whole-life contracts, and therefore the Statements 2 and 3 were of practical value and effect; but now that "other classes of assurance" bear so large a proportion to the total business, that value has ceased to exist; a statement of single life contracts, such as the retrospective valuation schedules would produce, would give now, I suggest, a better view of a Company's business, and would include about the same proportion of policies, as, in 1870, whole-life assurances bore to the total. For this reason, I think it unlikely that the Board of Trade would object to the substitution of statements of single life assurances for whole-life assurances.

Turning now to some of the advantages of the method, I have already drawn attention to the great convenience of being able to group together policies of different descriptions, maturing or terminating at various dates (or ages) and either paid up, or subject to premiums which may, or need not, continue throughout the duration of the assurance, and I have pointed out that the net premium is in every case *the measure of the risk;* it is the governing factor of the Retrospective Method of valuation.

I have also pointed out the readiness with which the method may be adapted to circumstances, by showing its application to a valuation using a loaded premium, and thus to valuation by mixed tables of mortality; it is therefore unnecessary to dwell upon these advantages, and I will pass at once to another which may possibly be thought of interest, even though it may not be generally regarded as of great practical importance. I refer to the suitability of the method for the valuation of benefits payable (or terminable) at fractional ages.

These commonly arise under certain methods of bonus application, the most usual being the somewhat troublesome application of a bonus (either cash or reversionary)—

(1) To make the sum assured under a whole-life policy payable in lifetime, and

(2) To reduce the endowment age under an endowment assurance.

Simple formulæ for obtaining the new endowment age may be derived from the new formulæ; thus—

(1) Let C be the cash bonus to be applied to convert a whole-life policy effected at age x, n years in force, into an endowment assurance. Writing $x+t$ for the endowment age to be found, we have—

$$C+A_{x+n}-\pi_x(1+a_{x+n})=A_{x+n}-(P_{x\overline{t}}-f)(1+a_{x+n})+\frac{k_{x\overline{t}}-f\mathrm{N}_{x+t}}{D_{x+n}}$$

The right-hand side of the equation consists of the value of the Endowment Assurance Policy; the premium payable is π_x and the theoretical premium is therefore reduced by f, which is $(P_{x\overline{t}}-\pi_x)$. The formula is derived by combining formulæ (A) and (C).

Writing $(P_{x\overline{t}}-\pi_x)$ for f, and omitting terms which cancel each other, we have—

$$C=\frac{(P_{x\overline{t}}-\pi_x)(\mathrm{N}_x-\mathrm{N}_{x+t})}{D_{x+n}},$$

$$\therefore \quad CD_{x+n}=\left(\frac{M_x+d\mathrm{N}_{x+t}}{\mathrm{N}_x-\mathrm{N}_{x+t}}-\frac{M_x}{\mathrm{N}_x}\right)(\mathrm{N}_x-\mathrm{N}_{x+t})$$

$$=M_x+d\mathrm{N}_{x+t}-M_x+\frac{M_x\mathrm{N}_{x+t}}{\mathrm{N}_x}$$

$$=\frac{\mathrm{N}_{x+t}(d\mathrm{N}_x-d\mathrm{N}_x+D_x)}{\mathrm{N}_x}$$

$$\therefore \quad CD_{x+n}(1+a_x)=\mathrm{N}_{x+t},$$

enabling the new endowment age to be obtained by inspection of a column of N_x.

In the same way at a subsequent declaration of bonus—

(2) Let ²C be the cash bonus to be applied to reduce the endowment age. Let $(x+m)$ be the new valuation age, and $(x+r)$ the new endowment age to be found.

Then—

$$^{2}C = A_{x+m} - \pi_x(1+a_{x+m}) + \frac{(P_{x\overline{r}} - \pi_x)(N_x - N_{x+r})}{D_{x+m}}$$

$$- A_{x+m} + \pi_x(1+a_{x+m}) - \frac{(P_{x\overline{t}} - \pi_x)(N_x - N_{x+t})}{D_{x+m}}$$

$$\therefore \quad {}^{2}CD_{x+m} = (P_{x\overline{r}} - \pi_x)(N_x - N_{x+r}) - (P_{x\overline{t}} - \pi_x)(N_x - N_{x+t})$$

$$= dN_{x+r} + \frac{M_x N_{x+r}}{N_x} - dN_{x+t} - \frac{M_x N_{x+t}}{N_x}$$

$$= \frac{N_{x+r} D_x}{N_x} - \frac{N_{x+t} D_x}{N_x}$$

$$\therefore \quad {}^{2}CD_{x+m}(1+a_x) = N_{x+r} - N_{x+t}$$

$$\therefore \quad {}^{2}CD_{x+m}(1+a_x) + N_{x+t} = N_{x+r},$$

$(x+t)$ being already known, $(x+r)$ may be obtained by inspection of a column of N_x as before.

When the bonus is reversionary, the substitution of BA_{x+n} for C and $^{2}BA_{x+m}$ for ^{2}C will give

$$BM_{x+n}(1+a_x) = N_{x+t},$$

and $$^{2}BM_{x+m}(1+a_x) + N_{x+t} = N_{x+r}.$$

It has always, I think, been felt to be a difficulty with this class of policy that the bonus rarely brought the new endowment date to an anniversary of the date of entry, and therefore it was necessary to deal with a fractional endowment age; a further inconvenience of this system of bonus allotment was the necessity of transferring the policies to the policy lists for their new endowment ages. Both these troubles disappear by the use of the new formulæ; the only new quantity we require for the valuation card is N_{x+t} and from it we can readily derive our new $k_{x\overline{t}}$ and fN_{x+t}, *i.e.*, the corrections of formulæ (A) and (C); it is not necessary that this N_{x+t} should be the number appearing opposite an age in our table, but the exact value of $CD_{x+n}(1+a_x)$ can be used, and the resulting endowment assurance premium (which may be conveniently obtained by the use of the formula $P_{x\overline{t}} = \dfrac{M_x + dN_{x+t}}{N_x - N_{x+t}}$) and policy-value will be correct for the fractional age $(x+t)$; the formula assumes that a proportionate

premium will be payable in the final (fractional) year of assurance. As there is only one policy-list for all policies of the same current age, no question of transferring the entry because of a change of endowment maturity age can arise.

This circumstance, that there need be only one policy-list for each current age, is the advantage of the Retrospective Method of valuation, and its practical convenience can hardly, I submit, be over-estimated. To decrease the number of groups of policies to be valued is to decrease the work of valuation, and I therefore venture to suggest the use of the Retrospective Method for

(1) annual valuations, and
(2) valuations by select tables of mortality.

This latter is, I suppose, the ideal valuation, but the labour involved in the use of select tables is so great that hitherto I have not heard of their being employed for the purpose of an office valuation. If, however, the great bulk of the policies of an office can be grouped together, according to current ages, the work is so greatly decreased that it should not be impossible to apply this test of valuation by select tables; a test which would be the nearest approach to scientific accuracy yet applied, and which, I think, would be generally more satisfactory to the actuarial profession than that hitherto adopted—the mis-named "net" premium valuation by combined tables of mortality, H^M and $H^{M(5)}$, or O^M and $O^{M(5)}$.

ADDENDUM.

Increasing and Decreasing Premiums.

It has been pointed out to me that I have not referred to the valuation of policies subject to increasing and decreasing premiums. To these cases formula (A) applies during the time of payment of the commencing rate of premium, and, on a change in the rate occurring, the difference in the premium is accumulated from the time of its commencement (*not* from entry) as a positive, or negative, additional reserve, formula (C) being used for this purpose.

For example, a Whole-Life Policy is effected at age x at an annual premium of—

P_x for the first m years,
P_x+r from the $(m+1)$th to the nth year,
P_x-s from the $(n+1)$th to the qth year,

after which the premium ceases.

(1) The value t years after entry, t being less than m will be

$$_{t}V'_{x}=A_{x+t}-P_{x}(1+a_{x+t})+\frac{k_{x}}{D_{x+t}}.$$

Substituting in k_x for $\pi_x N_x$ its value in terms of P_x, r, and s, *i.e.*, $P_x N_x + rN_{x+m} - (r+s)N_{x+n} - (P_x - s)N_{x\ q}$, this formula may be reduced to the prospective formula—

$$_{t}V'_{x}=A_{x+t}-P_{x}(1+a_{x+t\,.\,\overline{q-t-1}}) \\ -r\times{}_{m-t-1\ q-t-1}a_{x+t}+(r+s)\times{}_{n-t-1\ q-t-1}a_{x+t}.$$

(2) The value w years after entry, w being greater than m and less than n will be

$$_{w}V'_{x}=A_{x+w}-(P_{x}+r)(1+a_{x+w})+\frac{k_{x}+rN_{x+m}}{D_{x+w}}.$$

By substituting for $\pi_x N_x$ as before, this formula may be reduced to the prospective formula

$$_{w}V'_{x}=A_{x+w}-(P_{x}+r)(1+a_{x+w\,.\,\overline{q-w-1}})+(r+s)_{n-w-1\ q-w-1}a_{x+w}.$$

(3) The value z years after entry, z being greater than n and less than q, will be

$$_{z}V'_{x}=A_{x+z}-(P_{x}-s)(1+a_{x+z})+\frac{k+rN_{x+m}-(r-s)N_{x+n}}{D_{x+z}}.$$

By substituting for $\pi_x N_x$ as before, this formula may be reduced to the prospective formula

$$_{z}V'_{x}=A_{x+z}-(P_{x}-s)(1+a_{x+z\,.\,\overline{q-z-1}}).$$

Hence it will be seen that policies subject to increasing or decreasing premiums may, if the retrospective formula be used, be grouped according to the nearest ages at valuation, whatever the number of changes in the premiums, or whatever the ages at which those changes take effect, and that, if desired, they may be included in groups of, and valued with, other single-life policies.

* Since this Paper was in type, my attention has been directed to a Paper by Mr. Wm. D. Whiting, published in the *Transactions of the Actuarial Society of America*, No. 12, p. 427, and referred to by Mr. Todhunter (*J.I.A.*, vol. xxxv, p. 18), also to letters addressed to the Editor of the *J.I.A.* by Mr. Julius Altenburger, and published in the *J.I.A.*, vol. xxxiv, p. 150, and vol. xxxv, p. 332. I had not previously seen Mr. Whiting's Paper, or probably I should not have offered this Paper to the Institute, and I am afraid I must confess that Mr. Altenburger's letters had escaped my attention.

The formulæ given by both these writers are virtually, or actually, reproduced by me, but in order that I may not be open to any possible charge of plagiarism, I desire to state categorically that I do not claim to have produced any new formulæ; I have merely drawn attention to a convenient, and, as it proves, not original, re-arrangement of what Mr. Whiting describes as "Perhaps one of the most general and comprehensive formulæ, for grouped valuations, as well as the oldest."

0^M **2½ per-cent.** Log $_t k_x$

LOG OF CORRECTION-NUMERATOR *for* WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	4·62439	4·37930	4·17707	3·99245	3·81209	20
21	·61316	·36679	·16292	·97615	·79290	21
22	·60167	·35396	·14837	·95932	·77302	22
23	·58992	·34081	·13341	·94196	·75244	23
24	·57788	·32731	·11799	·92401	·73108	24
25	·56551	·31341	·10210	·90544	·70888	25
26	·55284	·29912	·08570	·88620	·68580	26
27	·53982	·28441	·06875	·86627	·66178	27
28	·52641	·26923	·05123	·84557	·63674	28
29	·51263	·25357	·03308	·82407	·61062	29
30	·49842	·23742	·01430	·80172	·58337	30
31	·48379	·22071	3·99481	·77846	·55487	31
32	·46872	·20346	·97462	·75427	·52510	32
33	·45317	·18561	·95367	·72906	·49394	33
34	·43713	·16714	·93191	·70277	·46131	34
35	·42059	·14801	·90933	·67539	·42713	35
36	·40350	·12822	·88583	·64677	·39126	36
37	·38585	·10771	·86142	·61690	·35363	37
38	·36761	·08645	·83601	·58568	·31409	38
39	·34876	·06442	·80956	·55303	·27252	39
40	·32927	·04154	·78202	·51887	·22877	40
41	·30910	·01780	·75330	·48308	·18268	41
42	·28821	3·99314	·72334	·44556	·13408	42
43	·26658	·96749	·69206	·40617	·08275	43
44	·24415	·94082	·65938	·36480	·02850	44
45	·22089	·91305	·62521	·32129	2·97108	45
46	·19674	·88412	·58944	·27550	...	46
47	·17165	·85394	·55195	·22722	...	47
48	·14556	·82244	·51262	·17627	...	48
49	·11842	·78955	·47134	·12246	...	49
50	·09015	·75515	·42794	·06551	...	50
51	·06066	·71912	·38226	...	...	51
52	·02989	·68138	·33413	...	...	52
53	3·99777	·64180	·28334	...	...	53
54	·96418	·60023	·22971	...	...	54
55	·92905	·55654	·17298	...	...	55
56	·89225	·51057	...	...	...	56
57	·85369	·46214	...	...	...	57
58	·81321	·41105	...	...	...	58
59	·77068	·35709	...	...	...	59
60	·72598	·30005	...	...	...	60
61	·67893	...	...	...	...	61
62	·62936	...	...	...	...	62
63	·57708	...	...	...	...	63
64	·52186	...	...	...	...	64
65	·46351	...	...	...	...	65

O[M] 2½ per-cent. $\frac{{}_t\mathbf{k}_x}{1000}$

CORRECTION-NUMERATOR *for*
WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS
DIVIDED BY 1000.

Age at Entry	Number of Payments					Age at Entry
	10	15	20	25	30	
20	42·110	23·950	15·034	9·8277	6·4877	20
21	41·036	23·270	14·552	9·4656	6·2073	21
22	39·964	22·592	14·072	9·1058	5·9295	22
23	38·897	21·918	13·596	8·7490	5·6551	23
24	37·834	21·248	13·122	8·3948	5·3837	24
25	36·771	20·578	12·650	8·0434	5·1154	25
26	35·714	19·912	12·181	7·6948	4·8507	26
27	34·659	19·249	11·715	7·3497	4·5897	27
28	33·605	18·588	11·252	7·0076	4·3325	28
29	32·556	17·930	10·791	6·6691	4·0796	29
30	31·508	17·275	10·335	6·3346	3·8315	30
31	30·464	16·623	9·8812	6·0043	3·5881	31
32	29·425	15·976	9·4324	5·6790	3·3504	32
33	28·390	15·332	8·9881	5·3587	3·1185	33
34	27·361	14·694	8·5489	5·0439	2·8927	34
35	26·338	14·062	8·1158	4·7358	2·6738	35
36	25·322	13·434	7·6883	4·4337	2·4618	36
37	24·314	12·815	7·2681	4·1390	2·2575	37
38	23·314	12·203	6·8550	3·8519	2·0611	38
39	22·323	11·599	6·4500	3·5730	1·8729	39
40	21·344	11·004	6·0537	3·3027	1·6934	40
41	20·375	10·418	5·6663	3·0414	1·5229	41
42	19·418	9·8433	5·2886	2·7897	1·3617	42
43	18·475	9·2788	4·9211	2·5478	1·2099	43
44	17·545	8·7261	4·5644	2·3163	1·0678	44
45	16·630	8·1856	4·2190	2·0955	·93558	45
46	15·730	7·6581	3·8854	1·8858	...	46
47	14·847	7·1440	3·5641	1·6874	...	47
48	13·982	6·6442	3·2555	1·5006	...	48
49	13·135	6·1596	2·9603	1·3257		49
50	12·307	5·6905	2·6788	1·1628	...	50
51	11·499	5·2375	2·4113	...	...	51
52	10·712	4·8015	2·1584	...	...	52
53	9·9488	4·3833	1·9202	...	...	53
54	9·2083	3·9832	1·6971	...	...	54
55	8·4928	3·6020	1·4893	...	...	55
56	7·8028	3·2402	...	...	...	56
57	7·1399	2·8983	...	...	...	57
58	6·5014	2·5766	...	...	...	58
59	5·8977	2·2756	...	...	...	59
60	5·3208	1·9955	...	...	...	60
61	4·7745	...	...	...	...	61
62	4·2595	...	...	...	...	62
63	3·7764	...	...	...	...	63
64	3·3255	...	...	...	...	64
65	2·9074	...	...	...	...	65

O^{M} $2\frac{3}{4}$ per-cent. **Log ${}_{t}k_{x}$**

LOG OF CORRECTION-NUMERATOR *for*

WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS.

Age at Entry	Number of Payments					Age at Entry
	10	15	20	25	30	
20	4·54207	4·29537	4·09143	3·90495	3·72257	20
21	·53068	·28270	·07712	·88818	·70323	21
22	·51906	·26974	·06243	·87152	·68321	22
23	·50716	·25645	·04733	·85403	·66248	23
24	·49498	·24280	·03178	·83593	·64096	24
25	·48218	·22878	·01575	·81722	·61861	25
26	·46966	·21435	3·99921	·79784	·59536	26
27	·45649	·19951	·98212	·77776	·57118	27
28	·44295	·18418	·96446	·75690	·54597	28
29	·42903	·16841	·94618	·73527	·51969	29
30	·41469	·15211	·92725	·71276	·49223	30
31	·39992	·13526	·90762	·68934	·46356	31
32	·38470	·11785	·88728	·66498	·43359	32
33	·36900	·09986	·86617	·63959	·40221	33
34	·35283	·08125	·84427	·61313	·36938	34
35	·33614	·06198	·82151	·58554	·33498	35
36	·31891	·04204	·79786	·55675	·29889	36
37	·30109	·02137	·77327	·52668	·26100	37
38	·28272	3·99996	·74769	·49525	·22122	38
39	·26371	·97777	·72107	·46240	·17938	39
40	·24407	·95473	·69333	·42801	·13536	40
41	·22375	·93082	·66442	·39199	·08899	41
42	·20271	·90599	·63428	·35423	·04008	42
43	·18093	·88018	·60280	·31461	2·98845	43
44	·15835	·85333	·56992	·27298	·93387	44
45	·13492	·82538	·53553	·22921	·87612	45
46	·11062	·79627	·49954	·18315	...	46
47	·08536	·76591	·46182	·13457	...	47
48	·05910	·73421	·42227	·08333	...	48
49	·03178	·70111	·38072	·02918	...	49
50	·00331	·66649	·33706	2·97191	...	50
51	3·97363	·63024	·29110	...	...	51
52	·94268	·59226	·24267	...	...	52
53	·91034	·55244	·19159	...	...	53
54	·87655	·51062	·13762	...	...	54
55	·84119	·46666	·08056	...	...	55
56	·80414	·42039	...	...	...	56
57	·76532	·37165	...	...	...	57
58	·72459	·32026	...	...	...	58
59	·68180	·26596	...	...	...	59
60	·63680	·20857	...	...	...	60
61	·58946	...	...	...	...	61
62	·53956	...	...	...	...	62
63	·48694	...	...	...	...	63
64	·43136	...	...	...	...	64
65	·37264	...	...	...	...	65

O^M $2\frac{3}{4}$ per-cent. $\frac{{}_t k_x}{1000}$

CORRECTION-NUMERATOR *for*
WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS
DIVIDED BY 1000.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	34·839	19·741	12·343	8·0343	5·2792	20
21	33·938	19·173	11·943	7·7354	5·0493	21
22	33·042	18·610	11·546	7·4391	4·8218	22
23	32·148	18·049	11·151	7·1455	4·5971	23
24	31·259	17·490	10·759	6·8538	4·3748	24
25	30·372	16·935	10·369	6·5648	4·1554	25
26	29·489	16·381	9·9818	6·2783	3·9388	26
27	28·608	15·831	9·5967	5·9946	3·7255	27
28	27·730	15·282	9·2143	5·7135	3·5154	28
29	26·855	14·737	8·8345	5·4359	3·3089	29
30	25·983	14·194	8·4577	5·1613	3·1062	30
31	25·114	13·654	8·0839	4·8904	2·9078	31
32	24·249	13·117	7·7140	4·6236	2·7139	32
33	23·388	12·585	7·3480	4·3610	2·5247	33
34	22·534	12·057	6·9867	4·1033	2·3409	34
35	21·684	11·534	6·6299	3·8507	2·1626	35
36	20·841	11·016	6·2786	3·6037	1·9902	36
37	20·003	10·504	5·9329	3·3626	1·8239	37
38	19·174	9·9991	5·5936	3·1279	1·6643	38
39	18·353	9·5010	5·2610	2·9000	1·5114	39
40	17·542	9·0101	4·9355	2·6792	1·3657	40
41	16·740	8·5275	4·6176	2·4660	1·2274	41
42	15·948	8·0536	4·3080	2·2606	1·0967	42
43	15·168	7·5889	4·0068	2·0635	·97376	43
44	14·400	7·1339	3·7147	1·8749	·85876	44
45	13·643	6·6893	3·4319	1·6952	·75183	45
46	12·901	6·2556	3·1589	1·5246	...	46
47	12·172	5·8332	2·8961	1·3632	...	47
48	11·458	5·4226	2·6441	1·2115	...	48
49	10·759	5·0247	2·4028	1·0695	...	49
50	10·077	4·6397	2·1730	·93737	...	50
51	9·4109	4·2682	1·9548	...	...	51
52	8·7635	3·9107	1·7485	...	...	52
53	8·1347	3·5681	1·5545	...	...	53
54	7·5258	3·2406	1·3728	...	...	54
55	6·9373	2·9286	1·2038	...	...	55
56	6·3700	2·6326	...	...	...	56
57	5·8253	2·3532	...	...	...	57
58	5·3038	2·0905	...	...	...	58
59	4·8062	1·8448	...	...	...	59
60	4·3331	1·6165	...	...	...	60
61	3·8856	...	...	...	...	61
62	3·4639	...	...	...	...	62
63	3·0686	...	...	...	...	63
64	2·7000	...	...	...	...	64
65	2·3585	...	...	...	...	65

O^{M} 3 per-cent. **Log $_{t}k_{x}$**

LOG OF CORRECTION-NUMERATOR *for*

WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS.

Age at Entry	Number of Payments					Age at Entry
	10	15	20	25	30	
20	4·46118	4·21279	4·00702	3·81860	3·63413	20
21	·44961	·19993	3·99253	·80195	·61460	21
22	·43779	·18679	·97767	·78482	·59440	22
23	·42573	·17333	·96240	·76715	·57348	23
24	·41335	·15950	·94668	·74888	·55177	24
25	·40069	·14532	·93048	·72999	·52924	25
26	·38770	·13072	·91378	·71045	·50581	26
27	·37436	·11571	·89653	·69019	·48143	27
28	·36064	·10022	·87869	·66916	·45602	28
29	·34654	·08427	·86023	·64733	·42954	29
30	·33204	·06779	·84113	·62463	·40188	30
31	·31708	·05078	·82133	·60102	·37299	31
32	·30170	·03321	·80081	·57647	·34279	32
33	·28583	·01504	·77951	·55088	·31120	33
34	·26947	3·99624	·75741	·52422	·27812	34
35	·25261	·97681	·73448	·49644	·24348	35
36	·23519	·95667	·71063	·46742	·20713	36
37	·21722	·93583	·68585	·43712	·16898	37
38	·19866	·91423	·66007	·40547	·12892	38
39	·17949	·89186	·63326	·37239	·08682	39
40	·15966	·86863	·60531	·33776	·04249	40
41	·13917	·84454	·57621	·30150	2·99582	41
42	·11796	·81953	·54584	·26348	·94661	42
43	·09599	·79352	·51415	·22360	·89464	43
44	·07323	·76648	·48104	·18169	·83972	44
45	·04963	·73834	·44642	·13764	·78162	45
46	·02513	·70902	·41019	·09127	...	46
47	3·99968	·67843	·37221	·04240	...	47
48	·97323	·61653	·33240	2·99083	...	48
49	·94571	·61320	·29059	·93635	...	49
50	·91706	·57836	·24665	·87872	...	50
51	·88717	·54187	·20039	...	...	51
52	·85598	·50364	·15166	...	...	52
53	·82344	·46355	·10025	...	...	53
54	·78941	·42146	·04596	...	...	54
55	·75381	·37721	2·98853	...	...	55
56	·71652	·33065	...	...	...	56
57	·67743	·28160	...	...	...	57
58	·63642	·22986	...	...	...	58
59	·59334	·17523	...	...	...	59
60	·54804	·11746	...	...	...	60
61	·50037	...	...	...	...	61
62	·45015	...	...	...	...	62
63	·39716	...	...	...	...	63
64	·34123	...	...	...	...	64
65	·28210	...	...	...	...	65

O^M 3 per-cent. $\frac{{}_t\mathrm{k}_x}{1000}$

CORRECTION-NUMERATOR *for* WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS DIVIDED BY 1000.

Age at Entry	Number of Payments: 10	15	20	25	30	Age at Entry
20	28·919	16·323	10·163	6·5857	4·3066	20
21	28·159	15·846	9·8295	6·3380	4·1172	21
22	27·402	15·374	9·4988	6·0928	3·9301	22
23	26·652	14·905	9·1706	5·8499	3·7452	23
24	25·903	14·438	8·8446	5·6089	3·5626	24
25	25·159	13·974	8·5208	5·3702	3·3825	25
26	24·417	13·512	8·1994	5·1339	3·2049	26
27	23·679	13·053	7·8801	4·8999	3·0299	27
28	22·942	12·596	7·5629	4·6683	2·8577	28
29	22·210	12·141	7·2482	4·4395	2·6887	29
30	21·480	11·689	6·9363	4·2134	2·5228	30
31	20·753	11·240	6·6272	3·9904	2·3604	31
32	20·031	10·795	6·3214	3·7711	2·2019	32
33	19·312	10·352	6·0188	3·5553	2·0474	33
34	18·598	9·9138	5·7202	3·3436	1·8972	34
35	17·890	9·4800	5·4260	3·1365	1·7518	35
36	17·187	9·0504	5·1361	2·9337	1·6111	36
37	16·490	8·6264	4·8512	2·7360	1·4756	37
38	15·800	8·2079	4·5716	2·5437	1·3456	38
39	15·118	7·7958	4·2979	2·3572	1·2213	39
40	14·443	7·3898	4·0300	2·1765	1·1028	40
41	13·777	6·9910	3·7689	2·0022	·99042	41
42	13·121	6·5998	3·5143	1·8343	·88432	42
43	12·474	6·2161	3·2670	1·6734	·78459	43
44	11·837	5·8409	3·0272	1·5195	·69139	44
45	11·211	5·4744	2·7952	1·3729	·60481	45
46	10·596	5·1171	2·5715	1·2339	...	46
47	9·9926	4·7690	2·3562	1·1026	...	47
48	9·4022	4·4313	2·1498	·97911	...	48
49	8·8249	4·1039	1·9525	·86367	...	49
50	8·2615	3·7876	1·7646	·75635	...	50
51	7·7121	3·4823	1·5863	...	...	51
52	7·1776	3·1889	1·4179	...	...	52
53	6·6595	2·9077	1·2597	...	...	53
54	6·1576	2·6391	1·1116	...	...	54
55	5·6730	2·3835	·97394	...	...	55
56	5·2062	2·1412	...	...	...	56
57	4·7581	1·9125	...	...	...	57
58	4·3293	1·6977	...	...	...	58
59	3·9205	1·4970	...	...	...	59
60	3·5322	1·3106	...	...	...	60
61	3·1650	...	...	...	...	61
62	2·8194	...	...	...	...	62
63	2·4955	...	...	...	...	63
64	2·1940	...	...	...	...	64
65	1·9147	...	...	...	...	65

O^{M} 3½ per-cent. **Log $_{t}k_{x}$**

Log of Correction-Numerator *for*

WHOLE-LIFE ASSURANCE by LIMITED PAYMENTS.

Age at Entry	Number of Payments					Age at Entry
	10	15	20	25	30	
20	4·30368	4·05165	3·84201	3·64943	3·46049	20
21	·29162	·03832	·82705	·63231	·44018	21
22	·27933	·02472	·81174	·61473	·41982	22
23	·26680	·01080	·79602	·59660	·39844	23
24	·25396	3·99653	·77985	·57789	·37627	24
25	·24083	·98189	·76322	·55855	·35326	25
26	·22739	·96686	·74607	·53857	·32937	26
27	·21361	·95141	·72840	·51787	·30452	27
28	·19943	·93549	·71010	·49638	·27861	28
29	·18491	·91910	·69122	·47410	·25165	29
30	·16994	·90219	·67166	·45094	·22349	30
31	·15455	·88473	·65143	·42687	·19408	31
32	·13872	·86672	·63046	·40184	·16336	32
33	·12241	·84810	·60872	·37577	·13124	33
34	·10561	·82887	·58617	·34862	·09761	34
35	·08831	·80899	·56276	·32033	·06239	35
36	·07046	·78843	·53847	·29081	·02547	36
37	·05204	·76714	·51321	·25999	2·98673	37
38	·03303	·74509	·48695	·22783	·94605	38
39	·01343	·72228	·45967	·19422	·90332	39
40	3·99317	·69860	·43124	·15903	·85835	40
41	·97227	·67408	·40165	·12221	·81102	41
42	·95063	·64861	·37079	·08364	·76112	42
43	·92822	·62214	·33859	·04314	·70844	43
44	·90504	·59465	·30498	·00065	·65281	44
45	·88099	·56604	·26982	2·95596	·59394	45
46	·85607	·53625	·23305	·90896	...	46
47	·83019	·50519	·19454	·85942	...	47
48	·80329	·47280	·15415	·80716	...	48
49	·77533	·43898	·11176	·75198	...	49
50	·74620	·40360	·06719	·69360	...	50
51	·71585	·36659	·02032	...	...	51
52	·68418	·32782	2·97092	...	...	52
53	·65113	·28715	·91882	...	...	53
54	·61660	·24447	·86383	...	...	54
55	·58046	·19960	·80565	...	...	55
56	·54261	·15240	...	...	...	56
57	·50297	·10269	...	...	...	57
58	·46136	·05025	...	...	...	58
59	·41765	2·99488	...	...	...	59
60	·37170	·93634	...	...	...	60
61	·32337	...	...	...	...	61
62	·27243	...	...	...	...	62
63	·21873	...	...	...	...	63
64	·16202	...	...	...	...	64
65	·10209	...	...	...	...	65

O^M $3\frac{1}{2}$ per-cent. $\frac{{}_tk_r}{1000}$

CORRECTION-NUMERATOR *for*
WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS
DIVIDED BY 1000.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	20·122	11·263	6·9504	4·4610	2·8873	20
21	19·571	10·922	6·7151	4·2885	2·7573	21
22	19·025	10·586	6·4825	4·1184	2·6292	22
23	18·484	10·252	6·2520	3·9500	2·5029	23
24	17·946	9·9204	6·0235	3·7835	2·3783	24
25	17·411	9·5916	5·7972	3·6187	2·2556	25
26	16·881	9·2653	5·5728	3·4560	2·1349	26
27	16·353	8·9415	5·3506	3·2951	2·0161	27
28	15·828	8·6197	5·1298	3·1360	1·8994	28
29	15·308	8·3004	4·9116	2·9792	1·7850	29
30	14·789	7·9834	4·6953	2·8245	1·6730	30
31	14·274	7·6688	4·4816	2·6722	1·5634	31
32	13·763	7·3573	4·2703	2·5226	1·4567	32
33	13·256	7·0486	4·0618	2·3756	1·3528	33
34	12·753	6·7433	3·8563	2·2316	1·2520	34
35	12·255	6·4415	3·6539	2·0909	1·1545	35
36	11·761	6·1437	3·4552	1·9535	1·0604	36
37	11·273	5·8498	3·2599	1·8197	·96991	37
38	10·790	5·5602	3·0687	1·6898	·88318	38
39	10·314	5·2757	2·8818	1·5639	·80042	39
40	9·8440	4·9957	2·6992	1·4422	·72169	40
41	9·3815	4·7215	2·5214	1·3250	·64717	41
42	8·9254	4·4526	2·3485	1·2124	·57693	42
43	8·4766	4·1893	2·1807	1·1044	·51102	43
44	8·0360	3·9323	2·0183	1·0015	·44958	44
45	7·6031	3·6816	1·8613	·90357	·39259	45
46	7·1791	3·4376	1·7102	·81089	...	46
47	6·7638	3·2003	1·5651	·72347	...	47
48	6·3576	2·9703	1·4261	·64145	...	48
49	5·9611	2·7478	1·2935	·56491	...	49
50	5·5744	2·5328	1·1673	·49386	...	50
51	5·1982	2·3259	1·0479	...	...	51
52	4·8326	2·1273	·93523	...	...	52
53	4·4785	1·9371	·82951	...	...	53
54	4·1362	1·7558	·73085	...	...	54
55	3·8059	1·5834	·63922	...	...	55
56	3·4883	1·4204	...	...	...	56
57	3·1840	1·2667	...	...	...	57
58	2·8931	1·1227	...	...	...	58
59	2·6161	·98828	...	...	...	59
60	2·3534	·86365	...	...	...	60
61	2·1056	...	...	...	...	61
62	1·8725	...	...	...	...	62
63	1·6547	...	...	...	...	63
64	1·4522	...	...	...	...	64
65	1·2650	...	...	...	...	65

O^{M} 4 per-cent. **Log ${}_{t}k_{x}$**

LOG OF CORRECTION-NUMERATOR *for*

WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	4·15177	3·89579	3·68193	3·48186	3·29112	20
21	·13905	·88184	·66637	·46714	·27050	21
22	·12614	·86762	·65046	·44896	·24923	22
23	·11298	·85310	·63415	·43025	·22724	23
24	·09955	·83825	·61740	·41096	·20449	24
25	·08582	·82302	·60019	·39105	·18090	25
26	·07179	·80741	·58247	·37049	·15641	26
27	·05741	·79140	·56123	·34922	·13097	27
28	·04268	·77491	·54540	·32716	·10447	28
29	·02756	·75795	·52594	·30430	·07688	29
30	·01203	·74048	·50584	·28059	·04812	30
31	3·99607	·72247	·48504	·25593	·01811	31
32	·97967	·70389	·46350	·23031	2·98675	32
33	·96281	·68473	·44119	·20365	·95399	33
34	·94544	·66194	·41808	·17591	·91972	34
35	·92760	·64153	·39414	·14703	·88386	35
36	·90918	·62340	·36925	·11690	·84625	36
37	·89022	·60156	·34343	·08548	·80682	37
38	·87069	·57897	·31661	·05269	·76546	38
39	·85054	·55561	·28875	·01846	·72202	39
40	·82976	·53141	·25975	2·98265	·67633	40
41	·80831	·50633	·22958	·94518	·62826	41
42	·78615	·48032	·19814	·90594	·57759	42
43	·76321	·45331	·16535	·86479	·52414	43
44	·73951	·42527	·13114	·82162	·46770	44
45	·71496	·39611	·09540	·77624	·40802	45
46	·68951	·36578	·05801	·72852	...	46
47	·66309	·33416	·01886	·67824	...	47
48	·63568	·30121	2·97783	·62525	...	48
49	·60718	·26681	·93479	·56927	...	49
50	·57752	·23086	·88955	·51011	...	50
51	·54663	·19324	·84198	...	...	51
52	·51442	·15386	·79188	...	...	52
53	·48080	·11255	·73904	...	...	53
54	·44569	·06923	·68327	...	...	54
55	·40897	·02369	·62430	...	...	55
56	·37050	2·97579	...	...	...	56
57	·33023	·92535	...	...	...	57
58	·28797	·87217	...	...	...	58
59	·24360	·81601	...	...	...	59
60	·19696	·75668	...	...	...	60
61	·14789	...	...	...	...	61
62	·09621	...	...	...	...	62
63	·04172	...	...	...	...	63
64	2·98419	...	...	...	...	64
65	·92344	...	...	...	...	65

0^{M} 4 per-cent. $\frac{{}_{t}\mathrm{k}_{r}}{1000}$

CORRECTION-NUMERATOR *for*
WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS
DIVIDED BY 1000.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	14·183	7·8667	4·8076	3·0539	1·9549	20
21	13·774	7·6180	4·6384	2·9318	1·8642	21
22	13·370	7·3726	4·4716	2·8116	1·7751	22
23	12·971	7·1302	4·3068	2·6931	1·6875	23
24	12·576	6·8905	4·1438	2·5761	1·6014	24
25	12·185	6·6530	3·9828	2·4607	1·5167	25
26	11·798	6·4182	3·8236	2·3469	1·4335	26
27	11·413	6·1859	3·6663	2·2347	1·3520	27
28	11·033	5·9554	3·5108	2·1240	1·2719	28
29	10·655	5·7273	3·3569	2·0151	1·1937	29
30	10·281	5·5015	3·2051	1·9081	1·1172	30
31	9·9099	5·2780	3·0552	1·8027	1·0426	31
32	9·5427	5·0570	2·9074	1·6995	·96995	32
33	9·1793	4·8387	2·7618	1·5983	·89948	33
34	8·8194	4·6232	2·6187	1·4994	·83123	34
35	8·4645	4·4109	2·4782	1·4029	·76535	35
36	8·1130	4·2015	2·3402	1·3089	·70183	36
37	7·7664	3·9954	2·2051	1·2175	·64094	37
38	7·4249	3·7929	2·0731	1·1290	·58272	38
39	7·0883	3·5943	1·9442	1·0434	·52725	39
40	6·7571	3·3995	1·8187	·96084	·47460	40
41	6·4315	3·2087	1·6966	·88141	·42487	41
42	6·1115	3·0222	1·5781	·80527	·37809	42
43	5·7971	2·8399	1·4634	·73247	·33430	43
44	5·4892	2·6624	1·3525	·66316	·29356	44
45	5·1875	2·4895	1·2457	·59737	·25587	45
46	4·8923	2·3216	1·1429	·53520	...	46
47	4·6035	2·1585	1·0444	·47669	...	47
48	4·3220	2·0008	·95023	·42194	...	48
49	4·0474	1·8485	·86058	·37091	...	49
50	3·7802	1·7016	·77544	·32368	...	50
51	3·5207	1·5604	·69499	...	...	51
52	3·2690	1·4251	·61927	...	...	52
53	3·0255	1·2958	·54833	...	...	53
54	2·7906	1·1728	·48225	...	...	54
55	2·5643	1·0561	·42102	...	...	55
56	2·3469	·94578	...	...	...	56
57	2·1391	·84207	..	...	...	57
58	1·9408	·74502	...	...	...	58
59	1·7523	·65465	...	...	...	59
60	1·5738	·57106	...	...	...	60
61	1·4057	...	...	...	...	61
62	1·2480	...	...	...	..	62
63	1·1008	...	...	...	.	63
64	·96425	...	...	...	..	64
65	·83838	...	...	...	...	65

0^{M} $2\frac{1}{2}$ per-cent. Log $k_{x\overline{t|}}$

Log of Correction-Numerator *for* ENDOWMENT ASSURANCE

secured by Annual Premium payable throughout the duration of the Assurance.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	4·42629	4·24593	4·06005	3·85928	3 63259	20
21	·43888	·25579	·06808	·86604	·63847	21
22	·45219	·26614	·07616	·87307	·64457	22
23	·46626	·27699	·08518	·88035	·65087	23
24	·48117	·28837	·09428	·88791	·65738	24
25	·49701	·30035	·10379	·89577	·66413	25
26	·51386	·31296	·11372	·90393	·67109	26
27	·53184	·32625	·12411	·91241	·67831	27
28	·55107	·34028	·13496	·92122	·68576	28
29	·57174	·35514	·14636	·93040	·69350	29
30	·59402	·37090	·15832	·93997	·70151	30
31	·61812	·38765	·17088	·94993	·70980	31
32	·64436	·40552	·18413	·96035	·71842	32
33	·67307	·42463	·19810	·97124	·72736	33
34	·70168	·44514	·21287	·98263	·73666	34
35	·73980	·46725	·22854	·99460	·74634	35
36	·77916	·49117	·24518	4·00716	·75643	36
37	·82379	·51720	·26292	·02037	·76695	37
38	·87513	·54569	·28190	·03432	·77795	38
39	·93530	·57709	·30227	·04907	·78947	39
40	5·00755	·61194	·32421	·06469	·80154	40
41	...	·65104	·34797	·08130	·81422	41
42	...	·69537	·37381	·09898	·82755	42
43	...	·74639	·40208	·11788	·84160	43
44	...	·80619	·43323	·13815	·85643	44
45	...	·87803	·46780	·15996	·87212	45
46	...	...	·50656	·18356	·88877	46
47	...	...	·55052	·20920	·90647	47
48	...	...	·60111	·23723	·92536	48
49	...	...	·66012	·26807	·94557	49
50	...	...	·73171	·30229	·96729	50
51	...	...	...	·34061	·99071	51
52	...	...	...	·38404	4·01611	52
53	...	...	...	·43400	·04381	53
54	...	...	...	·49259	·07425	54
55	...	...	...	·56301	·10793	55
56	...	...	...	...	·14561	56
57	...	...	...	...	·18828	57
58	...	...	...	...	·23730	58
59	...	...	...	...	·29476	59
60	...	...	...	...	·36387	60

For ENDOWMENT ASSURANCE by Single Premium.

Log $K_{x\overline{t|}}$ Log $K_{x\overline{t|}}$

At all Ages	4·06444	3·93275	3·78319	3·60918	3·40255	At all Ages

O[M] 2½ per-cent. $\dfrac{k_{x\overline{t|}}}{1000}$

CORRECTION-NUMERATOR *for*
ENDOWMENT ASSURANCE
SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE—DIVIDED BY 1000.

Age at Entry	Age at Maturity					Age at Entry
	45	50	55	60	65	
20	26·686	17·617	11·483	7·2324	4·2913	20
21	27·471	18·021	11·697	7·3458	4·3498	21
22	28·326	18·456	11·925	7·4657	4·4113	22
23	29·259	18·923	12·167	7·5919	4·4758	23
24	30·281	19·425	12·425	7·7252	4·5134	24
25	31·406	19·959	12·700	7·8663	4·6146	25
26	32·648	20·557	12·993	8·0155	4·6891	26
27	34·028	21·196	13·308	8·1735	4·7677	27
28	35·569	21·892	13·645	8·3410	4·8502	28
29	37·303	22·654	14·007	8·5192	4·9374	29
30	39·266	23·491	14·399	8·7090	5·0293	30
31	41·507	24·415	14·821	8·9111	5·1263	31
32	44·092	25·440	15·280	9·1275	5·2290	32
33	47·105	26·585	15·780	9·3592	5·3378	33
34	50·662	27·870	16·326	9·6079	5·4533	34
35	54·929	29·326	16·925	9·8764	5·5762	35
36	60·140	30·986	17·587	10·166	5·7073	36
37	66·648	32·900	18·320	10·480	5·8472	37
38	75·012	35·131	19·138	10·822	5·9972	38
39	86·159	37·765	20·057	11·196	6·1584	39
40	101·754	40·920	21·096	11·606	6·3320	40
41	...	44·775	22·283	12·059	6·5196	41
42	...	49·587	23·649	12·560	6·7228	42
43	...	55·769	25·239	13·118	6·9438	43
44	...	64·001	27·116	13·745	7·1851	44
45	...	75·514	29·363	14·453	7·4494	45
46	...	...	32·104	15·260	7·7405	46
47	...	...	35·524	16·188	8·0625	47
48	...	...	39·913	17·268	8·4209	48
49	...	...	45·753	18·538	8·8221	49
50	...	...	53·915	20·058	9·2745	50
51	...	...	...	21·908	9·7884	51
52	...	...	...	24·213	10·378	52
53	...	...	...	27·164	11·061	53
54	...	...	...	31·088	11·865	54
55	...	...	...	36·560	12·821	55
56	...	...	...	...	13·983	56
57	...	...	...	...	15·427	57
58	...	...	...	...	17·270	58
59	...	...	...	...	19·713	59
60	...	...	...	...	23·111	60

$\dfrac{K_{x\overline{t|}}}{1000}$ FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM. $\dfrac{K_{x\overline{t|}}}{1000}$

At all Ages	11·600	8·5655	6·0700	4·0689	2·7267	At all Ages

O^M $2\frac{3}{4}$ per-cent. **Log $k_{x\overline{t|}}$**

LOG OF CORRECTION-NUMERATOR *for*
ENDOWMENT ASSURANCE
SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	4·37696	4·19458	4·00653	3·80334	3·57397	20
21	·38916	·20408	·01422	·80979	·57956	21
22	·40205	·21404	·02223	·81647	·58534	22
23	·41572	·22451	·03061	·82343	·59133	23
24	·43021	·23551	·03935	·83065	·59753	24
25	·44563	·24710	·04849	·83817	·60396	25
26	·46206	·25930	·05805	·84598	·61061	26
27	·47962	·27220	·06806	·85412	·61750	27
28	·49842	·28583	·07855	·86258	·62463	28
29	·51866	·30028	·08956	·87141	·63203	29
30	·54049	·31563	·10114	·88061	·63971	30
31	·56415	·33196	·11332	·89022	·64767	31
32	·58993	·34940	·12616	·90027	·65594	32
33	·61818	·36808	·13973	·91078	·66454	33
34	·64934	·38816	·15410	·92181	·67349	34
35	·68399	·40983	·16936	·93339	·68283	35
36	·72287	·43332	·18559	·94557	·69256	36
37	·76703	·45891	·20292	·95840	·70273	37
38	·81788	·48694	·22148	·97196	·71337	38
39	·87756	·51788	·24142	·98631	·72452	39
40	·94932	·55228	·26294	4·00154	·73622	40
41	...	·59089	·28625	·01773	·74852	41
42	...	·63476	·31165	·03501	·76148	42
43	...	·68528	·33947	·05348	·77513	43
44	...	·74460	·37016	·07334	·78959	44
45	...	·81696	·40428	·09474	·80489	45
46	...	...	·44258	·11790	·82114	46
47	...	...	·48607	·14310	·83845	47
48	...	...	·53617	·17068	·85692	48
49	...	...	·59500	·20108	·87673	49
50	...	...	·66580	·23485	·89803	50
51	...	...	...	·27271	·92103	51
52	...	...	...	·31568	·94601	52
53	...	...	...	·36517	·97328	53
54	...	...	...	·42328	4·00328	54
55	...	...	...	·49322	·03652	55
56	...	...	...	...	·07376	56
57	...	...	...	...	·11597	57
58	...	...	...	...	·16453	58
59	...	...	...	...	·22152	59
60	...	...	...	...	·29016	60

FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM.

Log $K_{x\overline{t|}}$ **Log $K_{x\overline{t|}}$**

At all Ages	4·01151	3·90899	3·75559	3·57802	3·36719	At all Ages

0^{M} $2\frac{3}{4}$ per-cent. $\frac{k_{x\overline{t|}}}{1000}$

CORRECTION-NUMERATOR *for* ENDOWMENT ASSURANCE

SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE—DIVIDED BY 1000.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	23·821	15·652	10·151	6·3583	3·7495	20
21	24·500	15·999	10·333	6·4534	3·7980	21
22	25·238	16·370	10·525	6·5535	3·8489	22
23	26·045	16·769	10·730	6·6593	3·9024	23
24	26·928	17·199	10·918	6·7710	3·9585	24
25	27·902	17·664	11·181	6·8892	4·0175	25
26	28·977	18·168	11·430	7·0142	4·0795	26
27	30·173	18·715	11·697	7·1469	4·1448	27
28	31·508	19·312	11·983	7·2875	4·2134	28
29	33·011	19·965	12·290	7·4372	4·2858	29
30	34·713	20·684	12·622	7·5964	4·3622	30
31	36·656	21·476	12·981	7·7664	4·4429	31
32	38·898	22·356	13·371	7·9482	4·5284	32
33	41·513	23·339	13·795	8·1429	4·6189	33
34	44·601	24·413	14·259	8·3524	4·7151	34
35	48·305	25·694	14·769	8·5781	4·8176	35
36	52·829	27·122	15·332	8·8221	4·9267	36
37	58·483	28·768	15·956	9·0866	5·0435	37
38	65·748	30·686	16·653	9·3748	5·1686	38
39	75·433	32·952	17·435	9·6897	5·3030	39
40	88·986	35·668	18·321	10·036	5·4478	40
41	...	38·984	19·331	10·417	5·6043	41
42	...	43·128	20·495	10·840	5·7740	42
43	...	48·448	21·851	11·310	5·9584	43
44	...	55·539	23·451	11·840	6·1601	44
45	...	65·608	25·368	12·438	6·3810	45
46	...	...	27·706	13·119	6·6243	46
47	...	...	30·625	13·903	6·8937	47
48	...	...	34·369	14·814	7·1932	48
49	...	...	39·355	15·888	7·5289	49
50	...	...	46·323	17·173	7·9073	50
51	...	...	...	18·737	8·3374	51
52	...	...	...	20·686	8·8310	52
53	...	...	...	23·183	9·4033	53
54	...	...	...	26·502	10·076	54
55	...	...	...	31·133	10·877	55
56	...	...	...	...	11·851	56
57	...	...	...	...	13·061	57
58	...	...	...	...	14·606	58
59	...	...	...	...	16·654	59
60	...	...	...	...	19·506	60

$\frac{K_{x\overline{t|}}}{1000}$ FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM. $\frac{K_{x\overline{t|}}}{1000}$

At all Ages	11·079	8·1093	5·6962	3·7846	2·3291	At all Ages

0^M 3 per-cent. **Log $k_{x\overline{t|}}$**

LOG OF CORRECTION-NUMERATOR *for*
ENDOWMENT ASSURANCE
SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	4·32783	4·14336	3·95303	3·74735	3·51521	20
21	·33963	·15249	·96037	·75348	·52051	21
22	·35215	·16210	·96806	·75987	·52601	22
23	·36541	·17219	·97608	·76649	·53170	23
24	·37950	·18282	·98448	·77339	·53760	24
25	·39451	·19402	·99327	·78058	·54372	25
26	·41053	·20585	4·00247	·78807	·55007	26
27	·42767	·21836	·01213	·79588	·55665	27
28	·44605	·23160	·02225	·80400	·56347	28
29	·46585	·24564	·03289	·81248	·57055	29
30	·48725	·26059	·04409	·82134	·57791	30
31	·51048	·27652	·05589	·83060	·58555	31
32	·53581	·29355	·06836	·84029	·59349	32
33	·56362	·31181	·08154	·85046	·60176	33
34	·59432	·33146	·09552	·86112	·61038	34
35	·62850	·35270	·11037	·87233	·61937	35
36	·66693	·37576	·12621	·88414	·62877	36
37	·71060	·40090	·14313	·89659	·63858	37
38	·76099	·42849	·16127	·90977	·64888	38
39	·82017	·45897	·18079	·92373	·65967	39
40	·89144	·49291	·20188	·93856	·67101	40
41	...	·53107	·22477	·95436	·68295	41
42	...	·57446	·24973	·97124	·69554	42
43	...	·62451	·27711	·98932	·70883	43
44	...	·68334	·30736	4·00876	·72290	44
45	...	·75421	·34102	·02973	·73781	45
46	...	...	·37886	·05248	·75369	46
47	...	...	·42188	·07725	·77059	47
48	...	...	·47152	·10140	·78869	48
49	...	...	·52987	·13436	·80809	49
50	...	...	·60018	·16768	·82898	50
51	...	...	...	·20509	·85157	51
52	...	...	...	·24760	·87613	52
53	...	...	...	·29663	·90298	53
54	...	...	...	·35425	·93254	54
55	...	...	...	·42372	·96536	55
56	...	...	...	...	4·00215	56
57	...	...	...	...	·04391	57
58	...	...	...	...	·09202	58
59	...	...	...	...	·14855	59
60	...	...	...	...	·21671	60

FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM.

Log $K_{x\overline{t|}}$ **Log $K_{x\overline{t|}}$**

At all Ages	4·02133	3·88194	3·72468	3·54324	3·32848	At all Ages

O^M 3 per-cent. $\frac{k_{x\overline{t}|}}{1000}$

CORRECTION-NUMERATOR *for*
ENDOWMENT ASSURANCE
SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE—DIVIDED BY 1000.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	21·273	13·911	8·9749	5·5892	3·2750	20
21	21·859	14·207	9·1279	5·6687	3·3152	21
22	22·498	14·524	9·2909	5·7527	3·3575	22
23	23·196	14·866	9·4641	5·8410	3·4017	23
24	23·961	15·234	9·6489	5·9346	3·4483	24
25	24·803	15·632	9·8462	6·0336	3·4972	25
26	25·735	16·061	10·057	6·1386	3·5487	26
27	26·771	16·533	10·283	6·2500	3·6029	27
28	27·929	17·045	10·526	6·3680	3·6599	28
29	29·231	17·605	10·787	6·4935	3·7201	29
30	30·708	18·222	11·069	6·6274	3·7836	30
31	32·395	18·903	11·373	6·7702	3·8508	31
32	34·341	19·658	11·705	6·9229	3·9218	32
33	36·612	20·503	12·065	7·0870	3·9972	33
34	39·293	21·452	12·460	7·2631	4·0774	34
35	42·511	22·527	12·893	7·4530	4·1627	35
36	46·444	23·755	13·372	7·6584	4·2537	36
37	51·357	25·171	13·904	7·8812	4·3509	37
38	57·675	26·822	14·497	8·1240	4·4553	38
39	66·095	28·772	15·163	8·3894	4·5674	39
40	77·883	31·111	15·918	8·6808	4·6882	40
41	...	33·968	16·779	9·0024	4·8189	41
42	...	37·537	17·772	9·3592	4·9607	42
43	...	42·122	18·928	9·7571	5·1148	43
44	...	48·233	20·294	10·204	5·2832	44
45	...	56·782	21·929	10·709	5·4678	45
46	...	...	23·925	11·284	5·6714	46
47	...	...	26·417	11·947	5·8964	47
48	...	...	29·616	12·717	6·1474	48
49	...	...	33·874	13·626	6·4282	49
50	...	...	39·827	14·712	6·7450	50
51	...	...	...	16·036	7·1051	51
52	...	...	...	17·685	7·5185	52
53	...	...	...	19·798	7·9980	53
54	...	...	...	22·607	8·5613	54
55	...	...	...	26·529	9·2334	55
56	...	...	...	...	10·050	56
57	...	...	...	...	11·064	57
58	...	...	...	...	12·360	58
59	...	...	...	...	14·078	59
60	...	...	...	...	16·471	60

$\frac{K_{x\overline{t}|}}{1000}$ FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM. $\frac{K_{x\overline{t}|}}{1000}$

At all Ages	10·503	7·6198	5·3050	3·4933	2·1305	At all Ages

O^{M} $3\frac{1}{2}$ per-cent. **Log $k_{x\overline{t}|}$**

LOG OF CORRECTION-NUMERATOR *for*
ENDOWMENT ASSURANCE
SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE.

Age at Entry	Age at Maturity					Age at Entry
	45	50	55	60	65	
20	4·23023	4·04129	3·84613	3·63520	3·39729	20
21	·24129	·04974	·85285	·64075	·40205	21
22	·25305	·05864	·85989	·64654	·40700	22
23	·26554	·06803	·86726	·65258	·41214	23
24	·27886	·07794	·87500	·65888	·41749	24
25	·29308	·08841	·88312	·66545	·42304	25
26	·30830	·09951	·89165	·67231	·42882	26
27	·32463	·11126	·90061	·67918	·43182	27
28	·34219	·12374	·91004	·68697	·44106	28
29	·36116	·13703	·91998	·69181	·44755	29
30	·38171	·15118	·93046	·70301	·45430	30
31	·40407	·16632	·94153	·71160	·46133	31
32	·42855	·18255	·95326	·72062	·46866	32
33	·45546	·20000	·96569	·73009	·47630	33
34	·48527	·21883	·97892	·74006	·48428	34
35	·51855	·23923	·99300	·75057	·49263	35
36	·55606	·26144	4·00806	·76167	·50138	36
37	·59879	·28571	·02418	·77340	·51053	37
38	·64823	·31244	·04153	·78584	·52016	38
39	·70646	·34203	·06023	·79906	·53027	39
40	·77675	·37506	·08049	·81313	·54092	40
41	...	·41231	·10254	·82817	·55216	41
42	...	·45478	·12666	·84127	·56404	42
43	...	·50390	·15317	·86156	·57662	43
44	...	·56177	·18254	·88019	·58996	44
45	...	·63167	·21531	·90036	·60414	45
46	...	...	·25224	·92227	·61926	46
47	...	...	·29436	·94622	·63542	47
48	...	...	·34307	·97252	·65274	48
49	...	...	·40046	4·00161	·67135	49
50	...	...	·46981	·03405	·69145	50
51	...	...	...	·07058	·71325	51
52	...	...	...	·11219	·73699	52
53	...	...	...	·16029	·76301	53
54	...	...	...	·21699	·79174	54
55	...	...	...	·28550	·82370	55
56	...	...	...	...	·85961	56
57	...	...	...	...	·90048	57
58	...	...	...	...	·94770	58
59	...	...	...	...	4·00331	59
60	...	...	...	...	·07053	60

Log $K_{x\overline{t}|}$ FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM. **Log $K_{x\overline{t}|}$**

At all Ages	3·96746	3·82023	3·65516	3·46588	3·24322	At all Ages

0^{M} $3\frac{1}{2}$ per-cent. $\frac{k_{x\overline{t}|}}{1000}$

CORRECTION-NUMERATOR *for*
ENDOWMENT ASSURANCE
SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE—DIVIDED BY 1000.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	16·991	10·997	7·0167	4·3172	2·4963	20
21	17·430	11·213	7·1261	4·3727	2·5238	21
22	17·908	11·446	7·2425	4·4314	2·5527	22
23	18·431	11·696	7·3665	4·4935	2·5831	23
24	19·005	11·966	7·4989	4·5591	2·6151	24
25	19·637	12·258	7·6105	4·6286	2·6487	25
26	20·338	12·575	7·7920	4·7023	2·6842	26
27	21·117	12·920	7·9514	4·7806	2·7216	27
28	21·988	13·297	8·1291	4·8637	2·7610	28
29	22·970	13·710	8·3173	4·9523	2·8025	29
30	24·083	14·164	8·5204	5·0467	2·8464	30
31	25·355	14·666	8·7404	5·1475	2·8929	31
32	26·826	15·225	8·9797	5·2556	2·9421	32
33	28·540	15·849	9·2404	5·3714	2·9943	33
34	30·568	16·551	9·5262	5·4962	3·0499	34
35	33·003	17·347	9·8401	5·6308	3·1091	35
36	35·980	18·257	10·187	5·7766	3·1723	36
37	39·700	19·307	10·573	5·9347	3·2399	37
38	44·487	20·532	11·003	6·1072	3·3125	38
39	50·870	21·980	11·488	6·2959	3·3905	39
40	59·807	23·717	12·036	6·5032	3·4747	40
41	...	25·841	12·663	6·7324	3·5658	41
42	...	28·496	13·386	6·9867	3·6647	42
43	...	31·908	14·229	7·2704	3·7724	43
44	...	36·456	15·224	7·5891	3·8901	44
45	...	42·822	16·418	7·9499	4·0192	45
46	.	...	17·875	8·3612	4·1616	46
47	...	...	19·695	8·8353	4·3194	47
48	...	...	22·033	9·3869	4·4951	48
49	..	...	25·145	10·037	4·6919	49
50	...	...	29·499	10·816	4·9142	50
51	...	...	...	11·765	5·1671	51
52	...	...	...	12·948	5·4575	52
53	...	...	...	14·464	5·7944	53
54	...	...	...	16·481	6·1907	54
55	...	...	...	19·297	6·6635	55
56	...	...	...		7·2379	56
57	...	...	...	...	7·9521	57
58	...	...	...	...	8·8654	58
59	...	...	...	..	10·077	59
60	...	...	...	...	11·763	60

$\frac{K_{x\overline{t}|}}{1000}$ FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM. $\frac{K_{x\overline{t}|}}{1000}$

At all Ages	9·2781	6·6105	4·5202	2·9234	1·7507	At all Ages

O^M 4 per-cent. **Log $k_{x\overline{t}|}$**

LOG OF CORRECTION-NUMERATOR *for*
ENDOWMENT ASSURANCE
SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE.

Age at Entry	Age at Maturity					Age at Entry
	45	50	55	60	65	
20	4·13344	3·93970	3·73937	3·52284	3·27887	20
21	·14380	·94751	·74551	·52786	·28315	21
22	·15483	·95575	·75196	·53311	·28761	22
23	·16660	·96448	·75873	·53860	·29226	23
24	·17919	·97372	·76586	·54435	·29711	24
25	·19265	·98351	·77336	·55036	·30215	25
26	·20710	·99390	·78125	·55665	·30740	26
27	·22265	4·00495	·78957	·56324	·31287	27
28	·23943	·01671	·79835	·57014	·31857	28
29	·25759	·02926	·80762	·57737	·32451	29
30	·27732	·04268	·81743	·58496	·33071	30
31	·29886	·05706	·82782	·59294	·33718	31
32	·32249	·07251	·83884	·60132	·34393	32
33	·34854	·08918	·85056	·61015	·35100	33
34	·37749	·10722	·86307	·61947	·35839	34
35	·40989	·12682	·87643	·62932	·36615	35
36	·44649	·14820	·89073	·63974	·37428	36
37	·48832	·17164	·90609	·65078	·38281	37
38	·53682	·19750	·92266	·66252	·39181	38
39	·59411	·22624	·94059	·67504	·40129	39
40	·66344	·25840	·96005	·68839	·41129	40
41	...	·29475	·98129	·70269	·42187	41
42	...	·33630	4·00457	·71804	·43308	42
43	...	·38450	·03025	·73458	·44498	43
44	...	·44144	·05876	·75244	·45762	44
45	...	·51039	·09067	·77182	·47111	45
46	...	...	·12672	·79294	·48551	46
47	...	...	·16793	·81607	·50094	47
48	...	...	·21572	·84154	·51751	48
49	...	...	·27219	·86980	·53539	49
50	...	...	·34059	·90138	·55472	50
51	...	...	...	·93704	·57573	51
52	...	...	...	·97776	·59868	52
53	...	...	...	4·02496	·62390	53
54	...	...	...	·08073	·65180	54
55	...	...	..	·14831	·68293	55
56	...	...	...	...	·71799	56
57	...	...	...	...	·75800	57
58	...	...	...	...	·80433	58
59	...	...	...	...	·85904	59
60	...	...	...	...	·92535	60

FOR ENDOWMENT ASSURANCE BY

Log $K_{x\overline{t}|}$ SINGLE PREMIUM. **Log $K_{x\overline{t}|}$**

At all Ages	3·90606	3·75087	3·57787	3·38067	3·15003	At all Ages

O^{M} 4 per-cent. $\frac{k_{x\overline{t}|}}{1000}$

Correction-Numerator *for* ENDOWMENT ASSURANCE secured by Annual Premium payable throughout the duration of the Assurance—divided by 1000.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	13·597	8·7036	5·4874	3·3330	1·9005	20
21	13·925	8·8616	5·5656	3·3718	1·9193	21
22	14·283	9·0313	5·6488	3·4128	1·9391	22
23	14·676	9·2147	5·7376	3·4562	1·9600	23
24	15·107	9·4128	5·8326	3·5023	1·9820	24
25	15·583	9·6274	5·9342	3·5511	2·0052	25
26	16·110	9·8605	6·0430	3·6029	2·0296	26
27	16·697	10·115	6·1598	3·6580	2·0553	27
28	17·355	10·392	6·2856	3·7166	2·0824	28
29	18·096	10·697	6·4213	3·7789	2·1111	29
30	18·937	11·033	6·5680	3·8456	2·1415	30
31	19·900	11·404	6·7270	3·9169	2·1736	31
32	21·013	11·817	6·8999	3·9932	2·2076	32
33	22·312	12·279	7·0886	4·0752	2·2439	33
34	23·850	12·800	7·2958	4·1636	2·2824	34
35	25·697	13·391	7·5237	4·2591	2·3235	35
36	27·957	14·067	7·7755	4·3625	2·3674	36
37	30·784	14·847	8·0555	4·4749	2·4144	37
38	34·421	15·758	8·3687	4·5975	2·4650	38
39	39·274	16·836	8·7215	4·7319	2·5194	39
40	46·072	18·130	9·1212	4·8797	2·5780	40
41	...	19·713	9·5783	5·0430	2·6416	41
42	...	21·692	10·106	5·2244	2·7107	42
43	...	24·238	10·721	5·4273	2·7860	43
44	...	27·634	11·449	5·6551	2·8683	44
45	...	32·388	12·322	5·9132	2·9588	45
46	...	...	13·388	6·2078	3·0585	46
47	...	..	14·721	6·5474	3·1691	47
48	...	...	16·433	6·9429	3·2924	48
49	...	...	18·715	7·4097	3·4308	49
50	...	...	21·907	7·9686	3·5869	50
51	...	...	...	8·6505	3·7647	51
52	...	...	...	9·5008	3·9690	52
53	...	...	...	10·592	4·2063	53
54	...	...	...	12·043	4·4854	54
55	...	...	...	14·071	4·8187	55
56	...	...	...	...	5·2238	56
57	...	...	...	...	5·7280	57
58	...	...	...	...	6·3728	58
59	...	...	...	...	7·2284	59
60	...	...	...	...	8·4207	60

$\frac{K_{x\overline{t}|}}{1000}$ For ENDOWMENT ASSURANCE by Single Premium. $\frac{K_{x\overline{t}|}}{1000}$

At all Ages	8·0550	5·6347	3·7833	2·4025	1·4126	At all Ages

0^{M} $\frac{1000}{D_x}$

CORRECTION-DENOMINATOR

(Reciprocal of D_x)—multiplied by 1000.

Age at Valuation	$2\frac{1}{2}$ per-cent	$2\frac{3}{4}$ per-cent	3 per-cent	$3\frac{1}{2}$ per-cent	4 per-cent	Age at Valuation
20	·016989	·017837	·018725	·020630	·022717	20
21	·017484	·018402	·019365	·021438	·023721	21
22	·017996	·018987	·020030	·022282	·024773	22
23	·018526	·019594	·020720	·023161	·025876	23
24	·019074	·020223	·021437	·024079	·027031	24
25	·019612	·020875	·022183	·025038	·028244	25
26	·020230	·021553	·022959	·026040	·029515	26
27	·020840	·022257	·023767	·027086	·030850	27
28	·021474	·022990	·024608	·028182	·032253	28
29	·022131	·023751	·025485	·029328	·033727	29
30	·022814	·024543	·026400	·030528	·035277	30
31	·023524	·025369	·027355	·031785	·036907	31
32	·024263	·026230	·028351	·033103	·038623	32
33	·025032	·027127	·029392	·034485	·040430	33
34	·025832	·028063	·030480	·035936	·042333	34
35	·026667	·029040	·031619	·037459	·044340	35
36	·027537	·030061	·032808	·039058	·046458	36
37	·028445	·031127	·034055	·040738	·048690	37
38	·029392	·032242	·035362	·042506	·051049	38
39	·030381	·033409	·036731	·044366	·053539	39
40	·031417	·034631	·038166	·046324	·056173	40
41	·032499	·035913	·039675	·048389	·058959	41
42	·033634	·037256	·041259	·050566	·061912	42
43	·034823	·038668	·042928	·052865	·065036	43
44	·036071	·040153	·044685	·055294	·068357	44
45	·037385	·041715	·046535	·057867	·071880	45
46	·038766	·043361	·048492	·060591	·075626	46
47	·040222	·045102	·050559	·063480	·079618	47
48	·041761	·046942	·052748	·066551	·083872	48
49	·043390	·048890	·055072	·069823	·088417	49
50	·045114	·050958	·057544	·073308	·093284	50
51	·046948	·053160	·060176	·077030	·098493	51
52	·048905	·055506	·062984	·081018	·104095	52
53	·050989	·058015	·065989	·085302	·110123	53
54	·053220	·060702	·069214	·089904	·116626	54
55	·055617	·063593	·072685	·094868	·123663	55
56	·058200	·066707	·076429	·100240	·131294	56
57	·060987	·070067	·080483	·106065	·139595	57
58	·064008	·073719	·084882	·112405	·148655	58
59	·067295	·077694	·089670	·119328	·158572	59

O^{M} $\frac{1000}{D_x}$

CORRECTION-DENOMINATOR—*continued*

(RECIPROCAL OF D_x)—MULTIPLIED BY 1000.

Age at Valuation	$2\frac{1}{2}$ per-cent	$2\frac{3}{4}$ per-cent	3 per-cent	$3\frac{1}{2}$ per-cent	4 per-cent	Age at Valuation
60	·070882	·082034	·094913	·126916	·169471	60
61	·074811	·086798	·100667	·135263	·181492	61
62	·079139	·092039	·107011	·144486	·194799	62
63	·083928	·097847	·114034	·154715	·209600	63
64	·089238	·104295	·121846	·166116	·226132	64
65	·095166	·111496	·130576	·178881	·244684	65
66	·101821	·119580	·140384	·193252	·265618	66
67	·109321	·128700	·151458	·209507	·289360	67
68	·117826	·139053	·164039	·228014	·316436	68
69	·127530	·150873	·178415	·249203	·347512	69
70	·138671	·164452	·194947	·273613	·383392	70
71	·151545	·180158	·214087	·301932	·425116	71
72	·166528	·198456	·236401	·335020	·473979	72
73	·184094	·219920	·262612	·373972	·531660	73
74	·204864	·245333	·293660	·420221	·600312	74
75	·229616	·275642	·330754	·475579	·682641	75
76	·259383	·312139	·375460	·542476	·782473	76
77	·295552	·356519	·429886	·624181	·904568	77
78	·339928	·411066	·496845	·724900	1·055676	78
79	·395023	·478858	·580181	·850557	1·244694	79
80	·464231	·564111	·685166	1·009377	1·484208	80
81	·552303	·672766	·819135	1·212577	1·791601	81
82	·666001	·813273	·992654	1·476494	2·192117	82
83	·814996	·997606	1·220614	1·824485	2·721829	83
84	1·013490	1·243626	1·525274	2·290898	3·434184	84
85	1·282742	1·577859	1·939864	2·927829	4·409949	85
86	1·655273	2·041066	2·515470	3·814901	5·774339	86
87	2·181168	2·696145	3·330891	5·076142	7·720219	87
88	2·941782	3·645112	4·514265	6·912761	10·56435	88
89	4·068514	5·053568	6·273919	9·654373	14·82470	89
90	5·782352	7·199942	8·960573	13·85483	21·37803	90
91	8·476013	10·57988	13·19853	20·50735	31·79549	91
92	12·82545	16·04750	20·06783	31·33225	48·81382	92
93	20·15885	25·28509	31·69673	49·72898	77·85130	93
94	32·86123	41·31719	51·92108	81·85316	128·7581	94
95	56·13562	70·75639	89·12656	141·1951	223·1794	95
96	100·0250	126·3775	159·5838	254·0328	403·4698	96
97	189·1432	239·5554	303·2417	485·0601	774·1136	97
98	374·8126	475·8732	603·8283	970·5911	1556·420	98
99	768·3442	977·8995	1243·936	2009·243	3237·294	99

$O^{[M]}$ **$2\frac{1}{2}$ per-cent.** **Log $_{t}k_{[x]}$**

LOG OF CORRECTION-NUMERATOR *for*

WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	4·61385	4·36892	4·16706	3·98280	3·80276	20
21	·60076	·35450	·15094	·96450	·78157	21
22	·58742	·33979	·13446	·94569	·75969	22
23	·57395	·32487	·11768	·92648	·73726	23
24	·56022	·30963	·10049	·90673	·71406	24
25	·54626	·29406	·08289	·88642	·69013	25
26	·53206	·27821	·06485	·86554	·66540	26
27	·51758	·26199	·04636	·84404	·63983	27
28	·50283	·24539	·02739	·82188	·61336	28
29	·48779	·22842	·00789	·79903	·58590	29
30	·47238	·21102	3·98784	·77541	·55738	30
31	·45666	·19318	·96718	·75099	·52778	31
32	·44055	·17480	·94589	·72569	·49696	32
33	·42403	·15595	·92392	·69949	·46485	33
34	·40713	·13659	·90122	·67230	·43137	34
35	·38975	·11660	·87781	·64410	·39645	35
36	·37190	·09602	·85352	·61472	·35989	36
37	·35354	·07475	·82838	·58417	·32163	37
38	·33463	·05281	·80228	·55231	·28155	38
39	·31512	·03008	·77519	·51906	·23947	39
40	·29502	·00660	·74704	·48435	·19526	40
41	·27426	3·98223	·71775	·44804	·14877	41
42	·25280	·95698	·68722	·41002	·09975	42
43	·23062	·93078	·65543	·37018	·04809	43
44	·20761	·90350	·62219	·32835	2·99349	44
45	·18378	·87514	·58748	·28438	·93575	45
46	·15903	·84562	·55112	·23811	...	46
47	·13335	·81483	·51311	·18938	...	47
48	·10665	·78271	·47318	·13798	...	48
49	·07887	·74918	·43132	·08366	...	49
50	·04992	·71408	·38725	·02619	...	50
51	·01974	·67730	·34092	...	...	51
52	3·98824	·63880	·29208	...	...	52
53	·95535	·59845	·24060	...	...	53
54	·92096	·55606	·18619	...	...	54
55	·88495	·51143	·12864	...	...	55
56	·84727	·46455	...	...	...	56
57	·80772	·41508	...	...	...	57
58	·76624	·36295	...	...	...	58
59	·72264	·30789	...	...	...	59
60	·67679	·24963	...	...	...	60
61	·62853	...	...	...	...	61
62	·57767	...	...	...	...	62
63	·52405	...	...	...	...	63
64	·46738	...	...	...	...	64
65	·40750	...	...	...	...	65

O[M] $2\frac{1}{2}$ per-cent. $\dfrac{{}_t k_{[x]}}{1000}$

CORRECTION-NUMERATOR *for*

WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS

DIVIDED BY 1000.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	41·101	23·384	14·691	9·6117	6·3498	20
21	39·880	22·620	14·156	9·2151	6·0474	21
22	38·674	21·867	13·629	8·8245	5·7503	22
23	37·493	21·129	13·112	8·4427	5·4608	23
24	36·326	20·400	12·603	8·0673	5·1768	24
25	35·177	19·682	12·103	7·6987	4·8993	25
26	34·046	18·976	11·610	7·3374	4·6281	26
27	32·929	18·281	11·127	6·9830	4·3635	27
28	31·830	17·595	10·651	6·6356	4·1054	28
29	30.746	16·921	10·183	6·2955	3·8539	29
30	29·674	16·256	9·7239	5·9622	3·6089	30
31	28·619	15·602	9·2721	5·6362	3·3712	31
32	27·577	14·955	8·8286	5·3173	3·1402	32
33	26·548	14·320	8·3931	5·0060	2·9164	33
34	25·535	13·696	7·9656	4·7022	2·7000	34
35	24·533	13·080	7·5476	4·4066	2·4914	35
36	23·545	12·474	7·1371	4·1183	2·2903	36
37	22·570	11·878	6·7357	3·8386	2·0972	37
38	21·609	12·293	6·3128	3·5671	1·9123	38
39	20·660	10·717	5·9592	3·3042	1·7357	39
40	19·725	10·153	5·5852	3·0504	1·5677	40
41	18·804	9·5991	5·2210	2·8057	1·4085	41
42	17·898	9·0569	4·8665	2·5705	1·2582	42
43	17·007	8·5267	4·5230	2·3452	1·1171	43
44	16·129	8·0076	4·1898	2·1299	·98512	44
45	15·268	7·5014	3·8679	1·9248	·86248	45
46	14·422	7·0084	3·5573	1·7303	...	46
47	13·594	6·5287	3·2592	1·5466	...	47
48	12·784	6·0633	2·9729	1·3740	...	48
49	11·991	5·6128	2·6997	1·2124	...	49
50	11·218	5·1770	2·4392	1·0622	...	50
51	10·465	4·7566	2·1924	...	...	51
52	9·7328	4·3531	1·9592	...	...	52
53	9·0230	3·9669	1·7402	...	...	53
54	8·3360	3·5980	1·5353	...	...	54
55	7·6727	3·2466	1·3447	...	...	55
56	7·0351	2·9144	...	...	...	56
57	6·4227	2·6006	...	...	...	57
58	5·8377	2·3065	...	...	...	58
59	5·2801	2·0318	...	...	...	59
60	4·7511	1·7768	...	...	...	60
61	4·2514	...	...	...	...	61
62	3·7816	...	...	...	...	62
63	3·3423	...	...	...	...	63
64	2·9335	...	...	...	...	64
65	2·5556	...	...	...	...	65

$0^{[M]}$ $2\frac{3}{4}$ per-cent. **Log $_{t}k_{[x]}$**

LOG OF CORRECTION-NUMERATOR *for*

WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS.

Age at Entry	Number of Payments					Age at Entry
	10	15	20	25	30	
20	4·53285	4·28627	4·08263	3·89649	3·71412	20
21	·51938	·27149	·06616	·87783	·69286	21
22	·50576	·25648	·04940	·85875	·67070	22
23	·49196	·24122	·03230	·83920	·64793	23
24	·47794	·22571	·01480	·81916	·62444	24
25	·46369	·20989	3·99694	·79859	·60022	25
26	·44921	·19373	·97865	·77744	·57522	26
27	·43449	·17728	·95993	·75567	·54936	27
28	·41948	·16043	·94070	·73327	·52260	28
29	·40417	·14322	·92095	·71015	·49487	29
30	·38855	·12558	·90067	·68628	·46611	30
31	·37260	·10750	·87977	·66161	·43622	31
32	·35628	·08896	·85828	·63610	·40514	32
33	·33956	·06991	·83610	·60966	·37276	33
34	·32243	·05029	·81319	·58226	·33902	34
35	·30487	·03011	·78953	·55379	·30381	35
36	·28683	·00933	·76506	·52419	·26700	36
37	·26827	3·98789	·73971	·49342	·22848	37
38	·24918	·96573	·71340	·46130	·18809	38
39	·22952	·94287	·68611	·42785	·14573	39
40	·20923	·91918	·65774	·39289	·10124	40
41	·18829	·89462	·62825	·35632	·05441	41
42	·16666	·85919	·59751	·31806	·00512	42
43	·14429	·84279	·56549	·27796	2·95312	43
44	·12110	·81531	·53202	·23584	·89817	44
45	·09709	·78676	·49707	·19159	·84006	45
46	·07221	·75703	·46050	·14502	...	46
47	·04631	·72606	·42222	·09599	...	47
48	·01942	·69373	·38208	·04425	...	48
49	3·99143	·65994	·33990	2·98957	...	49
50	·96229	·62462	·29559	·93179	...	50
51	·93191	·58764	·24897	...	...	51
52	·90021	·54891	·19985	...	...	52
53	·86709	·50825	·14801	...	...	53
54	·83246	·46557	·09328	...	...	54
55	·79625	·42073	·03539	...	...	55
56	·75827	·37348	...	...	...	56
57	·71850	·32375	...	...	...	57
58	·67672	·27128	...	...	...	58
59	·63284	·21584	...	...	...	59
60	·58670	·15729	...	...	...	60
61	·53812	...	...		...	61
62	·48693	...	...	...	...	62
63	·43294	...	...	...	...	63
64	·37592	...	...	...	...	64
65	·31567	...	...	...	...	65

O$^{[M]}$ 2¾ per-cent. $\frac{{}_t\mathrm{k}_{[x]}}{1000}$

CORRECTION-NUMERATOR *for*
WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS
DIVIDED BY 1000.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	34·108	19·332	12·096	7·8793	5·1811	20
21	33·066	18·685	11·646	7·5480	4·9301	21
22	32·015	18·050	11·205	7·2235	4·6849	22
23	31·043	17·427	10·772	6·9056	4·4156	23
24	30·057	16·816	10·347	6·5942	4·2115	24
25	29·086	16·214	9·9298	6·2891	3·9831	25
26	28·133	15·622	9·5203	5·9902	3·7603	26
27	27·195	15·041	9·1186	5·6973	3·5429	27
28	26·271	14·469	8·7237	5·4109	3·3312	28
29	25·361	13·907	8·3359	5·1304	3·1251	29
30	24·465	13·353	7·9555	4·8560	2·9249	30
31	23·583	12·809	7·5818	4·5879	2·7304	31
32	22·713	12·273	7·2157	4·3261	2·5418	32
33	21·855	11·747	6·8565	4·0706	2·3592	33
34	21·010	11·228	6·5041	3·8217	2·1828	34
35	20·178	10·718	6·1593	3·5792	2·0128	35
36	19·357	10·217	5·8218	3·3434	1·8493	36
37	18·547	9·7250	5·4917	3·1147	1·6923	37
38	17·749	9·2412	5·1689	2·8927	1·5420	38
39	16·964	8·7674	4·8541	2·6782	1·3987	39
40	16·189	8·3019	4·5472	2·4711	1·2625	40
41	15·427	7·8155	4·2486	2·2715	1·1335	41
42	14·678	7·2309	3·9583	2·0800	1·0119	42
43	13·941	6·9629	3·6770	1·8965	·89768	43
44	13·216	6·5360	3·4012	1·7212	·79099	44
45	12·505	6·1201	3·1410	1·5545	·69193	45
46	11·809	5·7152	2·8874	1·3964	...	46
47	11·125	5·3218	2·6437	1·2474	...	47
48	10·457	4·9400	2·4103	1·1073	...	48
49	9·8046	4·5703	2·1873	·97627	...	49
50	9·1683	4·2133	1·9751	·85465	...	50
51	8·5489	3·8694	1·7741	...	...	51
52	7·9471	3·5392	1·5843	...	...	52
53	7·3636	3·2229	1·4061	...	...	53
54	6·7992	2·9213	1·2396	...	...	54
55	6·2553	2·6347	1·0849	...	...	55
56	5·7315	2·3631	...	...	...	56
57	5·2300	2·1074	...	...	...	57
58	4·7503	1·8676	...	...	...	58
59	4·2938	1·6438	...	...	...	59
60	3·8610	1·4364	...	...	...	60
61	3·4524	...	...	...	...	61
62	3·0685	...	...	..	...	62
63	2·7098	..	...	...	...	63
64	2·3764	...	...	...	...	64
65	2·0686	...	...	...	...	65

$O^{[M]}$ 3 per-cent. **Log $_tk_{[x]}$**

LOG OF CORRECTION-NUMERATOR *for*

WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	4·45327	4·20491	3·99943	3·81132	3·62712	20
21	·43946	·18981	·98266	·79235	·60524	21
22	·42548	·17450	·96556	·77293	·58277	22
23	·41131	·15886	·94812	·75304	·55961	23
24	·39693	·14299	·93027	·73263	·53577	24
25	·38237	·12685	·91210	·71175	·51123	25
26	·36758	·11043	·89352	·69030	·48591	26
27	·35255	·09365	·87445	·66823	·45970	27
28	·33724	·07651	·85495	·64552	·43264	28
29	·32167	·05904	·83493	·62214	·40463	29
30	·30582	·04117	·81442	·59800	·37559	30
31	·28956	·02279	·79326	·57306	·34538	31
32	·27298	·00401	·77148	·54726	·31400	32
33	·25605	3·98470	·74906	·52057	·28134	33
34	·23869	·96488	·72592	·49290	·24734	34
35	·22089	·94450	·70202	·46420	·21182	35
36	·20262	·92346	·67731	·43433	·17468	36
37	·18385	·90182	·65171	·40328	·13586	37
38	·16456	·87945	·62521	·37094	·09522	38
39	·14467	·85634	·59766	·33720	·05251	39
40	·12420	·83244	·56910	·30199	·00772	40
41	·10306	·80769	·53935	·26516	2·96059	41
42	·08122	·78204	·50839	·22661	·91094	42
43	·05864	·75541	·47609	·18620	·85857	43
44	·03528	·72777	·44211	·14383	·80332	44
45	·01106	·69900	·40721	·09928	·74483	45
46	3·98595	·66905	·37042	·05241	...	46
47	·95988	·63785	·33186	·00308	...	47
48	·93277	·60524	·29142	2·95096	...	48
49	·90458	·57126	·24898	·89598	...	49
50	·87520	·53566	·20436	·83783	...	50
51	·84462	·49847	·15747	...	...	51
52	·81269	·45944	·10802	...	...	52
53	·77931	·41854	·05583	...	...	53
54	·74444	·37558	·00074	...	...	54
55	·70798	·33040	2·94252	...	...	55
56	·66977	·28291	...	...	...	56
57	·62967	·23282	...	...	...	57
58	·58763	·17997	...	...	...	58
59	·54345	·12424	...	...	...	59
60	·49699	·06524	...	...	...	60
61	·44808	...	...	...	...	61
62	·39657	...	...	...	...	62
63	·34220	...	...	...	...	63
64	·28482	...	..	...	...	64
65	·22417	...	...	...	...	65

$O^{[M]}$ **3 per-cent.** $\frac{{}_t k_{[x]}}{1000}$

Correction-Numerator *for*
WHOLE-LIFE ASSURANCE by LIMITED PAYMENTS
divided by 1000.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	28·397	16·029	9·9869	6·4762	4·2376	20
21	27·508	15·481	9·6086	6·1994	4·0294	21
22	26·637	14·945	9·2376	5·9283	3·8262	22
23	25·782	14·417	8·8740	5·6629	3·6275	23
24	24·942	13·899	8·5167	5·4029	3·4338	24
25	24·120	13·392	8·1677	5·1493	3·2451	25
26	23·312	12·895	7·8256	4·9012	3·0613	26
27	22·519	12·407	7·4895	4·6583	2·8820	27
28	21·739	11·926	7·1606	4·4210	2·7079	28
29	20·973	11·456	6·8380	4·1893	2·5388	29
30	20·222	10·994	6·5226	3·9628	2·3746	30
31	19·479	10·539	6·2124	3·7416	2·2150	31
32	18·749	10·093	5·9085	3·5258	2·0606	32
33	18·032	9·6538	5·6113	3·3157	1·9113	33
34	17·326	9·2232	5·3201	3·1110	1·7674	34
35	16·630	8·8004	5·0352	2·9121	1·6286	35
36	15·945	8·3842	4·7567	2·7185	1·4951	36
37	15·270	7·9766	4·4845	2·5309	1·3673	37
38	14·607	7·5762	4·2190	2·3493	1·2451	38
39	13·953	7·1836	3·9597	2·1737	1·1285	39
40	13·311	6·7989	3·7077	2·0044	1·0179	40
41	12·678	6·4223	3·4622	1·8415	·91325	41
42	12·056	6·0540	3·2240	1·6850	·81459	42
43	11·446	5·6939	2·9929	1·5353	·72205	43
44	10·846	5·3428	2·7696	1·3926	·63580	44
45	10·258	5·0003	2·5539	1·2568	·55569	45
46	9·6817	4·6671	2·3465	1·1283	...	46
47	9·1176	4·3436	2·1471	1·0071	...	47
48	8·5658	4·0294	1·9562	·89322	...	48
49	8·0275	3·7261	1·7741	·78701	...	49
50	7·5024	3·4329	1·6009	·68838	..	50
51	6·9923	3·1512	1·4370	...	...	51
52	6·4967	2·8803	1·2824	...	...	52
53	6·0160	2·6214	1·1372	...	...	53
54	5·5519	2·3745	1·0017	...	...	54
55	5·1048	2·1399	·87603	...	...	55
56	4·6749	1·9183	...	...	...	56
57	4·2626	1·7093	...	...	...	57
58	3·8693	1·5135	...	...	...	58
59	3·4950	1·3312	...	...	...	59
60	3·1404	1·1621	..	...	...	60
61	2·8060	...	...	...	...	61
62	2·4921	...	...	...	...	62
63	2·1989	...	...	...	...	63
64	1·9267	...	...	...	...	64
65	1·6756	...	...	...	...	65

$O^{[M]}$ $3\frac{1}{2}$ per-cent. **Log $_t k_{[x]}$**

LOG OF CORRECTION-NUMERATOR *for*

WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	4·29850	4·01645	3·83698	3·64463	3·45592	20
21	·28380	·03045	·81934	·62482	·43319	21
22	·26892	·01423	·80138	·60154	·40984	22
23	·25396	3·99787	·78317	·58388	·38591	23
24	·23875	·98118	·76151	·56269	·36128	24
25	·22344	·96431	·74564	·54108	·33597	25
26	·20794	·94717	·72637	·51895	·30993	26
27	·19218	·92968	·70664	·49616	·28301	27
28	·17619	·91189	·68643	·47278	·25524	28
29	·15995	·89376	·66575	·44869	·22653	29
30	·14340	·87522	·64456	·42390	·19676	30
31	·12660	·85630	·62284	·39836	·16592	31
32	·10942	·83688	·60049	·37194	·13388	32
33	·09190	·81702	·57748	·34465	·10054	33
34	·07394	·79660	·55375	·31633	·06583	34
35	·05564	·77570	·52931	·28705	·02968	35
36	·03684	·75418	·50404	·25661	2·99191	36
37	·01757	·73199	·47793	·22499	·95243	37
38	3·99773	·70969	·45081	·19201	·91106	38
39	·97736	·68546	·42278	·15770	·86771	39
40	·95640	·66110	·39370	·12192	·82221	40
41	·93479	·63584	·36342	·08448	·77439	41
42	·91249	·60970	·33192	·04533	·72404	42
43	·88945	·58263	·29912	·00432	·67096	43
44	·86557	·55443	·26485	2·96128	·61487	44
45	·84094	·52522	·22911	·91611	·55566	45
46	·81536	·49475	·19176	·86857	...	46
47	·78878	·46299	·15258	·81850	...	47
48	·76122	·42991	·11159	·76573	...	48
49	·73257	·39542	·06856	·71002	...	49
50	·70273	·35929	·02332	·65111	...	50
51	·67162	·32153	2·97574	...	...	51
52	·63916	·28193	·92559	...	...	52
53	·60529	·24042	·87274	...	...	53
54	·56990	·19687	·81693	...	...	54
55	·53284	·15106	·75790	...	...	55
56	·49407	·10288	...	...	...	56
57	·45339	·05212	...	...	...	57
58	·41073	2·99859	...	...	...	58
59	·36588	·94205	...	...	...	59
60	·31878	·88232	...	...	...	60
61	·26918	...	...	...	...	61
62	·21693	...	...	...	...	62
63	·16184	...	...	...	...	63
64	·10368	...	...	...	...	64
65	·04220	...	...	...	...	65

$O^{[M]}$ $3\frac{1}{2}$ per-cent. $\frac{{}_tk_{[x]}}{1000}$

CORRECTION-NUMERATOR *for*
WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS
DIVIDED BY 1000.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	19·884	11·129	6·8704	4·4119	2·8571	20
21	19·222	10·726	6·5969	4·2152	2·7114	21
22	18·575	10·333	6·3297	4·0229	2·5694	22
23	17·946	9·9511	6·0697	3·8360	2·4317	23
24	17·328	9·5759	5·8149	3·6533	2·2976	24
25	16·728	9·2111	5·5672	3·4760	2·1676	25
26	16·141	8·8546	5·3256	3·3033	2·0414	26
27	15·566	8·5051	5·0891	3·1344	1·9187	27
28	15·003	8·1638	4·8577	2·9702	1·7999	28
29	14·453	7·8300	4·6318	2·8099	1·6847	29
30	13·912	7·5027	4·4112	2·6540	1·5731	30
31	13·384	7·1829	4·1960	2·5024	1·4653	31
32	12·865	6·8688	3·9856	2·3547	1·3611	32
33	12·357	6·5618	3·7799	2·2113	1·2605	33
34	11·856	6·2604	3·5789	2·0717	1·1637	34
35	11·367	5·9662	3·3831	1·9366	1·0707	35
36	10·885	5·6778	3·1918	1·8056	·98154	36
37	10·413	5·3950	3·0056	1·6788	·89625	37
38	9·9479	5·1179	2·8236	1·5560	·81482	38
39	9·4921	4·8469	2·6472	1·4378	·73741	39
40	9·0448	4·5825	2·4757	1·3241	·66406	40
41	8·6058	4·3235	2·3090	1·2147	·59483	41
42	8·1750	4·0710	2·1474	1·1100	·52971	42
43	7·7526	3·8250	1·9912	1·0100	·46877	43
44	7·3379	3·5845	1·8401	·91470	·41197	44
45	6·9333	3·3514	1·6948	·82435	·35947	45
46	6·5367	3·1243	1·5551	·73887	...	46
47	6·1487	2·9040	1·4210	·65842	...	47
48	5·7706	2·6910	1·2930	·58308	...	48
49	5·4022	2·4855	1·1710	·51289	...	49
50	5·0435	2·2871	1·0552	·44783	...	50
51	4·6948	2·0967	·94567	...	...	51
52	4·3567	1·9139	·84254	...	...	52
53	4·0299	1·7395	·74600	...	...	53
54	3·7145	1·5735	·65604	...	...	54
55	3·4107	1·4160	·57266	...	...	55
56	3·1194	1·2673	...	...	...	56
57	2·8405	1·1275	...	...	...	57
58	2·5747	·99676	...	...	...	58
59	2·3221	·87508	...	...	...	59
60	2·0834	·76264	...	...	...	60
61	1·8586	...	...	...	...	61
62	1·6479	...	...	...	...	62
63	1·4516	...	...	...	...	63
64	1·2696	...	...	...	...	64
65	1·1020	...	...	...	...	65

$0^{[M]}$ 4 per-cent. **Log $_t k_{[x]}$**

LOG OF CORRECTION-NUMERATOR *for*

WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	4·14932	3·89323	3·67948	3·48259	3·28902	20
21	·13353	·87618	·66083	·46173	·26524	21
22	·11768	·85903	·64192	·44052	·24096	22
23	·10171	·84164	·62271	·41891	·21606	23
24	·08558	·82408	·60322	·39685	·19055	24
25	·06937	·80632	·58346	·37439	·16435	25
26	·05290	·78823	·56325	·35128	·13735	26
27	·03633	·76996	·54271	·32773	·10965	27
28	·01953	·75138	·52174	·30357	·08104	28
29	·00248	·73243	·50032	·27871	·05152	29
30	3·98518	·71316	·47836	·25314	·02094	30
31	·96757	·69344	·45584	·22679	2·98929	31
32	·94967	·67334	·43277	·19964	·95645	32
33	·93149	·65281	·40911	·17166	·92241	33
34	·91283	·63172	·38467	·14260	·88691	34
35	·89388	·61019	·35959	·11265	·85002	35
36	·87444	·58798	·33365	·08152	·81147	36
37	·85449	·56514	·30686	·04917	·77121	37
38	·83413	·54170	·27920	·01559	·72917	38
39	·81312	·51744	·25049	2·98057	·68503	39
40	·79156	·49247	·22072	·94406	·63873	40
41	·76942	·46670	·18990	·90600	·59018	41
42	·74654	·43994	·15777	·86615	·53901	42
43	·72295	·41225	·12435	·82444	·48514	43
44	·69850	·38351	·08946	·78067	·42822	44
45	·67332	·35368	·05311	·73482	·36818	45
46	·64718	·32266	·01508	·68654	...	46
47	·62007	·29035	2·97529	·63571	...	47
48	·59196	·25669	·93363	·58217	...	48
49	·56275	·22158	·88992	·52568	...	49
50	·53235	·18187	·84401	·46595	...	50
51	·50064	·14641	·79568	...	...	51
52	·46763	·10623	·74484	...	...	52
53	·43320	·06408	·69124	...	...	53
54	·39717	·01985	·63463	...	...	54
55	·35953	2·97334	·57481	...	...	55
56	·32010	·92445	...	...	...	56
57	·27878	·87296	...	...	...	57
58	·23544	·81865	...	...	...	58
59	·18995	·76131	...	...	...	59
60	·14212	·70079	...	...	...	60
61	·09177	...	...	...	...	61
62	·03878	...	..	...	...	62
63	2·98288	...	...	...	...	63
64	·92389	...	...	...	...	64
65	·86159	...	...	...	...	65

$O^{[M]}$ 4 per-cent. $\dfrac{{}_t\mathrm{k}_{[x]}}{1000}$

CORRECTION-NUMERATOR *for* WHOLE-LIFE ASSURANCE BY LIMITED PAYMENTS DIVIDED BY 1000.

Age at Entry	Number of Payments 10	15	20	25	30	Age at Entry
20	14·103	7·8204	4·7806	3·0380	1·9454	20
21	13·600	7·5193	4·5796	2·8955	1·8418	21
22	13·112	7·2282	4·3845	2·7575	1·7416	22
23	12·639	6·9415	4·1948	2·6237	1·6446	23
24	12·178	6·6693	4·0107	2·4937	1·5508	24
25	11·732	6·4021	3·8323	2·3680	1·4600	25
26	11·295	6·1409	3·6581	2·2153	1·3720	26
27	10·873	5·8879	3·4891	2·1268	1·2872	27
28	10·460	5·6413	3·3246	2·0117	1·2051	28
29	10·057	5·4005	3·1646	1·8998	1·1260	29
30	9·6645	5·1661	3·0086	1·7912	1·0494	30
31	9·2805	4·9367	2·8565	1·6857	·97564	31
32	8·9057	4·7135	2·7088	1·5836	·90459	32
33	8·5406	4·4958	2·5651	1·4848	·83639	33
34	8·1814	4·2827	2·4248	1·3887	·77074	34
35	7·8321	4·0756	2·2887	1·2961	·70798	35
36	7·4893	3·8724	2·1560	1·2065	·64784	36
37	7·1530	3·6740	2·0270	1·1199	·59049	37
38	6·8254	3·4810	1·9020	1·0365	·53601	38
39	6·5031	3·2919	1·7803	·95625	·48421	39
40	6·1881	3·1079	1·6623	·87914	·43524	40
41	5·8806	2·9289	1·5485	·80538	·38921	41
42	5·5788	2·7538	1·4380	·73477	·34595	42
43	5·2838	2·5837	1·3315	·66748	·30559	43
44	4·9946	2·4183	1·2287	·60349	·26805	44
45	4·7132	2·2578	1·1301	·54303	·23344	45
46	4·4379	2·1021	1·0353	·48589	...	46
47	4·1694	1·9514	·94469	·43223	...	47
48	3·9080	1·8059	·85828	·38209	...	48
49	3·6538	1·6656	·77610	·33549	...	49
50	3·4068	1·5306	·69825	·29238	...	50
51	3·1669	1·4009	·62471	...	...	51
52	2·9351	1·2771	·55570	...	...	52
53	2·7114	1·1590	·49118	...	...	53
54	2·4956	1·0468	·43115	...	...	54
55	2·2884	·94046	·37567	...	...	55
56	2·0898	·84033	...	...	...	56
57	1·9001	·74638	...	...	...	57
58	1·7196	·65864	...	..	...	58
59	1·5486	·57722	...	...	...	59
60	1·3871	·50210	...	...	...	60
61	1·2353	...	...	...	...	61
62	1·0934	...	...	...	...	62
63	·96135	...	...	...	...	62
64	·83925	...	...	...	...	64
65	·72709	...	...	...	...	65

$O^{[M]}$ $2\frac{1}{2}$ per-cent. Log $k_{[x]\overline{t|}}$

LOG OF CORRECTION-NUMERATOR *for* ENDOWMENT ASSURANCE

SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	4·40599	4·22595	4·04046	3·84020	3·61430	20
21	·41796	·23518	·04782	·84629	·61950	21
22	·43063	·24485	·05552	·85262	·62488	22
23	·44411	·25506	·06360	·85922	·63051	23
24	·45841	·26582	·07204	·86613	·63634	24
25	·47366	·27719	·08090	·87332	·64241	25
26	·48994	·28920	·09019	·88083	·64874	26
27	·50738	·30189	·09997	·88868	·65530	27
28	·52610	·31536	·11025	·89690	·66215	28
29	·54628	·32969	·12208	·90548	·66927	29
30	·56812	·34494	·13251	·91448	·67671	30
31	·59183	·36120	·14455	·92393	·68446	31
32	·61767	·37862	·15729	·93383	·69255	32
33	·64606	·39730	·17079	·94422	·70098	33
34	·67741	·41742	·18513	·95514	·70979	34
35	·71233	·43918	·20039	·96668	·71903	35
36	...	·46280	·21663	·97879	·72865	36
37	...	·48856	·23402	·99163	·73873	37
38	...	·51680	·25265	4·00516	·74933	38
39	...	·54800	·27267	·01950	·76042	39
40	...	·58274	·29432	·03476	·77207	40
41	...	...	·31779	·05100	·78436	41
42	...	...	·34337	·06832	·79728	42
43	...	...	·37142	·08687	·81089	43
44	...	...	·40239	·10677	·82529	44
45	...	...	·43686	·12822	·84056	45
46	...	...	...	·15149	·85676	46
47	...	...	...	·17683	·87499	47
48	...	...	...	·20457	·89242	48
49	...	...	...	·23519	·91217	49
50	...	...	...	·26924	·93340	50
51	...	...	...	...	·95633	51
52	...	...	...	...	·98127	52
53	...	...	...	...	4·00855	53
54	...	...	...	...	·03863	54
55	...	...	...	...	·07204	55

FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM.

Log $K_{[x]\overline{t|}}$ Log $K_{[x]\overline{t|}}$

At all Ages	4·04019	3·90813	3·75839	3·58477	3·37831	At all Ages

$O^{[M]}$ $2\frac{1}{2}$ per-cent. $\frac{k_{[x]\overline{t|}}}{1000}$

CORRECTION-NUMERATOR *for*

ENDOWMENT ASSURANCE

SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE—DIVIDED BY 1000.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	25·468	16·825	10·976	6·9215	4·1143	20
21	26·179	17·186	11·164	7·0192	4·1639	21
22	26·954	17·573	11·364	7·1223	4·2158	22
23	27·804	17·991	11·577	7·2314	4·2708	23
24	28·735	18·443	11·804	7·3473	4·3285	24
25	29·762	18·932	12·048	7·4700	4·3894	25
26	30·899	19·463	12·308	7·6003	4·4539	26
27	32·165	20·040	12·588	7·7389	4·5217	27
28	33·581	20·671	12·890	7·8868	4·5936	28
29	35·179	21·364	13·246	8·0441	4·6695	29
30	36·993	22·128	13·568	8·2126	4·7502	30
31	39·069	22·972	13·949	8·3932	4·8357	31
32	41·461	23·912	14·364	8·5868	4·9266	32
33	44·265	24·963	14·818	8·7947	5·0232	33
34	47·578	26·147	15·315	9·0186	5·1261	34
35	51·562	27·490	15·863	9·2615	5·2364	35
36	...	29·027	16·468	9·5234	5·3537	36
37	...	30·801	17·140	9·8091	5·4794	37
38	...	32·870	17·892	10·120	5·6147	38
39	...	35·318	18·736	10·459	5·7600	39
40	...	38·260	19·693	10·833	5·9166	40
41	...	...	20·787	11·246	6·0864	41
42	...	...	22·048	11·704	6·2702	42
43	...	...	23·519	12·214	6·4698	43
44	...	...	25·257	12·787	6·6879	44
45	...	...	27·344	13·434	6·9272	45
46	...	...	...	14·174	7·1905	46
47	...	...	...	15·026	7·4988	47
48	...	...	...	16·017	7·8058	48
49	...	...	...	17·187	8·1690	49
50	...	...	...	18·588	8·5783	50
51	...	...	...	...	9·0434	51
52	...	...	...	...	9·5779	52
53	...	...	...	...	10·199	53
54	...	...	...	...	10·930	54
55	...	...	...	...	11·804	55

$\frac{K_{[x]\overline{t|}}}{1000}$ FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM. $\frac{K_{[x]\overline{t|}}}{1000}$

At all Ages	10·970	8·0934	5·7331	3·8439	2·3895	At all Ages

$0^{[M]}$ $2\frac{3}{4}$ per-cent. **Log $k_{[x]\overline{t|}}$**

Log of Correction-Numerator *for*
ENDOWMENT ASSURANCE
secured by Annual Premium payable throughout the duration of the Assurance.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	4·35660	4·17453	3·98681	3·78413	3·55552	20
21	·36818	·18339	·99385	·78990	·56044	21
22	·38047	·19270	4·00119	·79592	·56553	22
23	·39350	·20255	·00893	·80220	·57086	23
24	·40742	·21293	·01701	·80878	·57638	24
25	·42224	·22389	·02552	·81564	·58214	25
26	·43813	·23550	·03445	·82282	·58815	26
27	·45513	·24783	·04387	·83033	·59441	27
28	·47345	·26091	·05378	·83821	·60093	28
29	·49320	·27482	·06425	·84644	·60775	29
30	·51458	·28967	·07528	·85511	·61486	30
31	·53783	·30553	·08695	·86417	·62227	31
32	·56326	·32251	·09929	·87371	·63002	32
33	·59118	·34077	·11241	·88375	·63813	33
34	·62208	·36046	·12635	·89430	·64660	34
35	·65653	·38177	·14119	·90545	·65547	35
36	...	·40196	·15703	·91719	·66475	36
37	...	·43026	·17398	·92962	·67450	37
38	...	·45807	·19219	·94278	·68472	38
39	...	·48882	·21183	·95674	·69545	39
40	...	·52308	·23303	·97159	·70674	40
41	...	...	·25606	·98744	·71864	41
42	...	...	·28121	4·00434	·73119	42
43	...	...	·30881	·02248	·74443	43
44	...	...	·33932	·04199	·75846	44
45	...	...	·37334	·06301	·77332	45
46	...	...	...	·08585	·78914	46
47	...	...	..	·11075	·80599	47
48	...	...	...	·13805	·82398	48
49	...	...	..	·16821	·84330	49
50	...	..	..	·20181	·86414	50
51	.	.	...	...	·88667	51
52	...	..	...	...	·91119	52
53	...	..	...	...	·93804	53
54	...	...	...	..	·96766	54
55	...	...	...	...	4·00067	55

For ENDOWMENT ASSURANCE by Single Premium.

Log $K_{[x]\overline{t|}}$ **Log $K_{[x]\overline{t|}}$**

At all Ages	4·02026	3·88436	3·73078	3·55329	3·34292	At all Ages

$O^{[M]}$ $2\frac{3}{4}$ per-cent. $\frac{k_{[x]\overline{t}}}{1000}$

CORRECTION-NUMERATOR *for* ENDOWMENT ASSURANCE

SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE—DIVIDED BY 1000.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	22·730	14·946	9·7009	6·0832	3·5935	20
21	23·344	15·254	9·8594	6·1645	3·6345	21
22	24·014	15·585	10·027	6·2506	3·6773	22
23	24·746	15·942	10·208	6·3416	3·7227	23
24	25·552	16·328	10·399	6·4384	3·7703	24
25	26·439	16·745	10·605	6·5409	3·8207	25
26	27·424	17·199	10·826	6·6500	3·8739	26
27	28·519	17·694	11·063	6·7660	3·9302	27
28	29·747	18·235	11·318	6·8899	3·9896	28
29	31·131	18·829	11·594	7·0217	4·0528	29
30	32·702	19·484	11·893	7·1632	4·1196	30
31	34·501	20·208	12·217	7·3143	4·1905	31
32	36·581	21·014	12·569	7·4767	4·2660	32
33	39·010	21·916	12·954	7·6516	4·3464	33
34	41·887	22·933	13·377	7·8397	4·4320	34
35	45·345	24·086	13·842	8·0436	4·5235	35
36	...	25·407	14·356	8·2640	4·6211	36
37	...	26·931	14·927	8·5039	4·7261	37
38	...	28·712	15·566	8·7656	4·8386	38
39	...	30·819	16·287	9·0519	4·9596	39
40	...	33·349	17·101	9·3668	5·0903	40
41	...	...	18·033	9·7149	5·2317	41
42	...	...	19·108	10·100	5·3851	42
43	...	...	20·362	10·531	5·5518	43
44	...	...	21·843	11·015	5·7340	44
45	...	...	23·623	11·561	5·9336	45
46	...	...	...	12·186	6·1538	46
47	...	...	...	12·905	6·3972	47
48	...	...	...	13·742	6·6678	48
49	...	...	...	14·730	6·9711	49
50	...	...	...	15·915	7·3137	50
51	...	...	...	...	7·7032	51
52	...	...	...	...	8·1506	52
53	...	...	...	...	8·6704	53
54	...	...	...	...	9·2824	54
55	...	...	...	...	10·015	55

$\frac{K_{[x]\overline{t}}}{1000}$ FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM. $\frac{K_{[x]\overline{t}}}{1000}$

At all Ages	10·477	7·6623	5·3799	3·5751	2·2025	At all Ages

$O^{[M]}$ 3 per-cent. **Log $k_{[x]\overline{t|}}$**

LOG OF CORRECTION-NUMERATOR *for* ENDOWMENT ASSURANCE

SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	4·30741	4·12321	3·93320	3·72799	3·49661	20
21	·31859	·13172	·93989	·73347	·50124	21
22	·33050	·14071	·94694	·73920	·50607	22
23	·34317	·15017	·95433	·74517	·51109	23
24	·35668	·16019	·96208	·75141	·51634	24
25	·37113	·17078	·97026	·75797	·52182	25
26	·38658	·18202	·97885	·76481	·52753	26
27	·40317	·19393	·98789	·77201	·53347	27
28	·42107	·20664	·99745	·77956	·53970	28
29	·44040	·22016	4·00753	·78747	·54621	29
30	·46136	·23461	·01819	·79578	·55299	30
31	·48416	·25005	·02950	·80451	·56010	31
32	·50914	·26664	·04147	·81370	·56752	32
33	·53663	·28450	·05421	·82337	·57529	33
34	·56707	·30376	·06775	·83357	·58345	34
35	·60105	·32466	·08218	·84436	·59198	35
36	...	·34739	·09765	·85574	·60093	36
37	...	·37226	·11421	·86778	·61032	37
38	...	·39962	·13201	·88058	·62021	38
39	...	·42991	·15121	·89415	·63057	39
40	...	·46373	·17197	·90863	64152	40
41	...	...	·19460	·92406	·65306	41
42	...	...	·21929	·94056	·66523	42
43	...	...	·24645	·95830	·67811	43
44	...	...	·27652	·97738	·69176	44
45	...	...	·31008	·99802	·70623	45
46	...	..	...	4·02043	·72167	46
47	...	...	...	·04489	·73815	47
48	...	...	...	·07177	·75575	48
49	...	...	...	·10148	·77469	49
50	...	...	...	·13463	·79509	50
51	...	...	...	...	·81722	51
52	...	...	...	...	·84131	52
53	...	...	...	...	·86774	53
54	...	...	...	...	·89694	54
55	...	...	...	...	·92950	55

Log $K_{[x]\overline{t|}}$ FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM. **Log $K_{[x]\overline{t|}}$**

At all Ages	3·99709	3·85731	3·69986	3·51848	3·30418	At all Ages

$O^{[M]}$ 3 per-cent. $\frac{k_{[x]\overline{t|}}}{1000}$

CORRECTION-NUMERATOR *for*

ENDOWMENT ASSURANCE

SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE—DIVIDED BY 1000.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	20·296	13·280	8·5743	5·3455	3·1377	20
21	20·825	13·543	8·7074	5·4134	3·1713	21
22	21·404	13·826	8·8499	5·4853	3·2068	22
23	22·038	14·131	9·0018	5·5612	3·2441	23
24	22·734	14·461	9·1639	5·6417	3·2835	24
25	23·503	14·818	9·3381	5·7276	3·3252	25
26	24·355	15·206	9·5247	5·8185	3·3692	26
27	25·303	15·629	9·7250	5·9158	3·4156	27
28	26·368	16·093	9·9415	6·0195	3·4650	28
29	27·568	16·602	10·175	6·1301	3·5173	29
30	28·931	17·164	10·428	6·2486	3·5726	30
31	30·490	17·785	10·703	6·3754	3·6316	31
32	32·295	18·477	11·002	6·5118	3·6942	32
33	34·406	19·253	11·329	6·6584	3·7609	33
34	36·904	20·126	11·688	6·8166	3·8322	34
35	39·907	21·118	12·083	6·9881	3·9082	35
36	...	22·253	12·521	7·1736	3·9896	36
37	...	23·565	13·008	7·3753	4·0768	37
38	...	25·097	13·552	7·5959	4·1707	38
39	...	26·910	14·165	7·8370	4·2714	39
40	...	29·089	14·858	8·1027	4·3805	40
41	...	...	15·653	8·3958	4·4984	41
42	...	...	16·569	8·7209	4·6263	42
43	...	...	17·638	9·0845	4·7655	43
44	...	...	18·903	9·4925	4·9177	44
45	...	...	20·421	9·9545	5·0843	45
46	..	...	...	10·482	5·2683	46
47	...	...	...	11·089	5·4720	47
48	...	...	...	11·797	5·6984	48
49	...	...	...	12·632	5·9521	49
50	...	...	...	13·634	6·2386	50
51	...	...	...	...	6·5648	51
52	...	...	...	...	6·9392	52
53	...	...	...	...	7·3746	53
54	...	...	...	...	7·8875	54
55	..	...	...	...	8·5016	55

$\frac{K_{[x]\overline{t|}}}{1000}$ FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM. $\frac{K_{[x]\overline{t|}}}{1000}$

At all Ages	9·9332	7·1996	5·0102	3·2998	2·0146	At all Ages

$O^{[M]}$ $3\frac{1}{2}$ per-cent. **Log $k_{[x]\overline{t}|}$**

LOG OF CORRECTION-NUMERATOR *for* ENDOWMENT ASSURANCE SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	4·20968	4·02097	3·82608	3·61558	3·37837	20
21	·22017	·02882	·83218	·62050	·38250	21
22	·23134	·03711	·83859	·62564	·38680	22
23	·24325	·04588	·84535	·63104	·39130	23
24	·25598	·05521	·85247	·63672	·39602	24
25	·26965	·06509	·85998	·64268	·40093	25
26	·28435	·07562	·86790	·64894	·40610	26
27	·30012	·08679	·87631	·65551	·41149	27
28	·31719	·09873	·88517	·66242	·41716	28
29	·33572	·11152	·89457	·66969	·42308	29
30	·35583	·12517	·90451	·67737	·42929	30
31	·37779	·13984	·91510	·68544	·43580	31
32	·40190	·15563	·92634	·69395	·44260	32
33	·42850	·17269	·93835	·70295	·44977	33
34	·45804	·19113	·95113	·71248	·45728	34
35	·49114	·21120	·96481	·72255	·46518	35
36	...	·23309	·97949	·73324	·47349	36
37	...	·25710	·99526	·74458	·48224	37
38	...	·28358	4·01226	·75662	·49144	38
39	...	·31299	·03066	·76948	·50115	39
40	...	·34591	·05061	·78321	·51143	40
41	...	...	·07239	·79785	·52225	41
42	...	...	·09624	·81359	·53374	42
43	...	...	·12254	·83054	·54591	43
44	...	...	·15173	·84881	·55883	44
45	...	...	·18440	·86868	·57257	45
46	...	...	...	·89025	·58727	46
47	...	...	...	·91388	·60295	47
48	...	...	...	·93990	·61981	48
49	...	...	.	·96876	·63797	49
50	...	...	.	4·00101	·65761	50
51	...	...	...	...	·67894	51
52	...	..	...	...	·70221	52
53	...	...	...	...	·72782	53
54	...	...	...	...	·75618	54
55	...	...	...	...	·78787	55

FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM.

Log $K_{[x]\overline{t}|}$ **Log $K_{[x]\overline{t}|}$**

At all Ages	3·94323	3·79559	3·63031	3·44109	3·21887	At all Ages

$O^{[M]}$ $3\frac{1}{2}$ per-cent. $\frac{k_{[x]\overline{t}|}}{1000}$

Correction-Numerator *for* ENDOWMENT ASSURANCE.

SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE—DIVIDED BY 1000.

Age at Entry	Age at Maturity					Age at Entry
	45	50	55	60	65	
20	16·206	10·495	6·7001	4·1265	2·3898	20
21	16·602	10·686	6·7949	4·1735	2·4127	21
22	17·035	10·892	6·8959	4·2232	2·4367	22
23	17·509	11·114	7·0041	4·2760	2·4621	23
24	18·029	11·356	7·1198	4·3323	2·4890	24
25	18·606	11·617	7·2440	4·3922	2·5173	25
26	19·246	11·902	7·3773	4·4559	2·5474	26
27	19·958	12·212	7·5216	4·5239	2·5792	27
28	20·758	12·552	7·6766	4·5964	2·6131	28
29	21·663	12·928	7·8446	4·6740	2·6490	29
30	22·690	13·340	8·0262	4·7574	2·6871	30
31	23·867	13·799	8·2243	4·8466	2·7277	31
32	25·229	14·310	8·4100	4·9425	2·7708	32
33	26·823	14·883	8·6766	5·0460	2·8169	33
34	28·710	15·529	8·9357	5·1580	2·8660	34
35	30·984	16·263	9·2217	5·2790	2·9186	35
36	...	17·104	9·5387	5·4105	2·9750	36
37	...	18·076	9·8915	5·5537	3·0356	37
38	...	19·212	10·286	5·7098	3·1006	38
39	...	20·558	10·731	5·8814	3·1707	39
40		22·177	11·236	6·0703	3·2466	40
41		...	11·814	6·2784	3·3285	41
42			12·481	6·5101	3·4177	42
43		...	13·260	6·7692	3·5149	43
44			14·182	7·0601	3·6210	44
45			15·290	7·3906	3·7374	45
46			...	7·7669	3·8661	46
47				8·2012	4·0082	47
48				8·7076	4·1669	48
49				9·3059	4·3448	49
50				10·024	4·5458	50
51				...	4·7746	51
52				...	5·0374	52
53	...	...			5·3434	53
54	...		...	...	5·7040	54
55	...	...	...	...	6·1358	55

$\frac{K_{[x]\overline{t}|}}{1000}$ FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM. $\frac{K_{[x]\overline{t}|}}{1000}$

At all Ages	8·7746	6·2458	4·2688	2·7611	1·6553	At all Ages

$O^{[M]}$ **4 per-cent.** **Log $k_{[x]\overline{t}|}$**

Log of Correction-Numerator *for* ENDOWMENT ASSURANCE

secured by Annual Premium payable throughout the duration of the Assurance.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	4·11280	3·91923	3·71910	3·50297	3·25967	20
21	·12258	·92645	·72462	·50738	·26332	21
22	·13302	·93409	·73049	·51200	·26717	22
23	·14425	·94223	·73667	·51690	·27121	23
24	·15627	·95089	·74319	·52202	·27544	24
25	·16919	·96012	·75008	·52744	·27987	25
26	·18310	·96993	·75740	·53313	·28452	26
27	·19817	·98045	·76517	·53915	·28941	27
28	·21446	·99169	·77339	·54549	·29453	28
29	·23213	4·00371	·78213	·55216	·29991	29
30	·25145	·01665	·79143	·55923	·30559	30
31	·27258	·03057	·80132	·56671	·31154	31
32	·29586	·04560	·81190	·57460	·31782	32
33	·32162	·06190	·82317	·58297	·32439	33
34	·35030	·07955	·83525	·59185	·33134	34
35	·38250	·09881	·84821	·60127	·33864	35
36	...	·11986	·86214	·61129	·34635	36
37	...	·14304	·87714	·62192	·35447	37
38	...	·16868	·89340	·63332	·36307	38
39	...	·19724	·91098	·64545	·37214	39
40	...	·22927	·92977	·65843	·38177	40
41	...	...	·95112	·67240	·39195	41
42	...	...	·97416	·68737	·40275	42
43	...	...	·99963	·70358	·41425	43
44	...	...	4·02796	·72108	·42650	44
45	...	...	·05976	·74014	·43955	45
46	...	...	...	·76092	·45354	46
47	...	...	...	·78375	·46851	47
48	...	...	...	·80895	·48462	48
49	...	...	...	·83697	·50203	49
50	...	...	...	·86839	·52091	50
51	...	...	...	...	·54145	51
52	...	...	...	...	·56395	52
53	...	...	...	...	·58875	53
54	...	...	...	...	·61628	54
55	...	...	...	...	·64714	55

For ENDOWMENT ASSURANCE by Single Premium.

Log $K_{[x]\overline{t}|}$ **Log $K_{[x]\overline{t}|}$**

At all Ages	3·88184	3·72621	3·55299	3·35583	3·12563	At all Ages

$0^{[M]}$ 4 per-cent. $\frac{k_{[x]\overline{t}}}{1000}$

CORRECTION-NUMERATOR *for* ENDOWMENT ASSURANCE

SECURED BY ANNUAL PREMIUM PAYABLE THROUGHOUT THE DURATION OF THE ASSURANCE—DIVIDED BY 1000.

Age at Entry	Age at Maturity 45	50	55	60	65	Age at Entry
20	12·966	8·3029	5·2572	3·1840	1·8183	20
21	13·261	8·4421	5·3012	3·2165	1·8337	21
22	13·584	8·5919	5·3764	3·2509	1·8500	22
23	13·940	8·7545	5·4534	3·2878	1·8673	23
24	14·331	8·9308	5·5359	3·3267	1·8856	24
25	14·764	9·1226	5·6244	3·3685	1·9049	25
26	15·244	9·3310	5·7201	3·4130	1·9254	26
27	15·782	9·5598	5·8233	3·4606	1·9472	27
28	16·386	9·8105	5·9346	3·5115	1·9703	28
29	17·066	10·086	6·0552	3·5658	1·9948	29
30	17·842	10·391	6·1863	3·6243	2·0211	30
31	18·732	10·729	6·3288	3·6873	2·0490	31
32	19·763	11·107	6·4849	3·7549	2·0788	32
33	20·971	11·532	6·6553	3·8280	2·1105	33
34	22·403	12·010	6·8431	3·9071	2·1446	34
35	24·127	12·555	7·0503	3·9927	2·1809	35
36	...	13·178	7·2801	4·0859	2·2200	36
37	...	13·901	7·5360	4·1872	2·2619	37
38	...	14·746	7·8235	4·2985	2·3071	38
39	...	15·749	8·1467	4·4203	2·3558	39
40	...	16·954	8·5069	4·5544	2·4086	40
41	...	...	8·9355	4·7033	2·4658	41
42		...	9·4224	4·8682	2·5278	42
43			9·9915	5·0534	2·5957	43
44		...	10·665	5·2611	2·6699	44
45	...	...	11·475	5·4972	2·7514	45
46	...	...	...	5·7666	2·8414	46
47	...	...	...	6·0779	2·9411	47
48			...	6·4410	3·0522	48
49	...	...	...	6·8702	3·1771	49
50	...	...	...	7·3857	3·3183	50
51	...	...	...	...	3·4790	51
52	...			...	3·6640	52
53	...	...	...	...	3·8793	53
54	...	...	...	...	4·1331	54
55	...	...	...	...	4·4375	55

$\frac{K_{[x]\overline{t}}}{1000}$ FOR ENDOWMENT ASSURANCE BY SINGLE PREMIUM. $\frac{K_{[x]\overline{t}}}{1000}$

At all Ages	7·6180	5·3237	3·5726	2·2690	1·3355	At all Ages

$O^{[M]}$ ULTIMATE VALUES. $\frac{1000}{D_x}$

CORRECTION-DENOMINATOR

(RECIPROCAL OF D_x)—MULTIPLIED BY 1000.

Age at Valuation	$2\frac{1}{2}$ per-cent	$2\frac{3}{4}$ per-cent	3 per-cent	$3\frac{1}{2}$ per-cent	4 per-cent	Age at Valuation
20	·017253	·018115	·019017	·020951	·023071	20
21	·017803	·018737	·019718	·021829	·024154	21
22	·018371	·019382	·020447	·022745	·025290	22
23	·018958	·020051	·021204	·023702	·026480	23
24	·019566	·020744	·021990	·024700	·027728	24
25	·020195	·021463	·022808	·025743	·029039	25
26	·020846	·022208	·023657	·026831	·030414	26
27	·021520	·022983	·024541	·027969	·031856	27
28	·022217	·023786	·025461	·029158	·033370	28
29	·022941	·024620	·026418	·030401	·034961	29
30	·023691	·025487	·027414	·031701	·036631	30
31	·024468	·026387	·028452	·033060	·038388	31
32	·025275	·027324	·029533	·034484	·040233	32
33	·026112	·028298	·030661	·035974	·042176	33
34	·026983	·029313	·031838	·037536	·044218	34
35	·027888	·030369	·033066	·039173	·046369	35
36	·028828	·031471	·034348	·040890	·048636	36
37	·029809	·032620	·035689	·042693	·051026	37
38	·030830	·033819	·037092	·044585	·053548	38
39	·031895	·035074	·038561	·046577	·056208	39
40	·033007	·036385	·040099	·048669	·059018	40
41	·034169	·037759	·041714	·050875	·061989	41
42	·035384	·039197	·043408	·053200	·065134	42
43	·036658	·040707	·045192	·055651	·068470	43
44	·037995	·042294	·047068	·058245	·072005	44
45	·039400	·043964	·049044	·060987	·075758	45
46	·040876	·045723	·051133	·063890	·079745	46
47	·042434	·047583	·053339	·066970	·083998	47
48	·044078	·049547	·055676	·070244	·088527	48
49	·045817	·051626	·058156	·073730	·093371	49
50	·047662	·053836	·060790	·077447	·098551	50
51	·049620	·056183	·063597	·081413	·104098	51
52	·051706	·058689	·066596	·085660	·110061	52
53	·053932	·061365	·069798	·090228	·116480	53
54	·056316	·064234	·073239	·095129	·123407	54
55	·058872	·067313	·076941	·100420	·130899	55
56	·061626	·070631	·080932	·106141	·139024	56
57	·064599	·074223	·085244	·112347	·147863	57
58	·067820	·078113	·089936	·119097	·157505	58
59	·071322	·082345	·095039	·126469	·168062	59

$O^{[M]}$ ULTIMATE VALUES. $\frac{1000}{D_x}$

CORRECTION-DENOMINATOR—*continued*

(RECIPROCAL OF D_x)—MULTIPLIED BY 1000.

Age at Valuation	$2\frac{1}{2}$ per-cent	$2\frac{3}{4}$ per-cent	3 per-cent	$3\frac{1}{2}$ per-cent	4 per-cent	Age at Valuation
60	·075137	·086964	·100617	·134542	·179656	60
61	·079327	·092030	·106738	·143421	·192434	61
62	·083928	·097609	·113483	·153224	·206582	62
63	·089015	·103776	·120945	·164090	·222301	63
64	·094652	·110624	·129239	·176196	·239854	64
65	·100946	·118265	·138502	·189739	·259538	65
66	·107996	·126833	·148898	·204973	·281730	66
67	·115939	·136493	·160630	·222193	·306880	67
68	·124938	·147447	·173940	·241780	·335537	68
69	·135194	·159941	·189136	·264173	·368392	69
70	·146955	·174277	·206595	·289956	·406306	70
71	·160532	·190843	·226783	·319836	·450329	71
72	·176317	·210119	·250300	·354723	·501857	72
73	·194807	·232721	·277894	·395742	·562588	73
74	·216633	·259124	·310540	·444365	·634800	74
75	·242618	·291257	·349492	·502538	·721345	75
76	·273845	·329544	·396335	·572738	·826105	76
77	·311721	·376039	·453412	·658328	·954107	77
78	·358141	·433088	·523478	·763709	1·112236	78
79	·415679	·503905	·610538	·895095	1·309809	79
80	·487853	·592839	·720046	1·060749	1·559746	80
81	·579576	·706015	·859550	1·272443	1·880053	81
82	·697739	·852007	1·039890	1·546838	2·296528	82
83	·852297	1·043297	1·276471	1·907960	2·846327	83
84	1·057843	1·298044	1·592002	2·391143	3·584358	84
85	1·335987	1·643331	2·020406	3·049338	4·593055	85
86	1·719720	2·120486	2·613354	3·963378	5·998800	86
87	2·260245	2·793842	3·451608	5·260113	8·000000	87
88	3·039237	3·765769	4·663744	7·141837	10·91429	88
89	4·189886	5·204267	6·460783	9·941346	15·26648	89
90	5·935775	7·390983	9·197940	14·22212	21·94474	90
91	8·664010	10·81420	13·49073	20·96128	32·50024	91
92	13·06592	16·34842	20·44146	31·91931	49·72898	92
93	20·42108	25·61344	32·10788	50·37530	78·85813	93
94	33·18731	41·72752	52·43564	82·66512	130·0373	94
95	56·29046	70·94714	89·37349	141·5809	223·7887	95
96	100·0480	126·4079	159·6220	254·0909	403·5675	96
97	187·1503	237·0398	300·0480	479·9386	765·9900	97
98	370·2332	470·0573	596·4808	958·7728	1537·468	98
99	778·6949	991·0803	1260·589	2036·038	3280·840	99

Abstract of the Discussion.

Mr. W. P. Phelps, in opening the discussion, thanked the author for offering the Institute a paper which was both interesting and instructive, and one that would lend itself to discussion. At the same time he could not help thinking that the interest of the paper was purely academic. He did not think the system advocated by Mr. Bell was one that they would be able to adopt practically in the work of their offices. In the first place, it seemed to him that it was not a simple system. It was easy to follow, but when one worked it out it appeared to involve a tremendous amount of detail, which could not be deputed to anyone who was not skilled in the work of valuations. It did not seem to him a method of valuation which could be put into the hands of a clerical staff with the expectancy of their carrying it through with an ordinary amount of supervision. It struck him that every entry in the long list of policies of various classes required to be individually considered by someone who understood the whole principle of the system, and not a clerk who was only used to the working of it without understanding what was behind it. The title of the paper was, "On the Retrospective Method of Valuation." It did not appear to him to be a retrospective method. It was true that the author obtained the fundamental formula from the fundamental retrospective formula, but he might just as well have obtained it from the prospective. At the bottom of p. 19 the author gave the original retrospective formula—*i.e.*, accumulating the premiums and deducting accumulated claims—and he then stated that he preferred to use an alternative form. It appeared to him that the moment it was written in such alternative form it ceased to be retrospective. If the formula were examined it would be found that the first term on the right-hand side was the value of a whole-life sum assured. If the benefit to be valued was greater than the whole-life benefit an addition had to be made, and if it was less a subtraction had to be made. With regard to the premium, the actual premium for the risk was valued as if it were payable for the whole of life. It was all looking forward; it did not seem to him to be looking back in any way. If the premium were payable for less than whole-life a further correction had to be made; then these two corrections were combined, and it so happened that they came out in a very convenient form; the numerator was constant and fixed from the commencement, and it was only the denominator that varied with the valuation age. He did not think he need go into the other formulas that were given; they were all equally convenient, and they involved exactly the same principle. The disadvantages of the system were very great. He did not think it was at all desirable that all classes of insurance should be put into one group, because it was difficult to ensure accuracy if that were done. Very large groups were obtained, and if there were an error one had to go through every item to discover it. It was impossible by inspection, or any means of that sort, to say that the error was in a particular group of policies; they were quite in the dark, and depended entirely on the accuracy of the work. He thought that for small non-profit business it might be good, but for general business

he did not think it was a very workable scheme. He did not think the actuary's work would be over when once he obtained the corrections at the commencement. In the case of a limited payment policy, the moment the payment ceased they had to take off the premium and the correction, leaving the ordinary whole-life A_{x+n}. In the case of endowment assurances by limited premiums, it was further complicated, because not only had they to take off the correction in regard to the premium, but immediately after to put on another correction in regard to the sum assured.

The positive and negative corrections were not obvious; one had to look at every case to see whether it was positive or negative, and if by an error it was put in the wrong column, or the wrong coloured ink was used, they were never likely to discover it. If there were such a mistake on the books it might go on indefinitely. He did not think the system was available for keeping debtor and creditor class books; it would mean that the schedules would have to be written up pretty nearly every time. It was also very inconvenient not to know the liability under different classes of insurance. For instance, it was important to have a sort of idea of how the liability under endowment insurance was growing, and the actuary would not wish it to be included with every other sort of liability in the office. With regard to the corrections, he thought that they would not lend themselves to rough valuations. Some time ago he obtained a similar correction in connection with endowment assurances that he valued by Mr. Lidstone's method. He wanted to include limited payment policies in the groupings, and he obtained corrections similar in principle to those being discussed, but found the results were very unreliable. If the correction was used in the form the author gave in the paper, and it was valued at the true age, the result was absolutely correct; but if any approximate age was used, for instance, if they wanted to make a rough valuation by grouping the payments, a very slight alteration of the age made a very considerable difference in the correction, and he had to discard the system. It was impossible to know which way the error was going, and it was very treacherous to deal with such corrections unless they were dealt with at the exact age; it could not be done with an average age. He thought all knew what the objects of the Board of Trade returns were, and he also thought there was not very much doubt that the Board of Trade returns at the present time did not fulfil that object perfectly; but if valuation returns were made to the Board of Trade such as were suggested by the author, he very much doubted whether they would fulfil the objects of the Act at all. He did not think the returns suggested by Mr. Bell would tell even an actuary anything. He would not have anything to go on, except the statement of the corrections, and he did not see how that could be checked. Then he very much doubted whether the Board of Trade would be so obliging as the author seemed to think. As the author pointed out, on p. 57, a discretion was given, the schedule saying—"For the purpose of adapting them to the circumstances of such company, or of better carrying into effect the objects of this Act." Although it said "or", he thought the spirit of the Act was that a modification would only be made if it could be

shown that it equally carried through the objects of the Act, and he did not think the returns suggested by the author would carry through the objects of the Act in any sense. He thought the Institute had to thank the author for a very instructive paper, and he was sorry he did not think it was a practically useful one.

Mr. GEORGE KING said he did not propose to go into details, but he thought he would be doing well in following Mr. Phelps in expressing his views on the system advocated in the paper. In the first place, personally he had no rooted objection to the retrospective method of valuation; he thought he was probably the only actuary who had submitted a retrospective valuation to the Board of Trade and had it accepted. Where it was convenient, he thought it was right to employ it; and in a particular case where the whole of the business consisted of deferred annuities, he found that the most convenient way of valuing them, whether the premiums were returnable or not, was by the retrospective method, and the Board of Trade, after it had been explained to them, agreed, with the result that for the last fifteen or twenty years a retrospective method of valuation had found its place in the Blue Books. But it did not necessarily follow that, for ordinary life assurance business as distinguished from that very special class of business, the retrospective method would be good; personally he thought it would give much more trouble, and would very much obscure the issues, and he thought the returns as they were at present given in the Blue Books were most useful. Although the Act had its defects and its shortcomings, and although it might in some minor points be amended, it had been a most beneficial one, and he should be very sorry to see it fundamentally altered. It was passed on very good actuarial advice. He thought he was right in saying that the Government consulted the late Mr. W. P. Pattison with regard to it, and he believed he was also correct in saying that their friend Mr. R. P. Hardy had had a good deal to do with drawing up the schedules. The author said the Act was passed shortly after the failure of the Albert and the European, but if he remembered right, he thought the Albert failed in 1869 and the European in 1872, so that the Act came between the two, although it was understood at the time it was passed that the European was tottering. Apparently, from the voluminous tables prepared, the author seemed to think that the method should at present be applied to the three great classes of assurance, the ordinary whole-life, endowment assurances, and limited payments. He presumed, from there not being tables for others, that these others were, at any rate for the present, to be dealt with as hitherto. With regard to endowment assurances, personally he very much preferred Mr. Lidstone's most admirable method, and it would take a great deal to make him forsake that for what Mr. Bell called the retrospective method. He thought that the very advantages which the author claimed for his method were disadvantages from his (the speaker's) point of view. In fact the author's ideals and his (Mr. King's) seemed to differ fundamentally. In his opinion it was a very great advantage to have the endowment assurances separate, and it was also a very great advantage to tabulate, as Mr. Lidstone's method provided, the endowment assurances falling due in each

particular future year. It was then possible to see what was going to happen, and provision could be made for the inevitable. If they were all grouped according to current age, as the author suggested, that was lost sight of. Then Mr. Lidstone's method had another advantage. Having tabulated the endowment assurances according to their due date, irrespective of age, the Board of Trade accepted that instead of the total premiums paid from the commencement, and therefore they not only had a very simple method of valuation, which enabled them to value endowment assurances more quickly and easily than whole-life policies, but by that method they were saved all the trouble of returning the total premiums paid from the commencement, which was a very important element where there were such an immense mass of policies as, at the present day, we have in ordinary endowment assurances. That would not be obtained with the author's method; there would be more trouble with the valuation, and the additional trouble of stating the premiums paid would be left upon their shoulders. He was not quite clear as to how the limited payment policies were to be dealt with, but he gathered from Mr. Phelps' remarks that there was a complete gap between the points where the premiums were payable and where there were no more premiums. At that point the policies had to be transferred from one category to another. After all, limited payment policies were not a matter of supreme importance in the United Kingdom. Out of curiosity he had looked at the last Blue Book, and, taking all the valuations of British companies that appeared in it, he found that the limited payment, including paid-up, policies were only 3·98 per-cent of the whole business, so that it was not a large item in this country. It was quite true that in America it was a very large item, and possibly the retrospective method, done in groups, would be an advantage in the saving of labour there; but there was no chance whatever of getting the Americans to adopt it. In America they worked by tables of prepared values; every policy was valued separately every year, and, in view of the statutory valuations made by the State departments, he did not suppose they would ever make a change. He had only on one occasion had to deal with any large number of limited payment policies, and in that case the premiums all ceased at a given age, so that they could be valued in groups with great simplicity by the ordinary prospective method. He did not think the author had stated how he valued by his method policies with half-yearly and quarterly premiums. These had to be considered, because there were a great many of them. There were various ways of dealing with them, and he did not exactly see at the moment how the retrospective method could be applied. The author claimed that his retrospective method was useful by enabling them to value by select tables. He did not think they would, as a rule, value special policies by select tables, because the difference in reserve would usually be very small between the select reserve and the aggregate reserve; but it might, and he thought, would, be a very great advantage if they could value ordinary whole-life policies by select tables. Some years ago a formula occurred to him by which that could be done with hardly more trouble than that incurred in valuing such policies in groups by aggregate tables. He proposed to write

a short note on the subject, and put the method forward, because, now that there were such perfect select tables, it might be worth while to consider the subject. He had ventured to speak, not unfavourably of the paper, but against the method it advocated. In doing so he did not wish in any way to impugn the author's work; in fact, he should like to join Mr. Phelps in praising his ingenuity, thoroughness, and industry, and in thanking him for bringing the matter forward. It was a subject which it was quite right should be brought forward, and one that they might very properly discuss.

Mr. W. P. ELDERTON thought that the author had been very roughly handled by the previous speakers. If some of the formulas were re-worded, and were looked at prospectively as Mr. Phelps suggested, simply treating a temporary benefit as one which was equal to a whole-term benefit less the deferred benefit, where the deferred benefit could be represented by a fraction in which the numerator was a constant, depending upon the age at which the term expired, and the denominator a variable depending upon the age at valuation, then it would be seen at once that the sums assured, the office premiums, and the net premiums could be valued independently, and an absolutely accurate valuation of each obtained. That was another way of stating the author's method of valuation, and in that form, which was practically the one Mr. Altenburger brought forward in his letters to the *Journal*, he (Mr. Elderton) had seen the method applied to a fairly large experience. Mr. Phelps was certainly wrong, so far as his (Mr. Elderton's) experience went, for the method was simple, and the details did not require to be supervised very much by a skilled individual. The constants were calculated by people who certainly had no actuarial training, but seemed to understand what they were doing. Of course, all classes were not put into one age group; endowment assurances, limited payments, and whole-life must be kept separately for the Board of Trade, even if that were not advisable for other purposes, and rough checks could be devised to show they were not far out in the result, just as could be done when Mr. Lidstone's method was adopted for endowment assurances. There were three constants for endowment assurances, and two for limited payments. Mr. Phelps, in his remarks with regard to limited payments, seemed to indicate that there was a difficulty, because the constant had to be taken off at the end of the term; but surely as they had to take off the premiums it was little extra trouble to take off something else that was standing by at the same time. He had never had any difficulty in the valuations or book-keeping, and so, although he had only seen the method used for a few years, it seemed worth while to put on record that it had not been the hopeless failure which one might imagine from the last two speeches, and certainly at the time of valuation he found it gave less trouble in working than valuations made by the last payment grouping did when an average age was taken for the policies in each group.

Mr. T. J. SEARLE said he rose to place before the meeting a totally opposite view to that which had been advocated by the two gentlemen who opened the discussion. When he read the paper he immediately came to the conclusion that it would effect a revolution in the valuing of special policies, and that in time to come the

valuable work of Mr. Lidstone would be referred to as a sort of antiquarian curiosity, which would be studied in order to see what were the ways that were pursued by actuaries in the dark ages. He spoke strongly, because Messrs. Phelps and King had laid down the law rather strongly, and it was quite right that they should speak strongly on both sides. He considered that the author had made a very remarkable discovery, and it was a very strange thing that nobody had ever made it before. He had not had an opportunity since receiving the paper of referring to Mr. Whiting's paper, and he did not know how far that gentleman had gone, but he presumed he had not gone farther than the author. If the author had gone one step further he would have knocked out all the D's and N's, and he would have made a very important discovery. All great discoveries were simple, and in the present instance it was a very simple matter indeed to look at. It was perfectly true that everybody in assurance offices was used to the present system. He was afraid that some did not want to learn the new one, and that therefore it would be some little time before they would be able to apply it. The author had stated that if he had read Mr. Whiting's paper he would not have submitted his paper to the Institute. Had he followed that course, the author would, in his opinion, have committed a crime by keeping back the secret which he had discovered. What Mr. Bell had discovered was this—he did not know that the author fully realized it himself—that for every special policy on a single life, without exception, an ordinary whole-life policy might be substituted, together with a simple endowment fully paid-up; and the two things taken together would make any special policy they liked. They knocked out the D's and N's, and by so doing, took away the difficulty of the last term in the form on page 21, and it put the whole thing in a simple and easy form. If there was any student in the immediate future who was not able to understand that, he did not think he ought to be passed as an actuary. Every special policy could be made equivalent to a corresponding whole-life policy of the same sum assured, together with a simple endowment fully paid up for an amount which could be very easily ascertained. That was a general principle of great importance, and he could not understand why any of the members should say it was not practicable before they had tried it. In the formula given at the bottom of page 21 some difficulty arose through the P_x being valued as if it were payable through the whole of life, but the theory of the formula was that the cessation of the premium before the time of death was part of the special benefit to be secured, and so they had an endowment or other special assurance. If they wished to compare that with the corresponding whole-life assurance, part of the advantage obtained was the acceleration of the payment of the sum assured: another part of the advantage was the earlier cessation of the premium, *i.e.*, premium P was valued for the whole of life, and the earlier cessation of P was part of the benefit to be secured by the payment of P. If the formula were looked at, it would be seen that they were all pure premiums, and therefore the annual premiums were the exact equivalent of the benefits that were to be given for them. The π_x was the exact equivalent of the ordinary whole-life policy: the

annual premium was exactly equivalent to the sum assured. Similarly, the annual premium P payable throughout life was the exact equivalent of the special policy, together with the continuance of that P until the time of death. Therefore, the difference between P and π was the present value of all the benefits obtained under the special policy, over and above those obtained under the ordinary whole-life policy; and therefore the formula was practically proved. The difference between P and π was an annual premium throughout life. In order to ascertain at the date of the policy what that value was worth, they had to multiply the P and π by the annuity-due, and he then suggested that it should be turned into a simple endowment at some advanced age, and if that were to be done it could be valued very easily indeed for every successive valuation. It was a small step in which he wished to go further than the author, but it would throw a flood of light upon the method, so that the members would be able to see the applicability of the new formula. A formula in that shape was very suitable for valuation, because what happened between the initiation of the policy and the valuation was this:—The assurance A_x was subject to payment out in the event of death; at the valuation it became A_{x+n}. P_x was subject to cessation in the event of death. The endowment, which he suggested ought to be there, was subject only to being lost in the event of death. In the event of death the endowment would be lost and that was precisely what would happen to all special benefits under the special policy. If a man was insured under an endowment assurance, and died within the first few years, he obtained no more than if he were insured under a whole-life policy; he had simply paid the additional premium and lost it, and therefore an endowment was a very suitable function indeed to represent all classes of deferred benefits, such as were given under any form of special policy, over and above the benefits given under the ordinary whole-life policy.

He was of opinion that a great deal too much had been made of the question of the Board of Trade Returns, because there were several ways which could be suggested for filling up the schedules as they stood. The proposed endowment could easily be divided into two pure endowments, one of which would apply to the sum assured and the other to the premiums, so that they would be able to fill up the Board of Trade returns precisely as they did before, and with very little trouble—with very much less trouble than with some of the present systems of valuing special policies. The author's method had one very remarkable advantage. In the present day offices made their valuations in haste and filled up their returns at leisure; therefore anything which would enable them to close their books at four o'clock in the afternoon, and publish the figures at 10 o'clock the next morning was a thing to be desired. That was one of the reasons why the method would be adopted as soon as it was understood, because there was only one total for each age throughout the whole of the business, and the valuation could be worked out in a few hours. Later on they might want to split it into whole-life, endowment assurances, and other classes, and that might be done at leisure, either for the Board of Trade or for other purposes. Later on they might want to split it into the value of the premiums

and the value of the sum assured. That might be done at their leisure. The result could be worked out in the course of a few hours, so rapidly and easily, in fact, that annual valuations for the use of the offices would become quite common. It would be found that it was quite an easy thing to keep up the class books from year to year, and to make the valuations; and he should be very much surprised indeed if, before long, it did not come to be the only method in use for the valuing of special policies.

Mr. R. P. Hardy said it was a matter of legitimate pride to the Institute, and he trusted it would ever remain one of its characteristics, that it was prepared to extend a respectful welcome, and accord a patient hearing, to every shade of opinion, notwithstanding that such might not be accepted—or even might be rejected—by the majority, since such might contain a dormant, though unrecognized, element of the vivifying virtue of truth. There would be found no better and no more generous judges of the devotion of any of the brethren to the cause of his profession than in that room, nor any better qualified to appreciate the consistency with which an idea had been worked out, nor any more ready to acknowledge the heavy labour that had been bestowed upon such a paper as they had before them. He would therefore join with his other friends in thanking Mr. Bell, and unreservedly concede that he had said—and well said—all that could be urged in support of his proposed method of valuation, and that, by his abundant illustrations, he had given them full opportunity of gauging its working adaptability. But at that point, with the wholesome fear of Mr. Searle before his eyes, he was reluctantly compelled to assume the ungrateful task of criticism, and to range himself in opposition to the principle which had been so valiantly advocated. At the outset, with Mr. Phelps, he could not admit that the method proposed was any true sample of the retrospective system; it would, he considered, be more truly described as a series of plus and minus adjustments upon the ordinary reserves—certainly an ingenious idea, and one pretty enough in its own way as a college exercise, but not one corresponding to any reality in the facts to be measured, or presenting sufficient conveniences to justify its substitution for the clear symmetry, the logical order, and the intelligibility of the figures shown upon the work-sheets in the older arrangement. He laid no stress upon the additional and irritating work of the adjustments, nor would he dwell upon the almost impossibility of applying any of those customary concise and summary checks, although no doubt the mechanical processes could be controlled. But the great objection which he took was that the results were blended in such an admired disorder that all check upon their consistency was lost, and their relation to the actual facts under measurement was obscured beyond recognition and beyond test. They had to be taken with the placid assurance of blind faith, which in that case was truly the "evidence of things not seen." To sum up, the net liability due to a heterogeneous mass of risks, when brought out in the lump, was no function of the combined sums assured, neither did it stand in any relation to the premium income. It was at once everything, because it was indiscriminately inclusive, and nothing, because it was attachable to none; it was shapeless and

soulless, belonging neither to Heaven nor earth, but, like Mahomet's coffin, it floated poised between. Even supposing the Board of Trade were to consent to stultify themselves, and to put aside the settled precedent of many years, it would still require a conviction on their part, deeper even than that born of the laudable enthusiasm that actuated Mr. Bell, and an eloquence even more seductive than had admittedly fallen from him that night, to persuade the profession to abandon the instructive simplicity of the analytical form of valuation prescribed by the wisdom of Parliament, and hitherto appreciatingly accepted by all, with the exception of a single critic.

The PRESIDENT (Mr. Henry Cockburn) said he was sure it would be the pleasure of the members to pass a hearty vote of thanks to Mr. Bell for his efforts. The paper must have involved an immense amount of labour, which alone was worthy of their hearty acknowledgment. The subject well exemplified the fact that, even in so every-day a matter as an office valuation, there was still much that might be brought forward for debate, and if the author had done nothing else—which he would not like to suggest—he had furnished material for a very interesting discussion. He had given the members something to think about, and nobody would wish to put out of mind the good work he had placed before them.

The vote was unanimously passed.

Mr. BELL, in reply, said he was sure that all the members would appreciate the fact that to reply to the criticisms which had been passed upon his paper was a very difficult task. It was a very unusual thing, he thought, for a paper to be read at the Institute, and to be promptly "sat upon" by some of its most able members. In the first place, he wished to thank those gentlemen who had criticized his paper adversely for the kind way in which they had spoken of the effort he had put forth to interest the Institute. In the second place, he wished to point out to them that his turn had come, and he had one reply, which had occurred to him while they had been speaking, which might possibly convince them that there might be something in the paper after all. His reply was that Mr. Elderton had spoken from a practical experience of the use of the method, and he thought he might fairly say that his practical experience had convinced that gentleman of the method's practical value. Further than that, he presumed most of the members had read Mr. Altenburger's letters published in volumes xxxiv and xxxv of the *Journal*. That gentleman had applied the method to endowment assurances, and stated in the letters that the system was very largely used in Germany. He had had the curiosity to write to Mr. Altenburger in order to bring his information up to date, and in a letter, dated during the current month, that gentleman said "This method of valuing endowment assurances is almost generally in use here." A method could not be so very bad if it was "almost generally in use" in Germany, because all must admit that German actuaries were not ignoramuses. He thanked Mr. Searle for his speech, and with regard to his proposal to substitute an endowment for what he (Mr. Bell) called the correction-numerator, that, it was almost obvious, had occurred to him in preparing the paper. He rather thought it was spoken of by some writer, either in Germany

or in America, but it seemed to him that $k_x=(P_x-\pi_x)N_x$ was an easily remembered and perfectly obvious formula, and after considering the matter he preferred to keep it as it was. He suggested that the reason his paper had been received with rather scanty welcome was perhaps not his fault. He should put the fault down entirely to Mr. Lidstone, who some years ago submitted a paper on the valuation of endowment assurances, and produced a very convenient and beautiful method of valuing them. It was not that his paper was bad, but that Mr. Lidstone's paper was so good. If he had read his paper at the time he proposed originally, some ten years ago, he would have been in front of Mr. Lidstone; he would have experienced a better reception, and would not have made the mistake of writing the paper without referring to Mr. Whiting's paper in the American "Transactions", because at that date Mr. Whiting's contribution to the American transactions had not been written. One of Mr. Phelps's objections was that he proposed to include policies of all classes in one group, but that he did not propose at all. What he said was, that they could be included in one group if the actuary wanted to include them, but personally he did not want to do so. In making the quinquennial valuation he should certainly not value all the policies together; but he did not think it at all impossible that, after he had valued in several groups, he should not combine the groups for the purpose of subsequent annual valuations, and he thought the possibility of doing this a very great convenience. With regard to limited premiums, one had naturally to alter the lists when the premium ceased, but in the ordinary (or prospective) method of valuation, when the premium ceased they had to take the policy out of the lists, and it mattered very little whether they took it out of one column or two. They had to strike out the premium in the prospective method, and in what he called the retrospective they took out the premium and the correction. The columns lay side by side, and it was a perfectly easy thing to do. It seemed to be suggested as a difficulty that the schedules required re-writing, but they did not require re-writing more than any other schedules; the only difference was that there was an additional column for the correction. In ordinary schedules, if bonus additions were made to the sums assured, those additions had to be given effect to; in the retrospective method exactly the same thing was done; there was no difference at all, so that if the objection applied to one method it applied equally to the other. He again thanked the gentlemen who had been good enough to discuss the paper, and he assured those members to whose criticisms he had replied that he fully appreciated the spirit in which they had been made, and thanked them for their courtesy.

CORRESPONDENCE.

INSURANCES AGAINST ISSUE.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In continuation of the Tables contributed by Dr. Sprague and myself,* giving certain particulars of Issue Insurances granted by British Life Offices, I annex a Table, the figures of which are taken from the Board of Trade Returns for the five years, 1896 to 1900, inclusive.

Combining these figures with those formerly given, we have the following results:

Years	No. of Policies	Net Sums Assured	Net Premiums received	Average Premium per-cent
		£	£	
1871–1875	261	875,558	62,238	6·75
1876–1880	383	1,264,166	97,495	7·51
1881–1885	539	1,696,747	116,704	6·82
1886–1890	698	2,076,859	133.200	6·08
1891–1895	987	2,836,634	139,137	4·75
1896–1900	1,310	3,530,173	156.561	4·39

The new figures show a considerable increase on previous results, especially in the case of the new premiums, although the average premium is again less than in the previous quinquennial periods.

I am, SIR,

Your obedient Servant,

Edinburgh, 6 *June* 1904. M. MACKENZIE LEES.

* See *J.I.A.*, xx, 151; xxiii, 220; xxvi, 391; xxix, 548; and xxxiv, 592.

Name	Date of Valuation	Number of Policies	Net Amounts, Deducting Re-Assurances: Sums Insured	Net Amounts, Deducting Re-Assurances: Premiums Received	Gross Amounts: Sums Insured	Gross Amounts: Premiums Received	Gross Amounts: Reserve	Gross Amounts: Average Premium per-cent
			£	£	£	£	£	
gal & General .	31 Dec. 1896	217	674,820	26,142	878,097	32,561	31,426	3·71
w Union .	31 ,, 1899	68	275,655	8,259	454,455	14,965	14,837	3·29
itish Empire .	31 ,, 1899	62	214,925	10,345	214,925	10,345	10,739	4·81
rwich Union .	30 Jun. 1896	59	210,141	4,540	277,591	6,424	7,851	2·31
w Life .	31 Dec. 1899	71	174,067	7,123	263,467	11,843	10,658	4·49
rth Brit. & Merc.	31 ,, 1895	46	142,953	7,515	215,641	9,476	7,159	4·39
uity & Law .	31 ,, 1899	50	138,785	9,155	194,261	13,264	13,264	6·83
gle .	31 ,, 1897	39	127,975	9,162	137,510	9,908	9,906	7·21
tional Mutual .	31 ,, 1898	75	127,318	4,091	160,094	4,692	4,692	2·93
ardian .	31 ,, 1899	67	122,385	6,350	217,685	11,390	10,993	5·23
ottish Equit. .	1 Mar. 1898	68	102,988	7,057	117,988	7,801	7,417	6·61
e Assocn. .	5 Apr. 1896	38	95,100	3,576	130,100	4,089	4,089	3·14
yal Exchange .	31 Dec. 1895	41	82,309	6,789	83,809	6,836	6,836	8·10
iance .	31 ,, 1898	25	80,928	2,953	112,928	4,297	4,297	3·81
ck .	31 ,, 1895	19	79,800	3,868	158,300	9,780	9,305	6·18
nmercial Union	31 ,, 1897	36	71,393	2,911	76,393	2,961	2,931	3·88
g. & Scot. Law	31 ,, 1895	28	66,314	1,888	84,814	3,376	3,376	3·98
perial .	31 ,, 1896	25	61,644	3,437	64,044	3,509	3,509	5·48
1 Life .	31 ,, 1896	6	60,025	707	60,025	707	797	1·17
as .	31 ,, 1899	26	53,320	2,531	83,220	4,500	4,500	5·41
ican .	31 ,, 1895	21	52,800	2,194	64,300	2,662	2,662	4·14
ndard .	31 ,, 1895	13	52,000	1,919	89,500	3,102	855	3·47
nomic .	31 ,, 1898	17	47,872	1,726	47,872	1,726	1,806	3·61
t. Union & Nat.	31 ,, 1899	14	37,683	1,496	47,683	1,946	1,946	4·08
edonian .	31 ,, 1897	24	36,845	2,435	41,345	2,577	1,889	6·23
nburgh .	31 ,, 1897	20	31,457	1,042	31,457	1,042	992	3·31
idon Assce. .	31 ,, 1895	9	30,640	2,533	30,640	2,533	1,928	8·27
nd in Hand .	31 ,, 1898	4	28,190	1,683	28,190	1,683	1,790	5·97
kshire .	1 Mar. 1900	12	26,525	1,298	27,500	1,400	1,478	5·09
ieral .	31 Dec. 1897	12	21,981	1,036	25,484	1,186	1,186	4·65
iversal .	31 ,, 1899	3	20,800	*1,544	*20,800	*1,544	1,544	7·46
'al .	31 ,, 1899	12	20,625	490	20,625	490	486	2·37
. Lon. & Globe	31 ,, 1898	9	18,700	1,223	18,700	1,223	1,162	6·53
vident .	31 ,, 1897	5	18,000	785	158,000	7,558	6,424	4·78
r. Med. & Gen.	30 Jun. 1896	6	17,907	1,121	17,907	1,121	1,121	5·25
iitable .	31 Dec. 1899	2	17,000	728	38,000	1,390	1,390	3·65
t. Metro. .	31 ,, 1898	15	13,790	616	29,050	1,876	1,536	6·45
thern .	31 ,, 1895	4	11,879	593	15,257	898	898	5·89
stminster & Gen.	31 ,, 1896	8	11,562	410	11,562	410	367	3·55
r .	31 ,, 1898	2	9,000	94	9,000	94	80	1·44
y of Glasgow .	31 ,, 1899	7	8,850	372	55,850	2,346	2,346	4·20
t. Life .	31 ,, 1896	7	8,600	169	26,600	349	291	1·31
riotic .	31 ,, 1899	4	7,500	611	7,500	611	205	8·15
. Temp. & Gen. Prov.	31 ,, 1895	2	6,680	693	6,680	693	693	10·37
cashire .	31 ,, 1899	2	5,500	110	5,500	110	72	2·00
v. Clerks .	31 ,, 1897	3	3,040	73	3,040	73	73	2·40
don & Lanc. .	31 ,, 1897	2	3,000	105	3,000	105	105	3·50
ted Kent .	25 Mar. 1897	1	2,896	525	2,896	525	577	18·13
don Life .	31 Dec. 1897	1	2,500	150	2,500	150	150	6·00
l. of Ireland .	31 ,, 1897	1	2,000	60	2,000	60	60	3·00
ine & Gen. .	31 ,, 1899	1	1,000	25	1,000	25	25	2·50
on .	31 ,, 1897	1	200	3	200	3	3	1·50
Offices .	...	1,310	3,530,173	156,561	4,874,991	214,238	204,722	4·39

* Approximated.

REVIEWS.

*Bunyon's Law of Life Assurance.**

Notwithstanding the fact that there are other—and, in many respects, admirable—treatises on Life Assurance law, actuaries and the profession generally have always had a particular regard for "Bunyon." The reasons for this are quite intelligible—in the first place, the work in its various editions has now been in use for fifty years, and, in the second place, it was originally written by one who combined great legal knowledge with practical experience of the work of a life office and familiarity with the kind of information most frequently required by officials as to the legal aspects of the business. For the present edition Mr. Vesey Fitzgerald is alone responsible, and although, as he is himself the first to admit in his Preface, he cannot lay claim to any great degree of familiarity with the practical work of a life office, his reputation as a lawyer is a sufficient guarantee that the new edition is, from a legal standpoint, successful in maintaining the high standard set by its predecessors. The second edition of "Bunyon" appeared in 1868, and the third, which appeared in 1891, had therefore to deal with great changes in the law—for instance, the Life Assurance Companies Acts, the Judicature Act and the Married Women's Property Acts. In the interval that has elapsed since 1891 there have been no such sweeping changes as were effected by the Acts mentioned, but, on the other hand, there have been numerous Statutes of less direct bearing upon the subject and also numerous decisions which it has been necessary to embody in the new edition. So far as a first inspection can show, the editor is fully justified in claiming that the work is up to date, and as to the value of the legal opinions expressed there can of course be no question.

The form of the treatise remains substantially the same, although some improvements have been introduced, *e.g.*, a further division of the work into Parts and the affixing of the dates to the cases contained in the Table of Cases Cited. It is, however, a matter of some regret that the index is not even fuller and more extended than is the case. Seeing the frequent necessity of quick reference to some particular point, the index to such a work well repays any amount of time and trouble that may be spent upon it, and it would certainly have been desirable that, either in the index or elsewhere, a Table of Statutes referred to should have been given. One or two omissions may be noted in this connection; *re Carter and Kenderdine*, referred to on p. 534, does not appear in the Table of Cases; *re Griffith's policy*, unlike *re Browne's policy*, is referred to p. 23 only instead of to p. 564 also. Such small defects are no doubt unavoidable, and, apart from the question of the index, if any adverse criticism could be passed upon the new edition in principle it would probably take the form of regret that a certain amount of matter which has become for all practical purposes

* *The Law of Life Assurance.* By C. J. Bunyon, M.A. Fourth Edition, by J. V. Vesey Fitzgerald, K.C. London: C. & E. Layton, 1904.

obsolete continues to be included, for instance, the remarks as to the remuneration of medical referees on p. 65.

It is not of course possible to deal exhaustively here with the whole contents of a volume of a length commensurate with its importance, but the following remarks, suggested by a perusal of certain of the chapters, may not be without interest. Part I deals with the Contract of Insurance and, *inter alia*, with the questions of Insurable Interest, Proposal and other Forms, and the completion of the Policy. Considerable space is devoted to the discussion of what constitutes an insurable interest, although the tendency of modern practice is to take wider views on this question than formerly, provided that the office is fully satisfied as to the *bonâ fides* and standing of the parties. Various instances of a legal insurable interest are given, but that of an employer in the life of his servant is not referred to except as regards American practice. It may here be noted that the decision in the recent unreported case of *Simcock* v. *Scottish Imperial Insurance Co.* is at variance with the statement in Harvey's Lectures on Insurance Law that the value of the interest of an employer in the life of his servant does not depend upon the amount paid as wages but on the expectation of pecuniary advantage to the employer from the engagement. The statement appears, however, to be reasonable. and would, it is apprehended, be generally acted upon in practice. It would have been interesting, too, to have had a reference to a case *re a policy of the Scottish Equitable* (85 L. T. 720); this was the case of a policy effected by a man "for behoof of" his wife's sister, and the views of the editor would have been very welcome on the point whether, in English law, a man can effect a policy in his own name for the benefit of a third party (apart of course from assurances under the Married Women's Property Acts)—whether, in other words, the insertion of a "beneficiary clause" as in America is operative here, and, if so, whether and under what conditions the beneficiary can be changed. The chapters on Proposal and Policy necessarily do not contain much that is new, except that the important case of *Roberts* v. *Security Co.* is duly discussed. Mr. Fitzgerald might, it would seem, have greatly strengthened, by a reference to the recent case of *Scott* v. *Coulson* (referred to on p. 376), his remarks on p. 85 as to completion of a life of another proposal where the life has died between acceptance and completion unknown to either the office or the proposer. Chapters on Accident, Issue, and Guarantee Policies complete Part I, several decisions under the Workmen's Compensation Act of 1897 being duly noted.

Part II deals with the Insurer and the Constitution of Offices, and appears to be very well done. It would perhaps have been unreasonable to look for any reference to the legal position of sinking fund policies as a charge against the Life Assurance Fund (*see* pp. 225-6), as the point would occur only to those actively engaged in life office work, but a short reference might have been made to the requirements of the Companies Acts, 1900, as to certain returns to the Registrar.

Part III is entitled "Dealings with Policies", and the only criticisms that occur on a first reading are that nothing appears to be

said as to how far notices of assignment of policies may be withdrawn, that in the chapter on Voluntary Settlements, the case of *in re Gould* (*see J.I.A.*, xxxv, 371, note) does not seem to be referred to, and that the remarks on p. 445 as to the unsuitability of reversionary interests as security for advances will not meet with general acceptance, unless by the expression "reversionary interests", only reversionary *life* interests are meant (which appears from the context not to be the case). It is believed that at any rate the majority of Actuaries would not see any objection to advancing by way of ordinary mortgage upon absolute reversions, having regard to the powers of sale and foreclosure that should be conferred by the mortgage deed, and even in the case of contingent reversions the opinion is widely held that advances by way of mortgage may safely be made, subject to due enquiry as to the position of the reversioner, and provided that the amount of the advance is fixed with proper regard to the possible necessity of commuting premiums at some future date, and with proper regard also to the preservation of a margin sufficient to enable the ordinary mortgagees' remedies to be resorted to without loss if occasion should arise.

Part IV, "Interests under Policies", treats, *inter alia*, of the payment of claims and of miscellaneous subjects such as Stamp Duties and Income Tax. The Life Assurance Companies Payment into Court Act, 1896, is duly noted, though the Chancery proceedings in *Harrison* v. *Alliance Assurance Co.*, subsequent to the King's Bench and Appeal decisions do not appear to have come before the editor. As an illustration of the care taken to bring the references up to date, it may be mentioned that the chapter on Stamp Duties contains a reference to the very recent case of *Prudential Assurance Co.* v. *Commissioners.* It is stated, p. 681, that a receipt for the policy money, or for the surrender-value, endorsed on a duly stamped policy is not considered to require a receipt stamp; this, however, is a point upon which varying rulings have been given by the Revenue authorities from time to time. The chapter on Income Tax appears to be very well done, although the recent extension of 16 and 17 Vict. c. 34, § 54, to Colonial Companies is not noted.

In conclusion, it may be pointed out that to write, or to edit, a text-book on the Law of Life Assurance is no easy matter. There is no Law of Life Assurance standing by itself; the Law of Contract, the Law of Companies, the Law of Mortgage, and numerous other branches of the law, not to mention fiscal provisions, have all to be brought under review. This fact increases the debt of gratitude which the profession owes both to the original author of this text-book and to his collaborator and successor, and it may confidently be predicted that, in spite of a few minor imperfections, the new edition of "Bunyon" will be heartily welcomed and appreciated.

J. E. F.

British Offices Life Tables, 1893, *Select Tables, Participating and Non-Participating Assurances.**

THE book before us may well be called the eighth volume of tables published in connection with the completed investigation into the Mortality Experience of British Life Assurance Companies, undertaken by the Institute of Actuaries and the Faculty of Actuaries. It is not official, but it follows official lines, and has been compiled by the methods which were officially adopted by the Joint Committee. Moreover, the senior author, Mr. H. J. Baker, was officially responsible to the Joint Committee for the calculation of the Mortality and Monetary Tables based upon the Assurance Experience, and the Mortality Tables based upon the Annuity Experience, which have appeared in the official volumes. Nothing more need be said as to the general form of the present work, and as to its arithmetical accuracy. Messrs. Baker and Raisin are to be congratulated on being the pioneers in the private enterprise required in order to complete the canon of tables necessary to bring into everyday use the great experience which we owe to the public spirit of the two Chartered Societies, and of the British Companies. The Seventeen Offices' Experience was practically a failure in this country, only on account of the absence of subsidiary Monetary Tables. The Institute of Actuaries' Experience was a great success, thanks to the Institute itself for the volume of Institute of Actuaries' Life Tables, and to Mr. R. P. Hardy for his Valuation Tables. It is safe to prophesy that the British Offices' Experience will be at least an equal success, as a result of the industry of the Joint Committee, and of private individuals such as Messrs. Baker and Raisin.

The new volume deals entirely with "Select" Values, and therefore, coupled with the small book, "Select Tables", published a year ago by the Joint Committee, marks an epoch in the history of life assurance. Formerly, selection was ignored. Now it is accepted as the guiding principle; and, if only a reasonably simple method could be devised, we may be sure that in the near future valuations would be made, as well as premiums calculated, on a select basis.

The first section of the book consists of tables calculated on the Experience of Participating Policies. This section is merely a valuable supplement to the official "Select Tables." In the official volume we find tabulated at $2\frac{1}{2}$ per-cent, 3 per-cent, $3\frac{1}{2}$ per-cent, and 4 per-cent interest, fully extended tables of $D_{[x]+t}$ and $N_{[x]+t}$, for all values of t, including the ultimate values; also, as at date of assurance only, the values of $a_{[x]}$, $A_{[x]}$, $P_{[x]}$ and $a_{[xx]}$, and the temporary annuities, $a_{[x]:\overline{n}|}$ for all values of n. Messrs. Baker and Raisin do not repeat any of these tables, except that of $a_{[x]}$, but they give us $\log D_{[x]+t}$, and $\log N_{[x]+t}$ in complete form, and also, similarly complete, $C_{[x]+t}$, $M_{[x]+t}$, $R_{[x]+t}$, $\log M_{[x]+t}$ and $a_{[x]+t}$. There are valuable additions also for Endowment Assurances, in tables of $A_{[x]:\overline{n}|}$ and $P_{[x]:\overline{n}|}$ for maturity ages 45, 50, &c., up to 70; and for Limited Payment

* "*British Offices Life Tables*, 1893. Select Tables, deduced from the Graduated Experiences of Whole-Life Participating and Non-Participating Assurances on Male Lives." By Henry J. Baker, F.I.A., and Arthur H. Raisin, F.I.A. London: C. & E. Layton, 56, Farringdon Street, E.C.

Policies, in ${}_nP_{[x]}$, for values of n, 5, 10, &c., up to 40; all these being given as at date of assurance only.

At the rates of interest already named, the two volumes are complementary to each other, and in many calculations must be used simultaneously. This is a slight drawback, but is unavoidable. Messrs. Baker and Raisin, however, provide, at $2\frac{3}{4}$ per-cent and $3\frac{1}{4}$ per-cent interest, complete tables, embracing all the functions enumerated above, with the exception of $a_{[xx]}$, so that for these rates of interest reference to their own book alone is necessary.

As regards mortality functions without interest, the official publication supplied complete tables of $l_{[x]+t}$, $d_{[x]+t}$, $q_{[x]+t}$, and $\mu_{[x]+t}$, with their logarithms, and a table of uniform seniority. Messrs. Baker and Raisin, besides repeating the tables of $l_{[x]+t}$ and $\mu_{[x]+t}$, add the very useful functions, $p_{[x]+t}$ and $\log p_{[x]+t}$, as also the curtate expectation of life as at the date of assurance, and ten years after that date.

In this first section of the work we have a most welcome replenishment of our arsenal of actuarial weapons.

The second section of the book is devoted to tables based on the Select Experience of Non-Participating Assurances, and is thus entirely new, and very valuable, if for no other reason than that it provides the best measure yet extant of the risk in this class of business, including Temporary and Contingent Assurances. The Mortality Tables themselves were constructed by Mr. G. F. Hardy in the way explained in his Memorandum given in the *Journal*, vol. xxxviii, p. 501. By Mr. Hardy's ingenious process, although select, they involve Makeham's law, and therefore the property of uniform seniority is retained. Moreover, the constant, c, being identical in all the assurance tables of the British Offices, the same seniority prevails throughout. In his Memorandum Mr. Hardy gave fully extended tables of $l_{[x]+t}$, $d_{[x]+t}$, $p_{[x]+t}$, $q_{[x]+t}$, and $\mu_{[x]+t}$, with their logarithms, and these are repeated, with permission, by Messrs. Baker and Raisin, who also add a table of the Curtate Expectation of Life as at the date of assuring, and five years thereafter.

The monetary tables are at $2\frac{1}{2}$ per-cent, 3 per-cent, and $3\frac{1}{2}$ per-cent interest, and comprise complete tables, including the ultimate values, of $D_{[x]+t}$, $\mathfrak{N}_{[x]+t}$, and $M_{[x]+t}$, with their logarithms, and $a_{[x]+t}$; while at the date of assuring we have also $A_{[x]}$ and $P_{[x]}$, and $a_{[x]}$ repeated; and, at 3 per-cent interest only, a complete table of temporary annuities, $a_{[x]:\overline{n}|}$.

All these are very useful, but an acquisition equally valuable is the table of single and annual premiums for temporary assurances of from one to ten years' duration, and the small table on page iii, Introduction, in which these new rates are compared with those derived from the $O^{[M]}$ select, and from the $H^{[M]}$ select tables respectively. The new $O^{[NM]}$ rates are, as was to be expected, higher throughout than the $O^{[M]}$, and also, except at the young ages, higher than the $H^{[M]}$. Probably this is the table of Term Premiums which will be most widely adopted in the near future. An expression in Mr. Hardy's Memorandum above named (*see* his paragraph 10, *J.I.A.*, xxxviii, 507), would seem to raise the question whether these rates for the terms of two and four years are to be trusted, but,

seeing that they correspond accurately with the rates for other terms, we do not think there need be any misgiving under this head.

The last feature to be noticed of the book under review is the table at 3 per-cent interest of single and annual premiums for Contingent Assurances. The rate of mortality for the assured life is taken by the new $O^{[NM]}$ Table, while that of the counter-life is taken by the British Offices Female Annuitant Select Table $O^{[af]}$. This is a combination which should ensure safe rates for such assurances, although the mortality of lives assured under contingent policies is notoriously high. Messrs. Baker and Raisin, in the Introduction, p. iv, give an interesting comparison of their annual premiums with those computed on various other bases, and it is satisfactory to note that our old friend the Carlisle Table gives rates which may be trusted, where the life assured is not more than 40 years of age, and is younger than the counter-life. These conditions generally prevail in practice.

THE INSTITUTE OF ACTUARIES.

OBITUARY NOTICES.

The President (Mr. Henry Cockburn), before delivering his inaugural address on 28 November 1904, referred in the following terms to the loss which the Institute had sustained by the death of two of its prominent members.

Mr. Archibald Day.

Since the last meeting the Institute had lost by death one who was a familiar figure in that room, their friend Mr. Archibald Day. Mr. Day was one of the very oldest members of the Institute; he was one of the candidates who presented themselves for examination on the first occasion an examination was held by the Institute, in the year 1850, when he passed the examination. During the long period of his membership his interest in the Institute had never abated. He filled in turn the offices of Honorary Secretary, Vice-President, and President, and he was a member of Council during no less than twenty-seven years of his membership. He contributed to the *Journal*, and for many years he attended the meetings regularly, and beyond all he was a man greatly esteemed and respected. Open-hearted and kindly, he was the friend not only of his contemporaries in the Institute, but also of other members junior to himself, many of whom were his debtors for assistance and encouragement, which he cheerfully gave them. The Institute had lost one of its staunchest supporters, and many of the members had lost a real friend, whose presence would long be missed, and whose name would be held in honourable remembrance.

Mr. David Deuchar.

They had also to deplore the loss by death of Mr. David Deuchar, of Edinburgh, an old friend of many of them, and a man of high purpose and ability in his profession. Mr. Deuchar's Fellowship dated from 1869. He served several times on the Council, and he was a member of the Joint Committee of the Institute and the Faculty on Mortality Investigation, in the work of which he took great interest. Shrewd and sympathetic, not sparing of himself, Mr. Deuchar, by his native qualities and by his good work, commanded the sincere respect of all who knew him.

EXAMINATION QUESTIONS.

By order of the Council, a pamphlet has been prepared containing a reprint of the Examination Questions for the years 1902, 1903, and 1904. Copies may be had from the Assistant Secretary, at the price of 1*s.* 6*d.* each.

SYLLABUS OF MESSENGER PRIZES, 1905.

The Council of the Institute of Actuaries have resolved to offer Two Prizes, of the value of *Fifty Guineas* and *Twenty-five Guineas* respectively, for the two best Essays on the following subject, namely:

> "*The Methods of ascertaining the Rates of Mortality amongst the General Population of a Country, District or Town, or amongst different classes of such population, by means of Returns of Population, Births, Deaths and Migration.*"

While Competitors will have complete freedom in their treatment of the subject, the following points are suggested as being very suitable for consideration. Due account will be taken of the degree of completeness with which the subject has been discussed.

1. Methods of collecting the data, including precautions for preventing errors likely to occur through carelessness or ignorance.

2. Methods of collating and summarizing the data. The Schedule System. The Card System, with or without mechanical means of sorting and enumerating.

3. The detection of, and corrections for, errors and misstatements of ages, &c.

4. The interpolation of the numbers for each year of age when the Returns are available for grouped ages only.

5. Allowance for the effect of migration in an old Country and in a new Country, and for fluctuations (diurnal and permanent) and shiftings of population.

6. Infantile Mortality.

7. The methods of deducing the Rates of Mortality.

8. The errors involved in making comparisons between the Rates of Mortality amongst different Sections of the community without proper allowance for differences of age distribution. The means of making proper comparisons. Rates of Mortality according to occupation.

9. The methods adopted in the principal Colonies and Foreign Countries.

Competitors will be expected to deal both with the theoretical basis and the practical application of the methods of calculation and the formulæ discussed, but extended numerical calculations are not desired, except so far as they may be necessary for purposes of illustration. New methods which may be suggested should be fully discussed and investigated.

CONDITIONS OF THE COMPETITION.

1. The Essays must be sent to the Honorary Secretaries of the Institute of Actuaries, at Staple Inn Hall, Holborn, London, W.C., not later than 30 September 1906.

2. The Adjudicators shall be the President and Vice-Presidents of the Institute at that date.

3. Each Competitor must send in his name, under seal, with a Motto corresponding to one to be prefixed to his Essay. Such Motto and Essay must not be in the handwriting of the Competitor, and should be, where practicable, type-written.

4. The Essay should (*a*) consist of numbered paragraphs, (*b*) if possible be divided into Sections under appropriate headings, (*c*) contain an extended Table of Contents, enabling ready reference to every important point.

5. Successful Essays shall become the property of the Institute.

6. Unsuccessful Essays will be returned, on application at the Institute, with the corresponding envelopes unopened.

7. No Prize will be awarded except to an Essay which the Adjudicators consider to be worthy of the distinction, and a prize will be divided if they so recommend.

8. The Competition shall be open to all Members of the Institute.

GEORGE TODD,
SAMUEL GEORGE WARNER, } *Honorary Secretaries.*

STAPLE INN HALL,
HOLBORN, LONDON, W.C.
January 1905.

LIST OF MEMBERS

OF

THE INSTITUTE OF ACTUARIES.

FOUNDED 1848.

INCORPORATED BY ROYAL CHARTER 29TH JULY, 1884.

(Corrected to 31 December, 1904.)

STAPLE INN HALL, HOLBORN, W.C.

G.P.O. TELEPHONE: No. 1710 CENTRAL.

LONDON:

PRINTED BY CHARLES & EDWIN LAYTON,

56, FARRINGDON STREET, E.C.

1905.

THE INSTITUTE OF ACTUARIES,

STAPLE INN HALL, HOLBORN, W.C.

PRESIDENT.

Henry Cockburn.

VICE-PRESIDENTS.

Thomas G. C. Browne.
George King.
Ernest Woods.
Frederick Schooling.

COUNCIL.

Thomas Gans Ackland.
*Henry James Baker.
Arthur Digby Besant, B.A.
James Blakey.
Thomas G. C. Browne.
Henry Cockburn.
George Stephen Crisford.
Gordon Douglas.
Joseph Ernest Faulks, B.A.
Duncan Cumming Fraser, M.A.
*Niel Ballingal Gunn.
George Francis Hardy.
*Ralph Price Hardy.
Charles Daniel Higham.
William Hughes.
George King.
*George James Lidstone.
Henry William Manly.
Arthur Ernest Molyneux.
*William Peyton Phelps, M.A.
Gerald Hemmington Ryan.
Frederick Schooling.
George Todd, M.A.
Ralph Todhunter, M.A.
Samuel George Warner.
Alfred William Watson.
James Douglas Watson.
Ernest Woods.
Frank Bertrand Wyatt.
Thomas Emley Young, B.A.

*Not Members of the last Council.

TREASURER.

Gerald Hemmington Ryan.

HONORARY SECRETARIES.

George Todd, M.A.
Samuel George Warner.

AUDITORS.

Bernard Woods.
William Macgeorge Monilaws.
George Andrew Brown.

LIBRARIANS.

Geoffrey Marks.
Henry Walsingham Andras.

EDITOR OF THE JOURNAL.

George King.

SUB-EDITOR.

Henry James Baker.

ASSISTANT SECRETARY.

S. H. Jarvis.

THE INSTITUTE OF ACTUARIES.

LIST OF MEMBERS.

HONORARY MEMBER.

1894 LIEUT.-COL. W. H. OAKES, Stanley House, Granville Road, Sevenoaks.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.

1876 †Ackland, Thomas Gans, F.S.S., Mem. Act. Soc. Amer., 5 & 6 *Clement's-inn, Strand*, W.C.

1871 †Addiscott, Francis, *Medical Sickness, Annuity & Life Assur. Soc.*, 33 *Chancery-ln.*, W.C.

1892 Adlard, Alfred Barton, *Law Life Assur. Soc.*, 187 *Fleet-street*, E.C.

1901 †Adlard, Howard Tindale, A.K.C., *The Equitable Life Assurance Society, Mansion-house-st.*, E.C.

1864 †Adler, Marcus Nathan, M.A., F.S.S., 22 *Craven-hill, Hyde-park*, W.

1894 †Aldcroft, William Hancock, *Refuge Assur. Co., Oxford-st., Manchester.*

1889 †Allen, Arthur Gregory, 13 *Fairfax-road*, N.W.

1897 †Allen, John Mayhew, c/o H. H. Hampson, Esq., 61–62 *Chancery-lane*, W.C.

1899 †Allin, Samuel John Henry Wallis, *Mutual Life Insurance Co. of New York*, 16, 17 & 18 *Cornhill*, E.C.

1889 †Anderson, John, *Hand-in-Hand Insurance Soc.*, 26 *New Bridge-street*, E.C.

1902 †Anderson, Thomas Frederic, F.F.A., *Royal Exchange Assurance Corporation, Royal Exchange*, E.C.

1891 †Anderson, William Smith, *Gresham Life Assurance Society, St. Mildred's-house, Poultry*, E.C.

1885 †Andras, Henry Walsingham, F.S.S. (LIBRARIAN), *Provident Life Office*, 50 *Regent-street*, W.

1885 †Ansell, Hubert, *Anglo-American Debenture Corporation, Ltd.*, 75 *Lombard-st.*, E.C.

1902 †Appleton, Frederick, *London Life Association, Ltd.*, 81 *King William-street*, E.C.

1896 †Archer, Joseph Alfred, *Ecclesiastical Commission*, 10 *Whitehall-place*, S.W.

1901 †Austin, Herbert Henry, *Prudential Assurance Company, Holborn-bars*, E.C.

1903 †Bacon, James, c/o *T. G. Ackland, Esq.*, 5 & 6 *Clement's-inn, Strand*, W.C.

1850 †Bailey, Arthur Hutcheson, F.S.S. (PAST PRESIDENT, 1878–82), 26 *Mount Ephraim-rd, Streatham*, S.W.

1896 †Baker, Henry James, *Metropolitan Life Assur. Soc.*, 13 *Moorgate-street*, E.C.

1885 †Barnes, Joseph Howard, F.S.S., *Pelican and British Empire Life Office*, 70 *Lombard-street*, E.C.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.	
1895	†Barraud, Arthur Rhys, *Prudential Assurance Company, Holborn-bars,* E.C.
1890	†Bearman, Harry, *Gresham Life Assur. Soc., St. Mildred's-house, Poultry,* E.C.
1889	†Bell, Frederick, *Alliance Assurance Co., Limited (Imperial Life Assurance Fund),* 47 *Chancery-lane,* W.C.
1886	†Berry, Berry Alfred, B.A., *London Life Association, Ltd.,* 81 *King William-street,* E.C.
1895	†Besant, Arthur Digby, B.A., *Clerical, Medical & General Life Assur. Soc.,* 15 *St. James's-square,* S.W.
1879	Besso, Marco, *Via Gregoriana* 54 *in the Villino Besso, Rome.*
1894	†Blackadar, Alfred Kimball, M.A., Mem. Act. Soc. Amer., *Government Insur. Department, Ottawa, Canada.*
1883	†Blakey, James, *National Debt Office,* E.C.
1897	†Bradshaw, Thomas, Mem. Act. Soc. Amer., *Imperial Life Assurance Co. of Canada, Toronto, Canada.*
1899	†Brown, Edward Harold, *Prudential Assurance Company, Holborn-bars,* E.C.
1904	†Brown, Henry, B.A., *Hand-in-Hand Insurance Soc.,* 26 *New Bridge-street,* E.C.
1901	†Brown, Hugh Wylie, F.F.A., *Scottish Union & National Insur. Company,* 35 *St. Andrew-square, Edinburgh.*
1875	Browne, Thomas G. C. (Vice-President), *Guardian Assurance Company,* 11 *Lombard-street,* E.C.
1887	Browne, Willis, *India Office,* S.W.
1901	†Buchanan, James, M.A., D.Sc., F.F.A., *Scottish Widows' Fund and Life Assurance Society,* 9 *St. Andrew-square, Edinburgh.*
1899	†Bull, Ernest James, *Mutual Life Association of Australasia,* 5 *Lothbury, Bank,* E.C.
1866	†Bumsted, David Alexander, *General Reversionary and Investment Co., Ltd.,* 26 *Pall-mall,* S.W.
1894	†Burn, Joseph, *Prudential Assurance Company, Holborn-bars,* E.C.
1887	†Byers, Frederick Timothy Mason, *Clergy Mutual Assurance Soc.,* 2 & 3 *The Sanctuary,* S.W.
1888	†Calderon, Henry Philip, *Law Accident Insurance Society, Limited,* 215 *Strand,* W.C.
1871	†Carment, David, F.F.A., Mem. Act. Soc. Amer., *Australian Mutual Provident Society, Sydney, Australia.*
1889	†Chatham, James, F.F.A., F.S.S., *Scottish Life Assurance Co.,* 19 *St. Andrew-sq., Edinburgh.*
1875	Cherriman, J. B., Prof., M.A., *c/o The Bank of Montreal, Abchurch-lane,* E.C.
1883	Chisholm, James, F.F.A., Mem. Act. Soc. Amer. *Crossfield, Alberta, N.W.T., Canada.*
1895	†Clarke, Arthur Harold, *Clerical, Medical and General Life Assurance Society,* 15 *St. James's-square,* S.W.
1863	Clirehugh, William Palin, F.S.S., *London and Lancashire Life Assurance Company,* 66 & 67 *Cornhill,* E.C.
1879	Cockburn, Henry, F.F.A., Mem. Act. Soc. Amer. (President), *North British and Mercantile Insurance Co.,* 61 *Threadneedle-street,* E.C.
1886	Cockburn, Henry Robertson, F.F.A., *University Club, Edinburgh.*

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.

1898 †Cockman, Arthur Charles Roadnight, *Liverpool & London & Globe Insurance Co.,* 1 *Dale-street, Liverpool.*

1884 †Colenso, Francis Ernest, M.A., *Eagle Insurance Company,* 79 *Pall-mall,* S.W.

1864 †Coles, John, J.P., F.S.S., 39 *Throgmorton-street,* E.C.

1903 †Collins, Frank Lakeman, *Clerical, Medical & General Life Assurance Soc.,* 15 *St. James's-square,* S.W.

1882 †Colquhoun, Ernest, *Legal and General Life Assur. Society,* 10 *Fleet-street,* E.C.

1875 †Cooke, Thomas Homans, *Glendower, Torre Vale, Torquay.*

1889 †Cooper, Walter George, 56 *Unthank-road, Norwich.*

1902 †Coutts, Charles Ronald Vawdrey, *Hand-in-Hand Insur. Society,* 26 *New Bridge-street,* E.C.

1878 †Crisford, George Stephen, *Rock Life Assurance Company,* 15 *New Bridge-street,* E.C.

1903 †Cross, Howard Turner, *Economic Life Assurance Soc.,* 6 *New Bridge-street,* E.C.

1889 †Cross, Robert, *Atlas Assurance Company,* 92 *Cheapside,* E.C.

1904 †Curjel, Harald Worthington, M.A., *Royal Insurance Co., Liverpool.*

1864 Curtis, Frank Allan, 3 *Ennismore-gardens, Salisbury-road, Dover.*

Under the Charter. Davies, Griffith, 11 *Freeland-road, Ealing,* W.

1898 †Dawson, Charles Pearl, *Alliance Assurance Co., Ltd. (Imperial Life Assurance Fund),* 47 *Chancery-lane,* W.C.

1904 †Dawson, Miles Menander, F.S.S., 11 *Broadway, New York, U.S.A.*

1885 †Day, Stanley, *Marine and General Mutual Life Assurance Society,* 14 *Leadenhall-street,* E.C.

1897 †Day, William Reginald, *Standard Life Association, Ltd.,* 28 *Elizabeth-street, Sydney, Australia.*

1903 †Denham, Walter, F.F.A., *City of Glasgow Life Assurance Co.,* 30 *Renfield-street, Glasgow.*

1883 Deuchar, John Jas. Walker, F.F.A., *Norwich Union Life Insurance Society, Norwich.*

1882 Dewey, Thomas Charles, *Prudential Assurance Company, Holborn-bars,* E.C.

1886 †Dickinson, Arthur Lowes, M.A., F.C.A., 54 *William-street, New York, U.S.A.*

1904 †Diver, Oswald Francis, M.A., *Clerical, Medical & General Life Assur. Soc.,* 15 *St. James'-sq.,* S.W.

1887 Douglas, Gordon, F.F.A., *Life Association of Scotland,* 82 *Princes-street, Edinburgh.*

1901 †Dunn, Spencer Græme, *Liverpool & London & Globe Insurance Co.,* 11 *Cornhill,* E.C.

1872 Eccles, Yvon Richard, *Scottish Amicable Life Assurance Society,* 1 *Threadneedle-st.,* E.C.

1897 †Elder, Kenneth William, *Atlas Assurance Company,* 92 *Cheapside,* E.C.

1901 †Elderton, William Palin, *Guardian Assurance Company,* 11 *Lombard-street,* E.C.

1898 †Elliott, Charles Alfred, *Australian Mutual Provident Society, Sydney, Australia.*

1889 †Faulks, Joseph Ernest, B.A., F.S.S., *Law Life Assurance Society,* 187 *Fleet-street,* E.C.

1897 †Fellows, Rowland Hill, F.S.S., *Pelican and British Empire Life Office,* 70 *Lombard-street,* E.C.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.

Under the Charter. Fisher, Richard Charles, *2 Walsingham-rd., Hove, Sussex.*

1892 †Foot, Herbert, B.A., *Northern Assurance Company, 1 Moorgate-street,* E.C.

1884 Frankland, Frederick William, F.S.S., Mem. Act. Soc. Amer., *"Okataina," Foxton, Manawatu, New Zealand.*

1900 †Fraser, Alexander, Jr., F.F.A., *Scottish Life Assur. Company, 19 St. Andrew-sq., Edinburgh.*

1897 †Fraser, Duncan Cumming, M.A., *Royal Insurance Co., Liverpool.*

1895 †Fulford, Frederick Wesley, *Prudential Assurance Company, Holborn-bars,* E.C.

1904 †Galer, Frederic Bertram, B.A., *Rock Life Assurance Company, 15 New Bridge-street,* E.C.

1902 †Gillies, George, *Union Assurance Society, 81 Cornhill,* E.C.

1887 Gillison, John Brotch, F.F.A., *National Mutual Life Association of Australasia, 76 & 77 Cornhill,* E.C.

1878 Gordon, Charles, F.F.A., *South African Mutual Life Assurance Society, Cape Town, South Africa.*

1901 †Gordon-Smith, Randolph, F.F.A., *Scottish Amicable Life Assur. Soc., 35 St. Vincent-pl., Glasgow.*

1882 †Graham, James, F.F.A., *Australian Widows' Fund Life Assurance Society, Collins-street-west, Melbourne, Australia.*

1904 †Grant, Milton Daniel, B.A., *Government Insurance Department, Ottawa, Canada.*

1886 Gunn, Niel Ballingal, F.F.A., *Scottish Amicable Life Assur. Soc., 35 St. Vincent-place, Glasgow.*

1864 Harben, Sir Henry, *Prudential Assurance Company, Holborn-bars,* E.C.

1880 †Hardy, George Francis, *7 Broad-street House,* E.C.

Date of becoming a Fellow.

1870 †Hardy, Ralph Price, *61 Addison-road,* W.

1893 †Harris, Arnold Stoughton, M.A., *Clerical, Medical & General Life Assurance Society, 36 Park-row, Leeds.*

1892 †Hart, James Robert, *Pelican and British Empire Life Office, 70 Lombard-street,* E.C.

1879 Harvey, Chas. J., *Colonial Life Insurance Co. of America, Jersey City, N.J., U.S.A.*

1888 †Hemming, Arthur George, F.S.S., *London Assurance Corporation, 7 Royal Exchange,* E.C.

1896 †Henderson, Robert, B.A., Mem. Act. Soc. Amer., *Equitable Life Assurance Soc. of the United States, 120 Broadway, New York, U.S.A.*

1864 Hendriks, Augustus, F.S.S., Mem. Act. Soc. Amer. (PAST PRESIDENT, 1892–94), *6 Observatory-gardens, Kensington,* W.

Under the Charter. Hendriks, Frederick, F.S.S., *7 Vicarage-gate, Kensington,* W.

1883 Hewat, Archibald, F.F.A., *Edinburgh Life Assurance Co., 22 George-street, Edinburgh.*

1874 †Higham, Charles Daniel, Mem. Act. Soc. Amer. (PAST-PRESIDENT, 1900–2), *London Life Association, Ltd., 81 King William-street,* E.C.

1898 †Hodgson, William Horsford, *Law Life Assurance Society, 187 Fleet-street,* E.C.

1899 †Holliday, John, M.A., F.S.S., *New York Life Insurance Co., 124 Rua du Ouvidor, Rio de Janeiro.*

1888 †Hopkins, William Raynes, *London and Lancashire Life Assur. Co., 66 & 67 Cornhill,* E.C.

1890 †Hovil, Lewis Frederick, *National Provident Institution, 48 Gracechurch-street,* E.C.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.

1871 †Hughes, William, Mem. Act. Soc. Amer. (Ex-President), *62 Palace-road, Tulse-hill,* S.W.

1894 †Hutcheson, William Anderson, F.F.A., Mem. Act. Soc. Amer., *Mutual Life Insurance Company of New York, Nassau-street, New York, U.S.A.*

1893 †Hutton, William, F.F.A., *Scottish Amicable Life Assur. Soc., 1 Threadneedle-street,* E.C.

1903 †Jarman, William Rees, B.A., *National Debt Office,* E.C.

1869 †Justican, Edwin, F.S.S., *Gresham Life Assurance Society, St. Mildred's-house, Poultry,* E.C.

1902 †Kenchington, Charles William, *Prudential Assurance Company, Holborn-bars,* E.C.

1897 †Kentish, Owen, *Economic Life Assurance Soc., 6 New Bridge-street,* E.C.

1874 †King, George, F.F.A., Mem. Act. Soc. Amer. (Vice-President), *15 Walbrook,* E.C.

1887 †Kyd, Thomas, F.F.A., *Northern Assurance Company, 1 Union-terrace, Aberdeen.*

1882 Lancaster, William John, J.P., *South Lynn, Putney-hill,* S.W.

1894 †Laughton, Alexander Millar, F.F.A., *National Mutual Life Assoc. of Australasia, Limited, Corner of Collins and Queen-streets, Melbourne, Australia.*

1887 †Lemon, William Kent, Barrister-at-Law, *1 Vanbrugh-terrace, Blackheath,* S.E.

1896 †Levine, Abraham, M.A., *National Mutual Life Assur. Soc., 39 King-st., Cheapside,* E.C.

1896 †Lewis, John Norman, F.F.A., *London Assurance Corporation, 7 Royal Exchange,* E.C.

1892 †Lidstone, George James, *Alliance Assurance Co., Limited, Bartholomew-lane,* E.C.

1901 †Little, James Fulton, *Mutual Life Association of Australasia, Sydney, Australia.*

1899 Low, George Macritchie, F.F.A., *Scottish Equitable Life Assur. Society, 28 St. Andrew-square Edinburgh.*

1899 †Lutt, Harold Edward William, *Northern Assurance Company, 1 Moorgate-street,* E.C.

1898 †Macaulay, Thomas Bassett, Mem. Act. Soc. Amer., *Sun Life Assurance Co. of Canada, Montreal, Canada.*

1874 McClintock, Emory, Mem. Act. Soc. Amer., *Mutual Life Insurance Company of New York, New York, U.S.A.*

1894 †McDonald, John, *Prudential Assurance Company Holborn-bars,* E.C.

1883 †McGowan, James, B.A., *The Treasury, Cape Town, South Africa.*

1885 Mackenzie, Alexander George, F.F.A., *47 York-terrace, Regent's-park,* N.W.

1900 †Macnaghten, Steuart Edye, A.C.A., *Equity & Law Life Assur. Soc., 18 Lincoln's-inn-fields,* W.C.

1901 †Macphail, Donald, F.F.A., *Yorkshire Insurance Company, York.*

1870 †Manly, Henry William, Mem. Act. Soc. Amer. (Past President 1898-1900), *Equitable Life Assurance Soc. Mansion-house-street,* E.C.

1890 †Marks, Geoffrey (Librarian) *National Mutual Life Assur. Soc. 39 King-street, Cheapside,* E.C.

1900 †Marr, Vyvyan, F.F.A., *Edinburgh Life Assurance Co., 22 George-street, Edinburgh.*

1902 †May, Basil, *National Mutual Life Assur. Soc., 39 King-street, Cheapside,* E.C.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.	
1897	†May, George Ernest, *Prudential Assurance Company, Holborn-bars,* E.C.
1897	†Miller, Neville, *London Assurance Corporation, 7 Royal Exchange,* E.C.
1893	†Milner, John William, *North British & Mercantile Insur. Co., 61 Threadneedle-street,* E.C.
1892	†Milton, Henry, M.A., *Law Debenture Corporation, Ltd., 41 Threadneedle-street,* E.C.
1899	†Moir, Henry, F.F.A., Mem. Act. Soc. Amer., *Provident Savings Life Assur. Soc., 346 Broadway, New York, U.S.A.*
1890	†Molyneux, Arthur Ernest, *Provident Clerks' and General Mutual Life Assurance Assoc., 27 & 29 Moorgate-street,* E.C.
1901	†Moorhouse, Alfred, *Friends' Provident Institution, Bradford, Yorkshire.*
1897	†Moors, Elphinstone MacMahon, M.A., *University of Sydney, Australia.*
1896	†Moran, Joseph Flack, *Marine & General Mutual Life Assurance Society, 14 Leadenhall-street,* E.C.
1900	†Morgan, Benjamin Charles, M.A., *Commercial Union Assur. Co., 24, 25 & 26 Cornhill,* E.C.
1895	†Muter, Percy, *New Zealand Government Life Insurance Department, Wellington, New Zealand.*
1888	†Nash, Willie Oscar, *Law Reversionary Interest Soc., Ltd., 24 Lincoln's-inn-fields,* W.C.
1883	Neison, Francis G. P., F.S.S., *19 Abingdon-st., Westminster,* S.W.
1888	†Newman, Philip Lewin, B.A., *Yorkshire Insurance Co., York.*
1865	Newton, Algernon, M.A., *c/o London & Westminster Bank, 94 & 96 High-st., Kensington,* W.

Date of becoming a Fellow.	
1887	†Nightingale, Harry Ethelston, *Royal Exchange Assurance Corporation, Royal Exchange,* E.C.
1903	†Norris, Charles Arthur, *National Mutual Life Association of Australasia, Limited, Melbourne, Australia.*
1901	†Norton, William Ernest, *National Provident Institution, 48 Gracechurch-street,* E.C.
1899	†Parker, Robert Peter, *Sun Life Assurance Society, 63 Threadneedle-street,* E.C.
1864	Pearson, Arthur, *Betchworth-house, The Bank, Highgate,* N.
1891	†Phelps, William Peyton, M.A., *Equity and Law Life Assur. Soc., 18 Lincoln's-inn-fields,* W.C.
Under the Charter.	Priestley, John George, *44 St. German's-road, Forest-hill,* S.E.
1891	†Pulley, William Pritchard, *Norwich Union Life Insur. Soc., 71 & 72 King William-st.,* E.C.
1903	†Rae, Joseph, *Finance Department, Town-hall, Upper-street,* N.
1899	†Raisin, Arthur Herbert, *Pelican and British Empire Life Office, 70 Lombard-street,* E.C.
1897	†Rees, Martin, *Law Reversionary Interest Soc., Ltd., 24 Lincoln's-inn-fields,* W.C.
1901	†Reeve, Charles Ernest, *Royal Exchange Assurance Corporation, Royal Exchange,* E.C.
1902	†Richmond, George William, *Scottish Widows' Fund and Life Assur. Society, 28 Cornhill,* E.C.
1904	† Rietschel, Hermann Julius, *Sun Life Assurance Society, 63 Threadneedle-street,* E.C.
1898	†Robinson, George Frederick, *Legal and General Life Assur. Society, 10 Fleet-street,* E.C.
1888	†Rusher, Edward Arthur, F.S.S., *Prudential Assurance Company, Holborn-bars,* E.C.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.	
1882	†Ryan, Gerald Hemmington, Mem. Act. Soc. Amer. (TREASURER), *Pelican and British Empire Life Office, 70 Lombard-street,* E.C.
1898	†Salmon, Richard George, F.S.S., *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.
1883	Saunders, Harris Charter Lindon, F.R.A.S., *"Marquise," Twickenham.*
1886	†Schooling, Frederick (VICE-PRES.), *Prudential Assurance Company, Holborn-bars,* E.C.
1901	†Searle, George Morley, *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.
1901	†Sharman, William Charles, *Prudential Assurance Company, Holborn-bars,* E.C.
1896	†Sim, William Abernethy, F.F.A., *Scottish Union and National Insurance Co., 35 St. Andrew-square, Edinburgh.*
1875	†Smither, Arthur, *Ince's, Scaynes-hill, Lindfield.*
1881	†Somerville, William Finlay, *Liverpool and London and Globe Insurance Co., 7 Cornhill,* E.C.
1877	†Sorley, James, F.F.A., C.A., F.R.S.E., 32 *Onslow-square,* S.W.
1898	†Spencer, John, *English and Scottish Law Life Assurance Assoc.,* 12 *Waterloo-place,* S.W.
1894	†Sprague, Alfred Ernest, B.Sc., M.A., F.F.A., *Edinburgh Life Assurance Co.,* 22 *George-street, Edinburgh.*
1857	Sprague, Thomas Bond, M.A., LL.D., Hon. F.F.A., F.S.S., F.R.S.E. (PAST PRESIDENT, 1882–86), 29 *Buckingham-ter., Edinburgh.*
1896	†Stahlschmidt, Louis, *Alliance Assurance Co., Limited, Bartholomew-lane,* E.C.
Unde the Charter	Stevens, Charles, *Aberdeen Ho., Preston, Brighton.*
1888	Stewart, John, F.F.A., *City of Glasgow Life Assur. Co.,* 30 *Renfield-street, Glasgow.*
1898	Stirling, Robert, F.F.A., *Law Union & Crown Insurance Co., 126 Chancery-lane,* W.C.
1892	†Straker, Edward Robert, *Pelican and British Empire Life Office, 70 Lombard-street,* E.C.
1878	†Straker, Frank Arthur, *Legal and General Life Assur. Society, 10 Fleet-street,* E.C.
1902	†Strong, William Richard, *London Guarantee & Accident Co., 61 Moorgate-street,* E.C.
1884	†Stuart, John Moody, F.F.A., *Leeds Permanent Benefit Building Society, Victoria-buildings, Park-lane, Leeds.*
1900	†Sutherland, John, M.A., *Australasian Temperance and General Mutual Life Assurance Society, Swanston-street, Melbourne, Australia.*
1889	†Tarn, Arthur Wyndham, *Westminster and General Life Assurance Association, 28 King-street, Covent-garden,* W.C.
1887	Teece, Richard, F.F.A., F.S.S., Mem. Act. Soc. Amer., *Australian Mutual Provident Society, Sydney, Australia.*
1872	Templeton, Col. John M., C.M.G., *National Mutual Life Association of Australasia, Melbourne, Australia.*
1864	†Terry, James, *Hernlee, Lyme Regis, Dorset.*
1889	†Thiselton, Herbert Cecil, F.F.A., Mem. Act. Soc. Amer., *Hand-in-Hand Insurance Soc., 26 New Bridge-street,* E.C.
1901	†Thodey, Robert, *Australian Mutual Provident Society, Sydney, Australia.*
1893	†Thomas, Ernest Charles, *Gresham Life Assurance Society, St. Mildred's-house, Poultry,* E.C.
1899	†Thomas, Robert Arthur Caradoc, *Pelican and British Empire Life Office, 12 Dalhousie-sq., Calcutta.*

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.	
1895	†Thomson, Herbert Archer, B.A., *Umberslade, Boscobel-road, St. Leonard's-on-Sea.*
1880	Thomson, Robert, *Colonial Mutual Life Assurance Society, Collins-street-west, Melbourne, Australia.*
1893	†Thorne, Alfred Charles, *Equity & Law Life Assur. Soc., 18 Lincoln's-inn-fields,* W.C.
1891	†Tilt, Robert Ruthven, *General Reversionary & Investment Co., Ltd., 26 Pall-mall,* S.W.
1902	†Tinner, Thomas, *Comptroller's Depart., London County Council, Spring-gardens,* S.W.
1881	†Todd, George, M.A. (Hon. Sec.), *Economic Life Assurance Society, 6 New Bridge-street,* E.C.
1894	†Todhunter, Ralph, M.A., *University Life Assur. Soc., 25 Pall-mall,* S.W.
1899	†Trouncer, Harold Moltke, M.A., *London Life Association, Ltd., 81 King William-street,* E.C.
1878	Turnbull, Andrew Hugh, F.F.A., F.R.S.E., *Scottish Widows' Fund and Life Assur. Soc., 9 St. Andrew-square, Edinburgh.*
1889	Wallace, Thomas, F.F.A., *North British & Mercantile Insurance Co., 64, Princes-street, Edinburgh.*
1888	†Warner, Samuel George (Hon. Sec.), *Law Union & Crown Insur. Co., 126 Chancery-lane,* W.C.
1893	†Watson, Alfred William, *Manchester Unity Friendly Soc., Nottingham.*
1895	†Watson, James Douglas, *English & Scottish Law Life Assr. Assoc., 12 Waterloo-place,* S.W.
1904	† Weatherill, Henry, *National Debt Office,* E.C.

Date of becoming a Fellow.	
1880	†Whittall, Wm. Joseph Hutchings, Mem. Act. Soc. Amer., *Clerical, Medical & General Life Assur. Soc., 15 St. James's-sq.,* S.W.
1864	Wilson, Robert, *44 Talfourd-rd., Camberwell,* S.E.
1888	†Wilson, Robert, Jr., *General Assurance Company, 103 Cannon-street,* E.C.
Under the Charter.	Winser, Thomas Boorman, F.R.G.S., F.R.N.S., *81 Shooter's-hill-road, Blackheath,* S.E.
1899	†Winter, Arthur Thomas, *Pelican and British Empire Life Office, 70 Lombard-street,* E.C.
1897	†Wintle, Lancelot Andrewes, *Economic Life Assurance Soc., 6 New Bridge-street,* E.C.
1904	†Wood, Arthur Barton, B.A., Mem. Act. Soc. Amer., *Sun Life Assurance Co. of Canada, Montreal, Canada.*
1884	†Woods, Ernest, Mem. Act. Soc. Amer. (Vice-Pres.), *Westminster and General Life Assurance Association, 28 King-street, Covent-garden,* W.C.
1902	†Woolmer, Alfred Henry, *Star Life Assurance Society, 32 Moorgate-street,* E.C.
1902	†Workman, William Arthur, *Equitable Life Assur. Society Mansion-house-street,* E.C.
1902	†Worthington, William, *Royal Insurance Co., Liverpool.*
1875	†Wyatt, Frank Bertrand, Mem. Act. Soc. Amer., *Clergy Mutual Assurance Soc., 2 & 3 The Sanctuary,* S.W.
1874	Young, Thomas Emley, B.A., F.R.A.S. (Past-President, 1896-8), Mem. Act. Soc. Amer., *108 Evering-road, Stoke Newington,* N.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1900 [2] Adams, Cecil Francis, *New Zealand Accident Insurance Company, Palmerston North, New Zealand.*

1869 [2] Adey, Theodore Henry, *Scottish Provident Institution,* 17 *King William-street,* E.C.

1899 [3] Adlard, Stanley, A.K.C., *London Life Association, Ltd.,* 81 *King William-street,* E.C.

1899 [2] Ansell, George Frederic, *National Debt Office,* E.C.

1904 [2] Ashley, Charles Henry, *Refuge Assurance Company, Oxford-street, Manchester.*

1883 [2] Ashley, John Geo., M.A., *War Office,* S.W.

1901 [2] Ashton, William Richard, *Hand-in-Hand Insurance Soc.,* 26 *New Bridge-street,* E.C.

1901 [2] Atkins, Leonard George, *Law Union & Crown Insurance Co.,* 126 *Chancery-lane,* W.C.

1881 [2] Ayling, Charles Stephen, *Commercial Union Assur. Co.,* 20 *New Bridge-street,* E.C.

1903 [2] Ball, Sidney Robertson, *English and Scottish Law Life Assurance Association,* 12 *Waterloo-place,* S.W.

1904 [2] Barrett, William Goodsman, *United Kingdom Temperance and General Provident Institution,* 1 *Adelaide-place, London-bridge,* E.C.

1885 Barton, Arthur, *Royal Insurance Company, Maidstone.*

1894 [3] Barton, Robert Whitchurch, *Clerical, Medical & General Life Assurance Society,* 15 *St. James's-square,* S.W.

1903 [2] Baxter, Edwin Herbert, *Scottish Provident Institution,* 17 *King William-street,* E.C.

1901 [2] Benjamin, Stanley O., *Australian Mutual Provident Society, Sydney, Australia.*

1881 Birks, Edmund Alfred, *Yorkshire Insurance Co., York.*

Date of becoming an Associate.

1898 (2) Blount, Edward Thos. J., F.F.A., F.S.S., *Standard Life Assurance Co., Shanghai, China.*

1873 [2] Boon, Gerald Inglis, *United Legal Indemnity Insur. Soc., Limited,* 222 *Strand,* W.C.

1889 (2) Bremner, Thomas William, F.F.A., *Mutual Life Insurance Co. of New York, Sydney, Australia.*

1896 (2) Brown, George Andrew (AUDITOR), *Clerical, Medical & General Life Assurance Society,* 1 *King William-street,* E.C.

1899 [2] Brown, Harold, *Scottish Union and National Insurance Co.,* 3 *King William-street,* E.C.

1886 Buckley, Thomas John Wesley, 9 *St. Andrew-street, Holborn-circus,* E.C.

1882 Burke, David, F.S.S., *Royal Victoria Life Insur. Co., Montreal, Canada.*

1900 [2] Burnley, Isaac, *Australian Mutual Prov. Society, Sydney, Australia.*

1895 [3] Butterfield, William Thos., A.C.A., 9 *Market-street, Bradford, Yorkshire.*

1876 Carter, Eric Mackay, 33 *Waterloo-street, Birmingham.*

1899 [2] Catchlove, Chas. Hamilton Leyland, *Australian Mutual Provident Society, Sydney, Australia.*

1904 (2) Cathles, Lawrence Maclagan, F.F.A., *Provident Savings Life Assur. Soc.,* 346 *Broadway, New York, U.S.A.*

1900 [3] Chandler, Thomas Richard, *London Assurance Corporation,* 7 *Royal Exchange,* E.C.

1898 [2] Coates, Thomas Linnaeus, *North British and Mercantile Insurance Co.,* 61 *Threadneedle-street,* E.C.

1904 [2] Collier, Charles Aubrey, 30 *Crockerton-road, Tooting,* S.W.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1871 Cook, Arthur James, M.J.I., *Victoria Mutual Assur. Society, Farringdon-street,* E.C.

1899 3 Cook, William Playfair, *Guardian Assurance Company,* 11 *Lombard-street,* E.C.

1897 2 Coop, Charles Rowland, *United Kingdom Temperance and General Provident Institution,* 28 *High-street, Birmingham.*

1891 2 Coote, Ernest Charles, *Alliance Assurance Co., Ltd., Bartholomew-lane,* E.C.

1900 2 Corbett, Edwin Somerville, *Australasian Temperance and General Mutual Life Assurance Soc., Adelaide, South Australia.*

1871 Coutts, Edwin Arthur, *North British and Mercantile Insurance Company, Victoria-street, Nottingham.*

1900 2 Covington, Oliver Henry, *Prudential Assurance Company, Holborn-bars,* E.C.

1884 Craig, Robert Alexander, *Abstainers' and General Assur. Co., City Buildings, Birmingham.*

1901 3 Culley, Alfred Benjamin, *Star Life Assurance Society,* 32 *Moorgate-street,* E.C.

1900 3 Curtis, William Allen, *Clerical, Medical & General Life Assurance Society,* 15 *St. James's-square,* S.W.

1904 2 Daman, Gerard William, B.A., *Hand-in-Hand Insurance Soc.,* 26 *New-bridge-street,* E.C.

1902 2 Denmead, John Charles, M.A., *Estate Duty Office, Somerset-house,* W.C.

1901 2 Diamond, George Frederick, *City Mutual Life Assurance Society, Hunter-street, Sydney, Australia.*

1855 Dix, James, *Hurstdale, Wood-la., Highgate,* N.

1901 (2) Donald, Alexander Graham, M.A., F.F.A., *Scottish Provident Institution,* 6 *St. Andrew-square, Edinburgh.*

1881 Donaldson, John, *Australian Widows' Fund Life Assurance Society, Collins-street-west, Melbourne, Australia.*

1899 2 Dougharty, Harold, F.S.S., F.C.I.S., *London and Lancashire Life Assurance Company,* 66 & 67 *Cornhill,* E.C.

1902 2 Doust-Smith, Ernest Charles, *Prudential Assurance Company, Holborn-bars,* E.C.

1881 Dovey, William Roadly, F.F.A., Mem. Act. Soc. Amer., 26 *Drylands-road, Crouch End,* N.

1870 Dowson, John, *Royal Insur. Company, Liverpool.*

1898 2 Doyle, Arthur James, 54 *Bourke-st., Sydney, Australia.*

1901 2 Earle, Arthur Percival, *Reliance Life Insurance Co., Farmers' Bank-buildings, Pittsburgh, Pa., U.S.A.*

1868 Eaton, Henry William, *Liverpool & London & Globe Insurance Company, William-street, New York, U.S.A.*

1904 2 Ebihara, Kaitaro, 41 *St. Stephen's - avenue, Shepherd's Bush,* W.

1904 2 Ecroyd, Cuthbert W., *Friends' Provident Institution, Bradford, Yorkshire.*

1872 2 Evans, William, F.F.A., F.R.S.E., 38 *Morningside-park, Edinburgh.*

1896 2 Featherstonehaugh, William Irwin, *Commercial Union Assurance Co.,* 24, 25 & 26 *Cornhill,* E.C.

1903 2 Ferguson, Colin C., B.A., *Canada Life Assurance Co., Toronto, Canada.*

1897 2 Findlay, Alexander Wynaud, LL.D., *Prudential Assurance Company, Holborn-bars,* E.C.

1902 2 FitzGerald, Charles R., *Home Life Association of Canada, Toronto, Canada.*

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1901 [2] FitzGerald, William George, B.A., 82 *Dupont-street, Toronto, Canada.*

1890 (2) Fox, Charles Edward, F.F.A., *Standard Life Assurance Co., 83 King William-street,* E.C.

1886 Fox, Morris, Mem. Act. Soc. Amer., *New Zealand Government Life Insurance Dept., Wellington, New Zealand.*

1894 [2] Fraser, Thomas John, *Australian Alliance Assurance Company, Melbourne, Australia.*

1901 (2) Gaff, William Robertson, C.A., F.F.A., 3 *Crown-court, Old Broad-st.,* E.C.

1873 [2] Gage, Uriah Woodard, *North British and Mercantile Insurance Company,* 61 *Threadneedle-street,* E.C.

1895 [2] Galwey, Charles Edmund, *New Zealand Government Life Insurance Dept., Wellington, New Zealand.*

1893 [2] Gardiner, Robert Edward, *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.

1885 [2] Gayford, Herbert Stannard, *Northern Assurance Co.,* 1 *Moorgate-street,* E.C.

1903 [3] Gemmill, William, 7 *Royal-terrace, Queen's-park, Glasgow.*

1899 [3] Gibb, James Burnett, F.F.A., *Penn Mutual Life Insce. Co. of Philadelphia,* 923 *Chestnut-st., Philadelphia, U.S.A.*

1871 [2] Glennie, William Gordon, *Scottish Union & National Insur. Co.,* 3 *King William-street,* E.C.

1897 [2] Goggs, Frank Sidney, *Scottish Metropolitan Life Assur. Co., Ltd.,* 25 *St. Andrew-sq., Edinburgh.*

1882 Goldman, Leopold, *North American Life Assurance Co., North American Life Building,* 112-118 *King-street-west, Toronto, Canada.*

Date of becoming an Associate.

1904 [2] Goodman, Gilbert, *Prudential Assurance Company, Holborn-bars,* E.C.

1897 [2] Goodwyn, John, Jr., *Norwich & London Accident Insurance Association, Sydney, Australia.*

1902 [2] Gray, Robert Alexander, B.A., 324 *Markham-street, Toronto, Canada.*

1898 [3] Green, George, M.A., *Union Assurance Society,* 81 *Cornhill,* E.C.

1868 Greig, John Andrew, *Sun Life Assurance Society,* 60 *Charing-cross,* S.W.

1869 Griffith, E. Clifton, 4 *Carlton-chambers,* S.W.

1903 [2] Hall, John Bertram, *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1893 [2] Hall, John Francis Edmund, *Eagle Insurance Company,* 79 *Pall-mall,* S.W.

1869 Hann, Robert George, Mem. Act. Soc. Amer., *Equitable Life Assur. Soc. of the United States,* 120 *Broadway, New York.*

1894 [2] Hardcastle, Edward Edgington, M.A., *Union Central Life Office, Cincinnati, Ohio, U.S.A.*

1900 [2] Harding, Harry Burnard, *Hand-in-Hand Insurance Soc.,* 26 *New Bridge-street,* E.C.

1896 [3] Harris, Frederick Joseph, *Australian Mutual Provident Society, Sydney, Australia.*

1904 [2] Harriss, Walter James, *Union Assurance Society,* 81 *Cornhill,* E.C.

1897 [2] Haycraft, William Melhuish, *Prudential Assurance Company, Holborn-bars,* E.C.

1897 [2] Hazell, James Stanley, *National Provident Institution,* 48 *Gracechurch-street,* E.C.

1895 [2] Heness, Leonard Thomas, *Prudential Assurance Company, Holborn-bars,* E.C.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute. Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1878 Henry, Alfred, F.C.A., *Throgmorton-house, Copthall-avenue,* E.C.

1900 3 Hicks, Arthur Joseph, *Law Life Assurance Society, 187 Fleet-street,* E.C.

1884 Higham, William Samuel, *Equitable Life Assurance Soc., Mansion-house-street,* E.C.

1894 2 Hollingworth, Albert Charles, *Australian Mutual Provident Society, Sydney, Australia.*

1883 Holt, Edward Hallett, *Law Life Assurance Society, 187 Fleet-street,* E.C.

1898 2 Howell, Chas. Edward, B.A., LL.D., *Standard Life Assurance Compy., 59 Dawson-street, Dublin.*

1899 2 Hudson, Alfred James, *Northern Assurance Company, 1 Moorgate-street,* E.C.

1903 2 Humphreys, Henry Thompson, *Sun Life Assurance Society, 63 Threadneedle-street,* E.C.

1875 Hunt, Richard Aldington, F.S.S., *Wesleyan & General Assur. Soc., Corporation-street, Birmingham.*

1893 (2) Hunter, Arthur, F.F.A., Mem. Act. Soc. Amer., F.S.S. *New York Life Insurance Co., 346 & 348 Broadway, New York, U.S.A.*

1902 2 Hunter, Robertson G., Mem. Act. Soc. Amer., *New York Life Insurance Co., New York Life Building, Chicago, Ill., U.S.A.*

1887 2 Hunter, Samuel, *Patriotic Assurance Company, 9 College-green, Dublin.*

1904 (2) Imrie, John Hamilton, M.A., F.F.A., *Life Association of Scotland, 82 Princes-street, Edinburgh.*

1889 (2) Jacobs, Frederick Job, *Australian Mutual Provident Society, Sydney, Australia.*

1876 2 James, George Trevelyan, *12 Waterloo-place,* S.W.

1871 Jellicoe, George Rogers, *Eagle Insurance Company, 79 Pall-mall,* S.W.

Date of becoming an Associate.

1883 Jerman, Richard, *Commercial Union Assurance Company, Exeter.*

1896 2 Jobson, Alexander, *Equitable Life Assurance Society of the United States, Sydney, Australia.*

1894 2 Johannessen, Nikolai Mikal, *Hygea Life Assurance Company, Bergen, Norway.*

1894 2 Johnston, Frederick H., *Prudential Life Insurance Co. of America, Newark, N.J., U.S.A.*

1903 2 Jones, Leonard Alexander Mouat, *Hand-in-Hand Insur. Society, 26 New-bridge-street,* E.C.

1903 2 Jones, Wallace Mouat, *General Reversionary & Investment Company, Limited, 26 Pall-mall,* S.W.

1898 2 Kaufman, Henry N., Assoc. Act. Soc. Amer., *Phœnix Mutual Life Insurance Co., Hartford, Connecticut, U.S.A.*

1876 Kearry, Joseph, *44 Charlwood-street, Belgrave-road,* S.W.

1899 2 Kelly, John Joseph, *Citizens' Life Assurance Co., Sydney, Australia.*

1897 2 Kemp, Julian Ernest Sandford, *Eagle Insurance Company, 79 Pall-mall,* S.W.

1902 2 Kilgour, David Errett, M.A., *North American Life Assurance Co., North American Life Building, 112-118 King-street-west, Toronto, Canada.*

1874 King, Arthur Thomas, I.S.O., *National Debt Office,* E.C.

1882 2 King, William Alfred, *Northern Assurance Company, 1 Moorgate-street,* E.C.

1902 2 Kitchin, Frederick Harcourt, B.A., *Corehythe, Kingston-lane, Teddington.*

1893 2 Laing, William Claud, *North British and Mercantile Insurance Company, 61 Threadneedle-street,* E.C.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1897 [2] Lane, Arthur Vere, B.A., *City of Glasgow Life Assurance Co.,* 1 *Princess-street, Albert-square, Manchester.*

1899 [2] Lawton, George Herbert, *Clerical, Medical & General Life Assurance Society,* 15 *St. James's-square,* S.W.

1885 Ledward, Archibald Prentice, B.Sc., 29 *Langland-gardens,* N.W.

1879 Leitch, Alexander, *Scottish Provident Institution,* 17 *King William-street,* E.C.

1897 [2] Le Maitre, Frank William, *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.

1885 Leveaux, Arthur Michael, F.S.S., *Registry of Friendly Societies, Central Office,* 28 *Abingdon-street, Westminster,* S.W.

1885 [2] Lidbury, Isaac Stephen, *Prudential Assurance Company, Holborn-bars,* E.C.

1868 Litchfield, Edward, 92 *St. Vincent-street, Glasgow.*

1876 [2] Lucey, Herbert, *General Assurance Company,* 103 *Cannon-street,* E.C.

1890 (2) Lugton, Hugh, F.F.A., *North British and Mercantile Insurance Co.,* 61 *Threadneedle-street,* E.C.

1900 [3] McArthur, Harry de C., *Economic Life Assur. Society,* 6 *New Bridge-street,* E.C.

1867 Macdonald, William Rae, F.F.A., *Scottish Metropolitan Life Assur. Co., Limited,* 25 *St. Andrew-square, Edinburgh.*

1882 [3] McDougald, Alfred, *Pelican and British Empire Life Office, Montreal, Canada.*

1884 Mackay, Alexander, *Law Union & Crown Insur. Co.,* 126 *Chancery-lane,* W.C.

1901 [2] Mackenzie, Michael Alexander, 1 *Bellwoods-pk., Toronto, Canada.*

Date of becoming an Associate.

1896 [2] Macmillan, John Campbell, *Northern Assur. Co. and Law Union and Crown Insurance Co., Apartado Postal No.* 872, *Mexico.*

1867 Macpherson, Ronald, *Law Union & Crown Insurance Co.,* 126 *Chancery-lane,* W.C.

1883 [2] Makeham, William Reed, *Alliance Assurance Co., Ltd.* (*Imperial Life Assurance Fund*), 47 *Chancery-lane,* W.C.

1883 Mannering, George Willsher, *London and Lancashire Life Assur. Co.,* 66 & 67 *Cornhill,* E.C.

1880 Manwaring, Henry, *National Debt Office,* E.C.

1896 [2] Martin, Sidney George, *National Mutual Life Assoc. of Australasia, Ltd.,* 295 *Queen-street, Brisbane, Australia.*

1897 [2] Mascall, Alfred John, *Standard Life Assurance Co.,* 3 *Pall-mall East,* S.W.

1904 [2] Maudling, Reginald G., *London and Lancashire Life Assur. Co.,* 66 & 67 *Cornhill,* E.C.

1900 [2] Maunder, George Harvard, *Standard Life Assurance Co.,* 83 *King William-street,* E.C.

1902 (2) Maxwell, Benjamin Bell, F.F.A., *Scottish Equitable Life Assur. Society,* 28 *St. Andrew-square, Edinburgh.*

1903 [3] May, Walter Thomas, *Scottish Amicable Life Assurance Society,* 1 *Threadneedle-st.,* E.C.

1899 [2] Meade, Gerald Willoughby, *North British & Mercantile Insurance Company,* 61 *Threadneedle-street,* E.C.

1896 [2] Merfield, Percy Henry, *Law Life Assurance Society,* 187 *Fleet-street,* E.C.

1874 Miller, John W., F.S.S., *Scottish Widows' Fund and Life Assur. Soc.,* 28 *Cornhill,* E.C.

1902 [3] Milligan, Charles Livingstone, *Provident Life Office,* 50 *Regent-street,* W.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1879 Monilaws, William Macgeorge, (AUDITOR), *Scottish Provident Institution, 17 King William-street,* E.C.

1877 Moon, James, *Prudential Assurance Company, 30 Dale-street, Liverpool.*

1877 Moon, John, *Prudential Assurance Company, 76 King-street, Manchester.*

1879 Moon, Sidney Norman Laming, *74, 76 & 78 William-street, New York, U.S.A.*

1903 2 Moore, George Cecil, *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1898 2 Moore, Joseph Patrick, *Citizens' Life Assurance Co., Sydney, Australia.*

1871 2 Moore, Roderick Mackenzie, *United Kingdom Temperance and General Provident Institution, 1 Adelaide-place, London-bridge,* E.C.

1893 2 Munro, Donald Alexander, *Brook-house, 10 Walbrook,* E.C.

1900 2 Nash, Alfred Charles, *Clerical, Medical and General Life Assurance Society, 15 St. James's-square,* S.W.

1903 2 Neill, Samuel Bennett, *London, Edinburgh & Glasgow Assurance Co., Ltd., Insurance-buildings, Farringdon-st.,* E.C.

1897 2 Newling, Sidney Wallis, B.A., *Woodleigh, South Woodford, Essex.*

1903 2 Nicholls, Arthur William, *Australian Mutual Provident Society, Brisbane, Australia.*

1884 Nicoll, John, F.F.A., *Life Association of Scotland, 82 Princes-street, Edinburgh.*

1900 3 Oakley, Henry John Percy, *North British and Mercantile Insurance Company, 61 Threadneedle-street,* E.C.

1883 Orr, Lewis P., F.F.A., *Scottish Life Assur. Co., Ltd., 19 St. Andrew-sq., Edinburgh.*

Date of becoming an Associate.

1886 Owen, Evan Frederick, F.S.S., *Actuary for Friendly Societies, Melbourne, Australia.*

1895 2 Pagden, Lionel King, *Union Assurance Society, 81 Cornhill,* E.C.

1864 Panton, Edward Henry, *50 Wood-vale, Forest Hill,* S.E.

1901 3 Papps, Percy Charles Herbert, *Manufacturers' Life Insurance Co., Toronto, Canada.*

1895 2 Paradice, William Henry, *Australian Mutual Provident Society, Sydney, Australia.*

1869 Park, David Francis, C.A., F.F.A., *Crédit Foncier of Mauritius (Limited), 39 Lombard-st.,* E.C.

1884 Park, Leslie John, *Colonial Mutual Life Assurance Society, Melbourne, Australia.*

1882 2 Paterson, William Brockie, F.F.A., Mem. Act. Soc. Amer., *Norwich Union Life Insurance Society, Norwich.*

1898 (2) Pearce, Henry John, F.F.A., *Scottish Amicable Life Assurance Soc., St. Vincent's-place, Glasgow.*

1899 2 Peele, Thomas, *Universal Insurance Company, 77 New Briggate, Leeds.*

1901 3 Penman, William, Jr., *Northern Assurance Company, 1 Moorgate-street,* E.C.

1900 2 Peters, Charles Furness, *L'pool. Victoria Legal Friendly Society, 18 St. Andrew-street,* E.C.

1895 (2) Pierson, Israel Coriell, Mem. Act. Soc. Amer., *141 Broadway, New York, U.S.A.*

1902 2 Pigrome, George Davey, *Prudential Assurance Company, Holborn-bars,* E.C.

1899 2 Pipe, Sidney Herbert, *Toronto Life Insurance Co., Cor., Roncesvalles and Fermanagh-streets, Toronto, Canada.*

1883 Pitts, Thomas, *Commercial Union Assurance Company, Exeter.*

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1876 Pound, Thomas James, *Clerical, Medical & General Life Assurance Soc., 15 St. James's-square,* S.W.

1890 [2] Powell, Alfred, *Alliance Assurance Company, Limited, Bartholomew-lane,* E.C.

1881 Price, William John, *Life Association of Scotland, 5 Lombard-street,* E.C.

1869 Pringle, James, C.A., F.F.A., *42 Drumsheugh-gardens, Edinburgh.*

1884 Pullar, James, F.F.A., *Colonial Mutual Life Assurance Society, Melbourne, Australia.*

1881 Purves, Thomas Peter, *New York Life Insurance Company, Sydney, Australia.*

1904 (2) Rankin, John Adam, F.F.A., *Edinburgh Life Assurance Co., 22 George-street, Edinburgh.*

1867 Rattray, Patrick, C.A., *115 St. Vincent-street, Glasgow.*

1874 [2] Ray, Charles Richard, *Hand-in-Hand Insurance Soc., 26 New Bridge-street,* E.C.

1885 Rea, Charles Herbert Edmund, F.R.A.S., F.S.S., *24 Charing Cross,* S.W.

1898 [2] Reid, Edward E., B.A., *London Life Insurance Co., London, Ontario, Canada.*

1901 [2] Rhodes, Francis, B.A., *Royal Insurance Co., Liverpool.*

1887 Richardson, Josephus Hargreaves, F.F.A., Mem. Act. Soc. Amer., *New Zealand Government Life Insurance Department, Wellington, New Zealand.*

1879 Roberts, Thomas B., *Australian Alliance Assurance Company, Collins-street, Melbourne, Australia.*

1904 (2) Robertson, Frederick William, F.F.A., *Standard Life Assurance Co., 3 George-street, Edinburgh.*

Date of becoming an Associate.

1904 (2) Robertson, James Leask, F.F.A., *Edinburgh Life Assurance Co., 22 George-street, Edinburgh.*

1878 Robertson, William, F.F.A., *29 Stafford-street, Edinburgh.*

1876 Robinson, Andrew, *Sunningdale-park, Sunningdale, Berks.*

1902 [3] Robinson, Hugh Thomas Kay, *Clergy Mutual Assur. Soc., 2 & 3 The Sanctuary,* S.W.

1885 Ronald, Thomas Robert, *Law Guarantee and Trust Soc., Ltd., 49 Chancery-lane,* W.C.

1904 [2] Rudd, Alfred James, *Australian Widows' Fund Life Assurance Society, Grenfell-street, Adelaide, South Australia.*

1897 [2] Ryley, Edmund, *Prudential Assurance Company, Holborn-bars,* E.C.

1896 [2] Sanderson, Frank, M.A., F.F.A., Mem. Act. Soc. Amer., F.S.S., *Canada Life Assurance Company, Toronto, Canada.*

1904 [2] Sare, Thomas Henry, *Commercial Union Assur. Co., 24, 25 & 26 Cornhill,* E.C.

1884 Schooling, John Holt, *Fotheringay-house, Montpelier-row, Twickenham.*

1899 [2] Schouten, Pieter, *Verzekering Maatschappij, "Arnhem," Stations-plein 17 Arnhem, Holland.*

1873 Scott, Ernest Willem, Mem. Act. Soc. Amer., *Algemeene Maatschappij van Levensverzekering en Lijfrente, Damrak, 74, Amsterdam.*

1904 [2] Searle, Arthur Joseph, *English & Scottish Law Life Assurance Association, Limited, 12 Waterloo-place,* S.W.

1861 [2] Searle, Thomas John, *Mansion - house - chambers, Bucklersbury,* E.C.

1900 [2] Searls, Edwin Richard, *Northern Assurance Company, 1 Moorgate-street,* E.C.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws rom the Examinations in Parts I and II.

Date of becoming an Associate.

1900 [2] Sharpe, Edgar Cecil Engledue, *London Life Association, Ltd.*, 81 *King William-street*, E.C.

1894 [3] Sheppard, Herbert Norman, B.A., Mem. Act. Soc. Amer., *Home Life Insurance Company*, 256 *Broadway, New York, U.S.A.*

1899 [3] Sherriff, Francis Henry, *Provident Clerks' and General Mutual Life Assurance Assoc.*, 27 *& 29 Moorgate-street*, E.C.

1897 [2] Shimmell, James Edward, 8 *Osborne-road, Handsworth, Birmingham.*

1896 [2] Shlager, Joseph, *Equitable Life Assurance Society of the United States, Mansion-house-chambers, Adderley-street, Cape Town, South Africa.*

1903 [2] Shovelton, Sydney Taverner, B.A., *Merton College, Oxford.*

1864 Smith, Howard Samuel, F.F.A., F.C.A., F.S.S., *Bank-chambers*, 11 *Waterloo-street, Birmingham.*

1898 [2] Smith, Robert Parker, *Royal Insurance Company, Liverpool.*

1884 Smithett, Edward Henry, *"Hillside," Fitzroy-park, Highgate*, N.

1871 Spencer, Robert James, F.S.S., 75 *King's-road, Southsea.*

1868 Spens, William George, *Scottish Amicable Life Assur. Soc.*, 35 *St. Vincent-pl., Glasgow.*

1902 [2] Spurgeon, Ernest Frank, *Prudential Assurance Company, Holborn-bars*, E.C.

1904 [2] Stamp, Horatio E., *"Appuldurcombe," Park Lane, Wallington, Surrey.*

1860 Stark, James, *Reversionary Interest Society*, 30 *Coleman-street*, E.C.

1866 Stark, William Emery, *Chapel-walks, Manchester.*

1878 Stevenson, Charles, 9 *Albert-square, Manchester.*

Date of becoming an Associate.

1880 Stock, Edward James, *National Mutual Life Assoc. of Australasia, Melbourne, Australia.*

1896 [2] Stuckey, Jos. James, M.A., *Salisbury Chambers*, 49*a King William-street, Adelaide, South Australia.*

1904 [2] Sugars, Robert Morrison, B.A., *Gresham Life Assurance Society, St. Mildred's-house, Poultry*, E.C.

1899 [3] Symmons, Frank Percy, *Prudential Assurance Company, Holborn-bars*, E.C.

1882 Tarn, Walter George, *Reversionary Interest Society*, 30 *Coleman-street*, E.C.

1904 (2) Tatlock, John, M.A., F.R.A.S., Mem. Act. Soc., Amer., *Mutual Life Insurance Co. of New York*, 32 *Nassau-street, New York, U.S.A.*

1893 [2] Taylor, Arthur, *Westminster and General Life Assurance Assoc.*, 28 *King-street, Covent-garden*, W.C.

1875 Taylor, J. Wilford, *North British and Mercantile Insur. Co.*, 61 *Threadneedle-st.*, E.C.

1898 [3] Thompson, Thomas Percy, B.A., *Pelican and British Empire Life Office*, 70 *Lombard-street*, E.C.

1904 (2) Thomson, John Walter, F.F.A., *Scottish Life Assurance Co.*, 19 *St. Andrew-square, Edinburgh.*

1883 [2] Titmuss, Walter George, *Provident Life Office*, 50 *Regent-street*, W.

1902 [2] Traversi, Antonio Thomas, *New Zealand Government Life Insurance Department, Wellington, New Zealand.*

1883 Tregaskis, George Alfred, *Hand-in-Hand Insurance Soc.*, 26 *New Bridge-street*, E.C.

1894 [2] Trenerry, Charles Farley, B.A., *University of London, South Kensington*, S.W.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1869 [2] Trew, Edward Bellingham, *Law Life Assurance Society*, 187 *Fleet-street*, E.C.

1891 [2] Turnbull, A. D. Lindsay, C.A., F.F.A., *Scottish Widows Fund and Life Assurance Society*, 9 *St. Andrew-square, Edinburgh.*

1884 Vian, William Collett, *Railway Passengers' Assurance Company*, 64 *Cornhill*, E.C.

1884 Vincent, Frederick James, F.S.S., *London, Edinburgh & Glasgow Assurance Co., Ltd., Insurance-buildings, Farringdon-street*, E.C.

1899 [2] Vokins, George Alfred, *Prudential Assurance Company, Holborn-bars*, E.C.

1883 [2] Walker, Davidson, F.F.A., *Norwich Union Life Insurance Society, Norwich.*

1879 Wall, Walter George, 3 *Shrewsbury-road, Birkenhead.*

1878 Walton, William Gandy, F.F.A., *Scottish Provident Institution.* 6 *St. Andrew-square, Edinburgh.*

1902 [3] Wandless, John Robert, *Northumbria, Hockley, Essex.*

1902 [3] Wares, Harold Wallace, *Yorkshire Insurance Company, York.*

1862 Waterhouse, Edwin, M.A., F.C.A., F.S.S., 3 *Frederick's-place, Old Jewry*, E.C.

1903 [2] Watherston, Charles F., B.A., *War Office*, S.W.

1883 [2] Watson, John Robertson, *British Law Fire Insurance Co.*, 176 *West George-st., Glasgow.*

1894 [2] Watt, George, *Royal Insurance Co., Liverpool.*

1900 (2) Watt, James, F.F.A., 18 *Moray-place, Edinburgh.*

1883 Weall, Bertram, 16 *Waldegrave-park, Twickenham.*

1902 [2] Weatherill, Charles, *Scottish Office*, S.W.

Date of becoming an Associate.

1894 (2) Weeks, Rufus Wells, Mem. Act. Soc. Amer., *New York Life Insurance Co.*, 346 & 348 *Broadway, New York, U.S.A.*

1898 [3] Whigham, Charles Frederick, F.F.A., C.A., *Messrs. Moncrieff & Horsburgh*, 46 *Castle-street, Edinburgh.*

1884 Whyte, Alexander, *c/o Messrs. Lever Bros., Ltd., Port Sunlight, Cheshire.*

1897 [2] Wickens, Charles H., *Government Statistician's Office, Perth, W. Australia.*

1896 [2] Wilkinson, Edward Berkeley, 24 *Maxilla-gardens, N. Kensington*, W.

1903 [2] Wilkinson, William Magnay, Jr., *Citizens' Life Assurance Co., Sydney, Australia.*

1904 [2] Williams, Frederick Alfred, F.S.S., *Hurstpierpoint, Hornchurch, Essex.*

1904 [2] Wilson, Arthur Benjamin, *Australian Mutual Provident Soc., Wellington, New Zealand.*

1900 [2] Wilson, George, *Standard Life Assurance Company*, 3 *George-st., Edinburgh.*

1870 [2] Wilson, Henry Edward, *Northern Assurance Co.*, 1 *Moorgate-street*, E.C.

1901 [3] Wilson, John Sydney, *Australian Widows' Fund Life Assce. Soc., Melbourne, Australia.*

1873 [2] Windett, Charles, *Legal & General Life Assurance Society*, 10 *Fleet-street*, E.C.

1903 [2] Wood, William Archibald Porter B.A., *Canada Life Assurance Co., Toronto, Canada.*

1883 Woodhouse, Lister, A.C.A., F.S.S., *City Comptroller, City-hall, Westminster*, S.W.

1877 [2] Woods, Arthur Biddle, *Rock Life Assurance Company*, 15 *New Bridge-street*, E.C.

1866 Woods, Bernard (Auditor), *Metropolitan Life Assurance Society*, 13 *Moorgate-street*, E.C.

B 2

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute. Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1875 Woods, Edward,
Victoria Life and General Insurance Co., Market-st., Collins-st.-west, Melbourne, Australia.

1879 Wornum, Thornton Selden,
Rock Life Assurance Company, 15 New Bridge-street, E.C.

1903 [2] Worth, Bertram Oliver,
Clerical, Medical & General Life Assurance Society, 15 St. James's-square, S.W.

Date of becoming an Associate.

1871 Yardley, John,
Prudential Assurance Company, Holborn-bars, E.C.

1873 Young, Alexander Hunter,
60 *Market-street, Melbourne, Australia.*

1900 [3] Young, Arthur Stanley,
Metropolitan Life Assurance Society, 13 *Moorgate-street,* E.C.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1892 [1] Aaron, David Hyam,
Sun Life Assurance Society, 63 *Threadneedle-street,* E.C.

1903 [1] Acum, Wilfred Harry,
15 *Lordship-lane, Wood Green,* N.

1904 [1] Addey, Leonard,
Clergy Mutual Assurance Soc., 2 & 3 *The Sanctuary,* S.W.

1904 [1] Allison, Sinclair E.,
Canada Life Assurance Co., Toronto, Canada.

1894 [1] Anderson, Adam Thomson,
Australian Mutual Provident Society, Sydney, Australia.

1904 [1] Armstrong, Charles Henry,
Imperial Life Assurance Co. of Canada, Toronto, Canada.

1886 Arnold, Thomas, Jr.,
British Equitable Life Assurance Company, Ltd., Queen-street-place, E.C.

1902 [1] Askwith, Thomas Nowell,
London Life Association, Ltd., 81 *King William-street,* E.C.

1904 [1] Ayscough, Ivan,
Equity and Law Life Assurance Soc., 18 *Lincoln's-inn-fields,* W.C.

1899 [1] Baber, Walter Crosbie,
Royal Victoria Life Insurance Co. of Canada, Montreal, Canada.

1903 [1] Baggs, Henry Ernest,
English and Scottish Law Life Assurance Association, 12 *Waterloo-place,* S.W.

Date of becoming a Student.

1903 [1] Bain, William Algernon,
Manufacturers' Life Insurance Co., Toronto, Canada.

1904 (1) Barford, Frederick William, M.A.,
The High School, Perth, Western Australia.

1899 [1] Barnett, Isaac,
North British and Mercantile Insurance Co., 61 *Threadneedle-street,* E.C.

1896 [1] Barry, David,
Royal Commission on the University of Melbourne, Supreme Court Library, Melbourne, Australia.

1900 [1] Bell, Henry Soady,
"Netherhall," The Drive, Sidcup, Kent.

1898 [1] Bennell, Samuel Thomas,
25 *Meath road, Ilford.*

1903 [1] Bennett, Reginald,
Refuge Assurance Co., Oxford-street, Manchester.

1898 [1] Bennett, Samuel,
National Deposit Friendly Soc., 37 *Queen-square,* W.C.

1902 [1] Biden, Norman Frederick,
Standard Life Association, 28 *Elizabeth-st., Sydney, Australia.*

1895 [1] Bigby, Robert Frederick Mitchell,
General Assurance Company, 103 *Cannon-street,* E.C.

1900 [1] Bingeman, Milton H.,
Great West Life Assurance Co., Winnipeg, Manitoba, Canada.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1903 [1] Binney, Charles Eardley-Wilmot, *Royal Exchange Assurance Corporation, Royal Exchange*, E.C.

1891 [1] Bird, Edward William, *Northern Assurance Company*, 1 *Moorgate-street*, E.C.

1898 [1] Blake, Frederick Edward, *Prudential Assurance Company, Holborn-bars*, E.C.

1901 [1] Blake, Francis Seymour, 62 *Oakhurst-grove, East Dulwich*, S.E.

1895 Blanch, Frederick William, *Sun Life Assurance Society*, 157 *Newington Causeway*, S.E.

1902 [2] Blanchard, Norman, B.A., *Equity & Law Life Assur. Soc.*, 18 *Lincoln's-inn-fields*, W.C.

1901 [1] Blehl, Ernest M., A.M., *Security Trust & Life Insurance Co., 26th and Broadway, New York, U.S.A.*

1887 Blossom, James, 186 *South-view-road, Sheffield.*

1904 [1] Boag, Harold, 14 *Avondale-terrace, Gateshead.*

1892 [1] Boddy, Henry Mitchell, *Manufacturers' Life Insurance Co., Cape Town, South Africa.*

1903 [1] Bodley, Rupert Frank, *Star Life Assurance Society*, 32 *Moorgate-street*, E.C.

1897 Bond, Frederic D., 122 *South 39th Street, Philadelphia, U.S.A.*

1900 [1] Borrajo, Edward Joseph William, *Prudential Assurance Company, Holborn-bars*, E.C.

1902 [1] Bowerman, Judah Philip, 51 *McAuley-place, Jamaica, New York City, U.S.A.*

1897 [1] Bowles, Francis Marsh, *Pearl Life Assurance Company, London-bridge*, E.C.

1891 [1] Boyd, Henry Norris, *City of Glasgow Life Assurance Company*, 21 *St. Andrew-square, Edinburgh.*

Date of becoming a Student.

1903 [1] Bradbury, Algernon Charles, *Australian Mutual Provident Society, Melbourne, Australia.*

1899 [1] Brady, John Francis, *Citizens' Life Assurance Co., Sydney, Australia.*

1897 [1] Brierley, William Ernest, *Refuge Assurance Company, Oxford-street, Manchester.*

1904 [1] Broad, Laurence C., *Prudential Assurance Company, Holborn-bars*, E.C.

1902 [1] Brook, Frank A., *Refuge Assurance Co., Oxford-street, Manchester.*

1894 [1] Brough, Frank, *Federal Life Assurance Company, Hamilton, Ontario, Canada.*

1904 [1] Brown, Arthur Ewart, *Scottish Widows' Fund and Life Assurance Society*, 28 *Cornhill*, E.C.

1891 [1] Brown, William Heron, *Gresham Life Assurance Society, Limited, St. Mildred's-house, Poultry*, E.C.

1889 Buckle, Frederick, *Aberfoyle, Valkyrie-road, Westcliff-on-Sea, Essex.*

1901 [1] Caldwell, Richard H., *North British and Mercantile Insurance Co., Birmingham.*

1904 [1] Canter, Harold, *National Provident Institution*, 48 *Gracechurch-street*, E.C.

1903 [1] Capon, Frank Christopher, *Prudential Assurance Company, Holborn-bars*, E.C.

1902 [1] Capon, Geoffrey William, *Norwich Union Life Insurance Society, Norwich.*

1903 [1] Carpenter, Thomas B. Boyd, *Clergy Mutual Assurance Society*, 2 & 3 *The Sanctuary*, S.W.

1901 [1] Carter, George Stanley, *Life Association of Scotland*, 5 *Lombard-street*, E.C.

1899 [2] Carter, Norman John, *Eagle Insurance Company*, 79 *Pall-mall*, S.W.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.	
1900	[1] Chambers, John Joseph, *Prudential Assurance Co.*, 138 *Lord-street, Southport.*
1902	[1] Chandler, Frederick Joseph, *Eagle Insurance Co.*, 79 *Pall-mall*, S.W.
1903	[1] Cheshire, Harold Frank, *Woodside, Shornden, St. Leonard's-on-Sea.*
1903	[1] Child, Robert Harold, *North British and Mercantile Insurance Company*, 61 *Threadneedle-street*, E.C.
1901	[1] Chubb, William, *Sun Life Assurance Company of Canada Montreal, Canada.*
1901	[1] Clarke, Eustace Edgar, *Pelican & British Empire Life Office*, 70 *Lombard-street*, E.C.
1897	[1] Clinton, George, *Prudential Assurance Company, Holborn-bars*, E.C.
1902	[2] Clinton, Louis Ernest, *Sun Life Assurance Society*, 63 *Threadneedle-street*, E.C.
1902	[2] Coates, Frederick George, *Hand - in - Hand Insurance Society*, 26 *New Bridge-street*, E.C.
1901	[1] Cockerton, John Leonard, *Pioneer Life Assurance Co., Ltd., Century - buildings*, 31 *North John-street, Liverpool.*
1895	Cogar, William Edward, *New York Life Insurance Co., Trafalgar-square*, W.C.
1899	[1] Collins, Patrick A., *Citizens' Life Assurance Co., Sydney, Australia.*
1902	[1] Collins, William Ernest, 61 *Osmond-terrace, Norwood, South Australia.*
1896	[1] Cook, Henry Milton, *Standard Life Assurance Company, Dalhousie-square, Calcutta, India.*
1900	[1] Cooper, Bernard Hugh, *Prudential Assurance Company, Holborn-bars*, E.C.
1903	[1] Cooper, John James, *Sun Life Assurance Co. of Canada, Montreal, Canada.*
1902	[1] Corbett, Archibald Gladstone, *Australian Mutual Provident Society, Collins-st., Melbourne, Australia.*
1903	[1] Cotton, Arthur Sparkes, *Scottish Office*, S.W.
1905	[1] Cotterill, William Ernest (Reinstated), *Mutual Life Association of Australasia Ltd., Sydney, Australia.*
1897	[2] Court, Alexander George Daeus, *Coningsby, Hyde-vale, Greenwich*, S.E.
1901	[1] Coventry, Cameron H., *Australasian Temperance and General Mutual Life Assurance Society, Melbourne, Australia.*
1904	[1] Cowdy, Henry Leslie, *Scottish Union and National Insurance Co.*, 3 *King William-street*, E.C.
1894	Cox, Edward William, *Canada Life Assurance Co., Toronto, Canada.*
1894	Cox, Herbert Coplin, *Canada Life Assurance Co., Toronto, Canada.*
1887	[1] Cross, Henry John, 3 *Park-rd., Wandsworth-common*, S.W.
1897	[1] Crump, Percy C., *Prudential Assurance Company, Holborn-bars*, E.C.
1904	[1] Cushing, Robertson Macaulay, *Sun Life Assurance Company of Canada, Montreal, Canada.*
1904	[1] Dalrymple, Alfred George, *Canada Life Assurance Company, Toronto, Canada.*
1897	[1] Dalton, John, *London Life Association, Ltd.*, 81 *King William-street*, E.C.
1889	[1] Davies, Hugh Myddleton, *Royal Insurance Co., Liverpool.*
1900	[1] Davies, William Allison, *Borough Treasurer's Office, Town Hall, Birkenhead.*
1899	[1] Davison, Horace Williams, 15 *Torrington-square*, W.C.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1891 [1] Dawson, Frank Aubrey, *Ecclesiastical Insurance Office, Limited,* 11 *Norfolk-street, Strand,* W.C.

1902 [1] Deck, James Gilbert, *National Provident Institution,* 48 *Gracechurch-street,* E.C.

1904 [1] Defries, Frederick, *Ecclesiactical Insurance Office,* 11 *Norfolk-street, Strand,* W.C.

1902 [1] Denmark, Robert John, *Norwich Union Life Insurance Society, Norwich.*

1901 [1] Dent, Ernest Edward, *London and Lancashire Life Assurance Company,* 66 & 67 *Cornhill,* E.C.

1896 [1] de Ville, Francis, *Clergy Pensions Institution,* 11 *Norfolk-street, Strand,* W.C.

1890 [1] Docker, Leslie, *North British and Mercantile Insurance Co.,* 61 *Threadneedle-street,* E.C.

1897 [1] Dorrian, John Christopher, *Citizens' Life Assurance Company, Sydney, Australia.*

1903 [1] Downes, Edward George, *c/o T. G. Ackland, Esq.,* 5 *&* 6 *Clement's-inn,* W.C.

1901 [2] Downes, Sidney Cecil, *Prudential Assurance Company, Holborn-bars,* E.C.

1904 [1] Drake, Charles Clifford Hall, *Prudential Assurance Company, Holborn-bars,* E.C.

1892 [1] Edwards, Edward Samuel, *Australian Mutual Provident Society, Sydney, Australia.*

1902 [1] Edwards, Thomas Baker, *Comptroller's Dept., London County Council, Spring-gardens,* S.W.

1892 [1] Eedy, Arthur Malcolm, *Citizens' Life Assurance Company, Sydney, Australia.*

1901 [1] Egleton, Harold Edward, *Prudential Assurance Company, Holborn-bars,* E.C.

Date of becoming a Student.

1900 [1] Elderton, Robert Lapidge, *National Provident Institution,* 48 *Gracechurch-street,* E.C.

1904 [1] Eldridge, Ernest Edward Booth, *Atlas Assurance Company, Ltd.,* 92 *Cheapside,* E.C.

1902 [1] Ellis, Reginald George Gregson, 12 *Manson-pl., Queen's-gate,* W.

1903 [1] Ellis, Thomas Barnes, *Public Works Loan Board, Old Jewry,* E.C.

1893 [1] Emery, John M., *Auditor of State's Office, Des Moines, Iowa, U.S.A.*

1904 [1] Esler, John, *Crown Life Insurance Company, Toronto, Canada.*

1904 [1] Falk, Oswald Toynbee, B.A., *Hand-in-Hand Insurance Soc.* 26 *New Bridge-street,* E.C.

1903 [1] Farmer, Ernest Chattock, *London, Edinburgh & Glasgow Assurance Company, Limited, Insurance-bldngs., Farringdon-street,* E.C.

1892 [1] Farrell, John, *Citizens' Life Assurance Co.,* 210 *Queen-street, Brisbane, Australia.*

1902 [1] Farrow, Alfred Ellis, *Yorkshire Insurance Company, York.*

1901 [1] Fielder, William Crowhurst, *Atlas Assurance Company, Ltd.,* 92 *Cheapside,* E.C.

1904 [1] File, Lorne K., *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1904 [1] Fippard, Richard Clift, 11 *Oakington-road, Paddington,* W.

1901 [1] Fisher, John William, B.A., *Crown Life Insurance Co., Toronto, Canada.*

1896 [1] Fisk, George William Victor, F.S.S., *Prudential Assurance Company, Holborn-bars,* E.C.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1904 [1] Fletcher, Andrew W. A. C., *Standard Life Assurance Co., 3 Pall Mall East*, S.W.

1904 [1] Foot, Alfred Helsdon, *Law Accident Insurance Society, 215 Strand*, W.C.

1901 [1] Franklin, Herbert Dare, *Australian Mutual Provident Society, Melbourne, Australia.*

1903 [1] Fulford, William John, *Prudential Assurance Company, Holborn-bars*, E.C.

1890 Gamman, Robert Ebenezer, *London Joint Stock Bank, Princes-street*, E.C.

1886 Garcke, Emile, F.S.S., M.I.E.E., *Ditton-house, near Maidenhead.*

1900 [1] Garner, James, *9 Arlington-gardens, Chiswick*, W.

1901 (1) Gerrish, Frank Wilfred, B.A., *Minerva-villa, Albert-rd.-south, Buckhurst-hill, Essex.*

1899 [1] Giles, Hylton Lloyd, *Pelican & British Empire Life Office, 70 Lombard-street*, E.C.

1895 [1] Gill, James Stewart, *Australian Widows' Fund Life Assurance Society, Sydney, Australia.*

1900 [1] Gillespie, Joseph Hugh Ross, M.A., *Manufacturers' Life Insurance Co., Toronto, Canada.*

1901 [1] Glassford, David Murray, *Mutual Life Association of Australasia, Sydney, Australia.*

1893 Glasson, George Cornish, *Economic Life Assurance Soc., 4 St. Stephen's-chbrs., Baldwin-street, Bristol.*

1902 [1] Gleave, Charles Sheldon, *Refuge Assurance Co., Oxford-street, Manchester.*

1893 [1] Gledstone, W. L., *Royal Exchange Assur. Corporation, Royal Exchange*, E.C.

1902 [1] Godsill, Richard Collis, *Liverpool Victoria Legal Friendly Soc., 18 St. Andrew-street*, E.C.

1894 [1] Golding, Arthur, *40 Allerton-road, Stoke Newington*, N.

1388 [1] Gooding, Harold John, *Law Guarantee and Trust Soc., Ltd., 56 Moorgate-street*, E.C.

1903 [1] Gopp, John Ive, *14 Church-hill-road, Walthamstow*, E.

1902 [1] Gordon, Walter Hamilton, *45 Braydon-road, Stamford-hill*, N.

1902 [1] Gorham, Edwin Arthur, *75 Albion-street-east, Brunswick, Melbourne, Australia.*

1897 [2] Gosset, Thorold, *16 Durham-road, Wimbledon*, S.W.

1902 [1] Gould, W. H., M.A., *Sovereign Life Assurance Co., Toronto, Canada.*

1886 Gover, Frederick Field, F.S.S., *10 Lee-park, Blackheath*, S.E.

1904 [1] Gray, Maurice Donald, *Australian Mutual Provident Soc., Wellington, New Zealand.*

1903 [1] Green, Walter, *Ecclesiastical Commission, 10 Whitehall-place*, S.W.

1886 Greening, Herbert Joseph, *Abstainers' & General Insur. Co., City-buildings, Birmingham.*

1899 [1] Grigg, Benjamin, *Sun Life Assur. Co. of Canada, Montreal, Canada.*

1901 [1] Hall, Arthur F., *North American Life Assurance Co., North American Life Building, 112-118 King-street-west, Toronto, Canada.*

1902 [2] Hallett, William Sebastian, B.A., *Equitable Life Assurance Soc., Mansion-house-street*, E.C.

1896 [1] Hallman, M. S., *Mutual Life Assurance Company of Canada, Waterloo, Ontario, Canada.*

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.	
1899	1 Halloran, George Henry, 20 *Castlereagh-street, Sydney, Australia.*
1901	1 Hamilton, George Powell, *North American Life Assurance Co., McLean Block,* 6 *Douglas-street, Guelph, Ontario, Canada.*
1902	1 Hammant, Francis Clive, *Prudential Assurance Company, Holborn-bars,* E.C.
1900	1 Hammond, Reginald, *British Equitable Life Assur. Co., Ltd., Queen-street-place,* E.C.
1892	Hancock, Arthur Tom, *Clerical, Medical & General Life Assurance Society,* 15 *St. James's-square,* S.W.
1903	1 Hancock, Edwin J., 72 *Tredegar-road, Bow,* E.
1902	1 Hardy, Reginald Herbert, 32 *Highfield-street, Leicester.*
1903	1 Harley, Brian, *Guardian Assurance Co.,* 11 *Lombard-street,* E.C.
1901	1 Harpell, James John, B.A., *North American Life Assurance Co., North American Life Building,* 112-118 *King-street-west, Toronto, Canada.*
1901	1 Harper, Henry, 103 *Waverley-road, Small Heath, Birmingham.*
1903	1 Harris, Ernest Arthur, 40 *Lambert-road, Brixton-hill,* S.W.
1889	1 Harris, Henry, *Friends' Provident Institution,* 17 *Gracechurch-street,* E.C.
1896	Haskins, George Frederick, A.C.A., 18 *Walbrook,* E.C.
1894	1 Hatten, David Leslie, *Standard Life Assurance Co.,* 83 *King William-street,* E.C.
1903	1 Hill, Frank Wilson, *Norwich Union Life Insurance Society, Norwich.*
1896	2 Hines, Walter Robert, *Norwich Union Life Insurance Society, Norwich.*
1897	(1) Hitchins, William Richmond, B.A., *Manufacturers' Life Insurance Company, Toronto, Canada.*
1902	1 Hodge, Cecil Wilfred, *Star Life Assurance Society,* 32 *Moorgate-street,* E.C.
1896	1 Hogg, Charles, 10 *Whitehall-place,* S.W.
1898	2 Hooper, George Duncan, *Prudential Assurance Company, Holborn-bars,* E.C.
1895	2 Horn, Ernest Frederick, *Clerical, Medical and General Life Assurance Society,* 1 *King William-street,* E.C.
1902	1 Houston, Charles Cornelius, *Metropolitan Asylums Board, Victoria-embankment,* E.C.
1901	1 Howell, Archibald Rennie, B.A., *Royal Insurance Co. of England, Montreal, Canada.*
1898	Hughes, Arthur J., *Crown Life Insurance Co., Toronto, Canada.*
1902	1 Hughes, Charles, *Insurance Department of the State of Connecticut, Hartford, Conn., U.S.A.*
1902	1 Hugill, Herbert, *"Briarfield," Keighley.*
1904	1 Humphreys, Harry Lewis, 9 *Court-road, West Norwood,* S.E.
1902	1 Humphreys, John A., *National Mutual Life Assurance Society,* 39 *King-street, Cheapside,* E.C.
1902	1 Humphry, Edmund William, *Life Association of Scotland,* 5 *Lombard-street,* E.C.
1891	Hunt, Arthur Leonard, *Wesleyan and General Assur. Society,* 101 *Finsbury-pavement,* E.C.
1902	(1) Jackson, Charles William, M.A., *c/o* M. M. Dawson, Esq., 11 *Broadway, New York, U.S.A.*
1902	2 Jackson, Herbert Moore, *Australian Mutual Provident Society, Sydney, Australia.*

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1890 [2] Jackson, Samuel, *Scottish Widows' Fund and Life Assurance Society, Liverpool.*

1903 [1] Jefferson, John Arthur, *c/o T. G. Ackland, Esq., 5 & 6 Clement's-inn,* W.C.

1895 [1] Jenkyn, John, *Squirrel's-heath, Romford, Essex.*

1896 [1] Jepps, John Blacklee, *English and Scottish Law Life Assurance Assoc., 12 Waterloo-place,* S.W.

1904 [1] Johnson, Frank Henry, *Law Life Assurance Society, 187 Fleet-street,* E.C.

1898 [1] Johnston, Arthur Edward, *3 Cumnor-road, Sutton.*

1902 [1] Jones, Edward Furnival, A.S.A.A., *Hearts of Oak Benefit Society, 17 Charlotte-street,* W.

1903 [1] Jones, Ernest Stephens, *National Debt Office,* E.C.

1896 [1] Jones, Richard Foxley, *Refuge Assurance Co., Oxford-street, Manchester.*

1893 [2] Kelham, Cyril Stephen, *Prudential Assurance Company, Holborn-bars,* E.C.

1902 [1] Kemper, J. M. de Bosch, *La Mutuelle Hollandais Insurance Company, 21 Avenue de l'Opera, Paris.*

1894 [2] Kingsbury, James William, *Australian Mutual Provident Society, Sydney, Australia.*

1900 [1] Kirkham, Alfred, *c/o Messrs. Thos. Mitchell & Co., Lonsdale-street, Melbourne, Australia.*

1903 [1] Kirsopp, Frederick, *Liverpool Victoria Legal Friendly Society, 18 St. Andrew-street,* E.C.

1895 [1] Knight, Alfred Murray, *Bank-house, Chapel-st., Devonport.*

Date of becoming a Student.

1902 [1] Lang, Frederick John, *Royal London Friendly Society, Finsbury-square,* E.C.

1902 [1] Langstaff, James Miles, *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1901 [1] Latham, Bertrand, *Australian Mutual Provident Society, Melbourne, Australia.*

1904 [1] Latham, Percy James, *Inland Revenue Department, Falkirk,* N.B.

1891 Layzell, Phillip Cuddington, *Prudential Assurance Company, Holborn-bars,* E.C.

1904 [1] Lee, Frank Sidney, *Ocean Accident and Guarantee Corporation, 36-44 Moorgate-street,* E.C.

1904 [1] Lee, Frederick, *Ecclesiastical Insurance Office, 11 Norfolk-street, Strand,* W.C.

1901 [1] Leigh, Samuel George, *Refuge Assurance Co., Oxford-street, Manchester.*

1894 Leonard, Maurice, *14 Sotheby-road, Highbury,* N.

1904 [1] Levey, Ralph, *Prudential Assurance Company, Holborn-bars,* E.C.

1904 [1] Lewty, Francis Arthur, *Equity and Law Life Ass. Soc., 18 Lincoln's-inn-fields,* W.C.

1896 [1] Ley, James, *Office of the Actuary for Friendly Societies, Melbourne, Australia.*

1889 [1] Lighton, Harold John, *Law Union & Crown Insurance Co., 126 Chancery-lane,* W.C.

1904 [1] Linzmeyer, Louis, Mem. Act. Soc. Amer., *Manhattan Life Insurance Co., 64-70 Broadway, New York, U.S.A.*

1895 [1] Littell, Lewis Lloyd, *Standard Life Assurance Co., 83 King William-street,* E.C.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1904 [1] Littlefair, James Taylor, *Refuge Assurance Co., Oxford-street, Manchester.*

1890 Love, Robert, *Ecclesiastical Insurance Office,* 11 *Norfolk-street, Strand,* W.C.

1903 [1] Macaulay, Frederick Robertson, 4007 *Dorchester-street, Westmount, Montreal, Canada.*

1888 [1] McConway, James Robert, *Royal Insurance Company, Liverpool.*

1903 [1] McDonald, Charles Joseph Angus, *Australian Mutual Provident Society, Wellington, New Zealand.*

1903 [1] Macdonald, Charles Strange, M.A., *Confederation Life Association, Toronto, Canada.*

1903 [1] Macdougall, Graham L., *Massachusetts Mutual Life Insurance Co., Springfield, Mass., U.S.A.*

1904 [1] Macfarlane, Edmond Seales, *Manufacturers' Life Insurance Company, Toronto, Canada.*

1902 [1] Macfarlane, James Allan, *North American Life Assurance Co., North American Life Building,* 112-118 *King-street-west, Toronto, Canada.*

1902 [1] McGee, Cyril H. *Box* 981, *St. Thomas, Ontario, Canada.*

1904 [1] McKechnie, James Baldwin, *Manufacturers' Life Insurance Company, Toronto, Canada.*

1902 [1] McKellar, John A., *Equitable Life Assur. Society of the United States,* 120 *Broadway, New York, U.S.A.*

1901 [1] Macmillan, Alexander, 27 *Westcliffe-grove, Harrogate.*

1903 [1] Macneill, Murray, *McGill University, Montreal, Canada.*

1897 [1] McPhail, Frederick Charles, *Colonial Mutual Life Assurance Society Limited, Melbourne, Australia.*

1904 (1) Makepeace, Francis Lucas, B.A., 29 *Deronda-road, Herne-hill,* S.E.

1903 [1] Maltby, Charles Hugh, *Scottish Widows' Fund and Life Assurance Society,* 28 *Cornhill,* E.C.

1903 [1] Manly, George William, B.A., *Clerical. Medical & General Life Assurance Society,* 15 *St. James's-square,* S.W.

1904 [1] Marlin, James Harold, *Ocean Accident and Guarantee Corporation,* 36-44 *Moorgate-street,* E.C.

1903 [1] Martin, Frederick Charles, *Prudential Assurance Company, Holborn-bars,* E.C.

1904 [1] Matheson, Donald, *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1895 [1] Mayhew, Percy Craske, 4 *Princess-road, Selhurst,* S.E.

1890 [1] Meikle, Henry George Watson, F.F.A., *Oriental Government Security Life Assurance Co., Limited, Bombay, India.*

1901 [1] Melville, Henry Edward, *Alliance Assurance Company, Ltd., Bartholomew-lane,* E.C.

1892 [1] Meyers, Henry Wilson, *National Mutual Life Association of Australasia,* 76 & 77 *Cornhill,* E.C.

1899 [2] Minns, Ernest Edwin, *Norwich Union Life Insurance Society, Norwich.*

1904 [1] Monilaws, William Barrington, *Scottish Provident Institution,* 17 *King William-street,* E.C.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1903 (1) Monkhouse, Charles Cosmo, B.A., *English & Scottish Law Life Assur. Association*, 12 *Waterloo-place*, S.W.

1900 [1] Moore, George Edward, *Australian Widows' Fund Life Assurance Society, Melbourne, Australia.*

1895 [1] Moore, Gerald Leslie, A.C.A., 1 *Rosebery-gardens, Muswell-hill*, N.

1902 [1] Moore, Hubert Fred, *London Assurance Corporation*, 7 *Royal Exchange*, E.C.

1903 [1] Moore, Roderick John, 34 *Cleveland-mansions, Widley-road, Elgin-avenue*, W.

1898 [1] Moore, Stanley, *Prudential Assurance Company, Holborn-bars*, E.C.

1904 [1] Moran, Albert James, *Sun Life Assurance Society*, 63 *Threadneedle-street*, E.C.

1902 [1] Morton, Francis, *Hand-in-Hand Insurance Soc.*, 26 *New-bridge-street*, E.C.

1902 [1] Muckle, Charles Park, *Union Life Assurance Co., Toronto, Canada.*

1904 [1] Mulcahy, Francis Benedict, *Citizens' Life Assurance Co., Sydney, Australia.*

1902 [1] Mullin, Alexander, B.A., 76 *Major-st., Toronto, Canada.*

1903 [1] Myers, Harry Duxbury, A.S.A.A., 64 *Devonshire-street, Keighley.*

1896 [1] Neale, Maurice Baldwin, *Alliance Assurance Company, Ltd., Bartholomew-lane*, E.C.

1903 [1] Neill, William Adam Hoyes, *Scottish Widows' Fund and Life Assurance Society*, 28 *Cornhill*, E.C.

1895 [1] Newnham, Ernest Whiffin, *Prudential Assurance Company, Holborn-bars*, E.C.

Date of becoming a Student.

1904 [1] Norris, Isaac Taylor, *Collegiate Institute, Ottawa, Canada.*

1903 [1] Northcott, John Arthur, *St. Andrew's College, Toronto, Canada.*

1901 [1] Nugent, James, *Cornwall, Ontario, Canada.*

1903 [1] Oates, Percy Tuckfield, 30 *High-street, Wimbledon*, S.W.

1902 (1) O'Connor, William, M.A., M.D., *Mutual Life Insurance Co. of New York*, 31, 32 & 33 *Canadian Bank of Commerce Building, Toronto, Canada.*

1892 [1] O'Reilly, Anthony James, *Government Insurance Department, Ottawa, Canada.*

1897 [1] Osborn, Nathaniel Banner Francis, 11 *Bruce-grove, Tottenham*, N.

1893 [1] Owen, Edgar Theodore, F.S.S., *Registrar of Friendly Societies and Government Actuary, Perth, Western Australia.*

1901 [1] Papworth, Frederick William, A.S.A.A., *Admiralty*, S.W.

1904 [1] Parker, John G., 82 *Harvard-avenue, Toronto, Canada.*

1904 [1] Parker, Walter Montgomery, *Prudential Assurance Company, Holborn-bars*, E.C.

1895 [1] Pascoe, William Yeoman Bennett, *Prudential Assurance Company, Holborn-bars*, E.C.

1901 [1] Paton, Albert George, *London Assurance Corporation*, 7 *Royal Exchange*, E.C.

1897 [1] Paton, Harry Arthur, *Royal Exchange Assurance Corporation, Royal Exchange*, E.C.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1896 [2] Penny, Charles Augustus, *Prudential Assurance Company, Holborn-bars*, E.C.

1901 [1] Petter, Herbert, *British Workman's and General Assurance Co., Broad-street-corner, Birmingham.*

1904 [1] Phillips, Walter, A.C.I.S., A.S.A.A., 8 *Riverdale-terrace, Richmond, Surrey.*

1898 Poort, Willem Anthonie, Phil. Nat. Doct., *Algemeene Friesche Levensverzekerings Maatschappij Leeuwarden, Leeuwarden, Holland.*

1904 [1] Portch, Albert Garfield, *Canada Life Assurance Company, Toronto, Canada.*

1903 (1) Porter, Frank, M.A., *Mansfield House, Canning Town*, E.

1893 [1] Pownall, Herbert Wilfred, *Australian Mutual Provident Society, Adelaide, Australia.*

1901 [1] Ramsay, Cecil Byron, *Mutual Life Insurance Co. of New York*, 16, 17 & 18 *Cornhill*, E.C.

1903 [1] Raynes, Harold Ernest, *Legal and General Life Assurance Society*, 10 *Fleet-street*, E.C.

1898 [1] Reynell, Guy Courtenay, *National Mutual Life Assurance Society*, 39 *King-st., Cheapside*, E.C.

1904 [1] Reyner, Harry Fane, *Refuge Assurance Company, Oxford-street, Manchester.*

1903 [1] Reynolds, William Daniel, *Prudential Assurance Company, Holborn-bars*, E.C.

1904 [1] Rice, George Ritchie, 58 *Croxted-road, West Dulwich*, S.E.

1894 [1] Richards, Gilbert P. A., *Oak Cottage, Bulwer-road, New Barnet.*

Date of becoming a Student.

1904 [1] Ridgway, Wulfric, *Sun Life Assurance Society*, 63 *Threadneedle-street*, E.C.

1902 [1] Robertson, Aubrey Charles, *London Assurance Corporation*, 7 *Royal Exchange*, E.C.

1901 [1] Robertson, A. W. L., *Guardian Assurance Co.*, 11 *Lombard-street*, E.C.

1903 [1] Robertson, Bernard, *Prudential Assurance Company, Holborn-bars*, E.C.

1903 [1] Robinson, Ernest William, *Standard Life Association, Ltd.*, 28 *Elizabeth-street, Sydney, Australia.*

1896 [1] Robinson, Frederick Charles, *Royal Exchange Assur. Corporation, Royal Exchange*, E.C.

1893 [1] Roll, Frederick James, *Pearl Life Assurance Company, London-bridge*, E.C.

1893 [1] Roodenburch, Bartholomeus Adrianus, *Verzekeringsbank Victoria*, 689 *Prinsengracht, Amsterdam.*

1895 [1] Ross, Christopher Watson, *c/o Messrs. M. Moss & Co., Flinder's-lane, Melbourne, Australia.*

1901 [1] Rountree, Arthur FitzGerald, *The Rectory, Stretford, near Manchester.*

1895 Rowley, James Edward, A.C.A., 7 *Waterloo-street, Birmingham.*

1899 [1] Rutter, Edward Valentine, *Pelican & British Empire Life Office*, 70 *Lombard-street*, E.C.

1904 [1] Sadler, Percy, *Prudential Assurance Company, Holborn-bars*, E.C.

1894 Salter, George Ferry, Mem. Act. Soc. Amer., 123 *N. 16th-street, E. Orange, N.J., U.S.A.*

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1892 [1] Savery, Robert S. B., *Gresham Life Assurance Society, Giselastrasse, No. 1, Vienna.*

1897 [1] Scott, Alexander Lewis, *Australian Mutual Provident Society, Melbourne, Australia.*

1888 Sewell, Richard, F.F.A., C.A., 63 *Threadneedle-street*, E.C.

1904 [1] Sharp, George Gilbert, 34 *Croft-street, Deptford*, S.E.

1902 [1] Shrubsole, Stanley Smith, *Prudential Assurance Company, Holborn-bars*, E.C.

1896 [1] Shute, Oxenham Bent, *National Provincial Bank of England*, 53 *Baker-street*, W.

1892 [1] Simpson, William Murray, *North British and Mercantile Insurance Company*, 61 *Threadneedle-street*, E.C.

1904 [1] Sinclair, William Alexander, *Canada Life Assurance Company, Toronto, Canada.*

1891 [1] Sindall, Alfred John, *London and Lancashire Life Assurance Co.*, 66 & 67 *Cornhill*, E.C.

1888 [2] Slimon, William James, F.F.A., 10 *Mayfield-terrace, Edinburgh.*

1902 [1] Smith, Septimus Wontner, *Equitable Life Assurance Soc., Mansion-house-street*, E.C.

1903 (1) Smith, Thomas Cooper, B.A., *Hand-in-Hand Insurance Soc.*, 26 *New Bridge-street*, E.C.

1903 [1] Smith, William, *Standard Life Association, Ltd.*, 28 *Elizabeth-street, Sydney, Australia.*

1902 [1] Smither, Herbert Buxton, *University Life Assurance Soc.*, 25 *Pall-mall*, S.W.

1903 [2] Sneddon, Andrew William, *Australian Mutual Provident Society, Sydney, Australia.*

Date of becoming a Student.

1900 [1] Somerville, Walter Harold, *Mutual Life Assurance Company of Canada, Waterloo, Ontario, Canada.*

1904 [1] Spring, Stanley Harold, *London Guarantee and Accident Company*, 61 *Moorgate-st.*, E.C.

1904 [1] Sprules, Alfred M., 29 *Manor-row, Bradford, Yorkshire.*

1903 [1] Stanford, Harold William, *London Assurance Corporation*, 7 *Royal Exchange*, E.C.

1903 [1] Stebbings, George Warne, 37 *Leigh-road, Highbury*, N.

1901 [1] Steffensen, Johan F., *Forsikringsraadet*, 1 *Christiansgade, Copenhagen.*

1898 [2] Stewart, Lionel William, *Alliance Assurance Co., Ltd., Bartholomew-lane*, E.C.

1886 [2] Stirling, James, *Scottish Imperial Insurance Co.*, 183 *West George-st., Glasgow.*

1903 [1] Story, Cyril Lionel William Steane, *Norwich Union Life Insurance Society*, 71 & 72 *King William-street*, E.C.

1888 [1] Stott, Walter, *Royal Insurance Co., Liverpool.*

1893 [1] Streeter, Theodore Edward, *P.O. Box* 1415, *Winnipeg, Manitoba, Canada.*

1903 [1] Strong, Allan Wilmot, *Sun Life Assurance Co. of Canada, Montreal, Canada.*

1904 [1] Strong, Gordon Gilbert, *Sun Life Assurance Society*, 63 *Threadneedle-street*, E.C.

1902 [1] Strong, William Boughton, *Prudential Assurance Company, Holborn-bars*, E.C.

1904 [1] Stuart, Arthur William, *National Provident Institution*, 48 *Gracechurch-street*, E.C.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute.
Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.	
1899	[1] Stuckey, Reginald Robert, *Australian Mutual Provident Society, Adelaide, S. Australia.*
1904	[1] Sturt, Arthur James, *Pelican and British Empire Life Office, 70 Lombard-street,* E.C.
1902	[1] Sturt, Herbert Rothsay, *Independent Order of Foresters, 24 Charing Cross,* S.W.
1903	[1] Suddaby, William A., *Liverpool Victoria Legal Friendly Society, 18 St. Andrew-street,* E.C.
1902	[1] Sudell, Alfred Henry, *Prudential Assurance Company, Holborn-bars,* E.C.
1904	[1] Tamkin, Walter Ellis, *Prudential Assurance Company, Holborn-bars,* E.C.
1895	[1] Thistlethwaite, William, *4 Warren-terrace, Wakefield.*
1904	[1] Thompson, William George, *Commercial Union Assurance Co., 24, 25 & 26 Cornhill,* E.C.
1900	[1] Thomson, Frederick Robert T., *Kent-house, Church-end, Finchley,* N.
1902	[1] Thwaites, Frederick George, *Norwich Union Life Insurance Society, Norwich.*
1897	[1] Tipping, Oswald, *Trustees', Executors', and Agency Co., Limited, 412 Collins-street, Melbourne, Australia.*
1901	[1] Todhunter, Joseph, *Prudential Assurance Company, Holborn-bars,* E.C.
1902	[1] Tope, Maurice William, *National Mutual Life Assurance Society, 39 King-street, Cheapside,* E.C.
1897	[1] Touzel, Philip Duncan, *Australian Mutual Provident Society, Melbourne, Australia.*
1903	[1] Townley, Ebenezer William, *National Mutual Life Assurance Society, 39 King-st., Cheapside,* E.C.
1897	[1] Townshend, Edward Villiers, *Scottish Widows' Fund and Life Assurance Society, 28 Baldwin-street, Bristol.*
1901	[1] Tregaskis, George, *Sun Insur. Office, 40 Chancery-lane,* W.C.
1902	[1] Tully, Arthur Patrick Thomas, *85 Park-lane, Croydon.*
1891	Tyler, Edgar Alfred, F.S.S., *9 Old Jewry-chambers, Bank,* E.C.
1904	[1] Underwood, Reginald, *Guardian Assurance Company, 11 Lombard-street,* E.C.
1904	[1] Wansbrough, Thomas Percival (Reinstated), *English and Scottish Law Life Assurance Association, 17 Queen Victoria-street,* E.C.
1904	[1] Warnock-Fielden, Francis Hugh, *Prudential Assurance Company, Holborn-bars,* E.C.
1904	[1] Warren, Cyril Ferdinand, *Prudential Assurance Company, Holborn-bars,* E.C.
1903	[1] Watson, Alexander R. D., *Devonport, Auckland, New Zealand.*
1900	[1] Watt, Arthur W., *Sun Life Assur. Co. of Canada, Montreal, Canada.*
1898	[1] Webb, Lloyd, *Hand-in-Hand Insurance Soc., 26 New-bridge-street,* E.C.
1902	[1] Wellisch, Frederick, *Australian Mutual Provident Society, Sydney, Australia.*
1893	[1] Welman, Arthur Joseph, *Legal & General Life Assurance Soc., 15 Tithebarn-st., Liverpool.*

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute
Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1904 [1] Wenn, Albert Edward
Prudential Assurance Company, Holborn-bars, E.C.

1902 [1] White, Wilfred Clare,
Manufacturers' Life Insurance Co., Toronto, Canada.

1886 [1] Williams, David,
181 *Queen Victoria-street*, E.C.

1895 [1] Williams, Henry Samuel Walter,
North British and Mercantile Insurance Co., Dunedin, New Zealand.

1900 (1) Williams, Lewis, B.A.,
Hand-in-Hand Insurance Soc., 26 *New Bridge-street*, E.C.

1901 [2] Wilton, Herbert George,
Norwich Union Life Insurance Society, Norwich.

1894 [1] Windett, Sydney V.,
Eagle Insurance Company, 79 *Pall-mall*, S.W.

1899 [1] Winstanley, Charles William,
North British & Mercantile Insurance Co., 61 *Threadneedle-street*, E.C.

1903 [2] Wolfenden, Edgar Sydney,
Australian Mutual Provident Society, Sydney, Australia.

1895 [1] Wood, David James,
Commercial Union Assurance Co., 24, 25 *&* 26 *Cornhill*, E.C.

Date of becoming a Student.

1901 [1] Wood, Roland Stuart,
Liverpool & London & Globe Insurance Co., 7 *Cornhill*, E.C.

1902 [1] Woodhouse, David Alfred,
Refuge Assurance Co., Oxford-street, Manchester.

1896 [1] Woodhouse, Hubert Allen,
Union Assurance Society, 81 *Cornhill*, E.C.

1900 [1] Woolston, Paul Livingston, B.S.,
Hartford Life Insurance Co., Hartford, Conn., U.S.A.

1894 [1] Wyatt, George Matthew,
Law Guarantee & Trust Society, 49 *Chancery-lane*, W.C.

1894 [1] Wylie, Samuel Brown, A.M.,
112 *N. Broad-st., Philadelphia, U.S.A.*

1886 Yeatman, Alexander Alfred,
2 *Coleman-street*, E.C.

1895 [1] Yeldham, William James,
Prudential Assurance Company, Holborn-bars, E.C.

1903 [1] Young, Henry J.,
Prudential Assurance Company, Holborn-bars, E.C.

1897 [1] Younger, R. Hugh,
Beech Lea, Springfield-road, Altrincham, Cheshire.

1904 [1] Zumstein, Herbert Christian,
Australian Mutual Provident Society, Melbourne, Australia.

*** *It is requested that any inaccuracy in the foregoing list may be pointed out to the* Assistant Secretary.

CORRESPONDING MEMBERS.

Belgium.

BRUSSELS.

M. George H. Adan,
Directeur-Général de la Royale Belge Compagnie Anonyme d'Assurances à Forfait sur la vie et contre les Accidents, Rue Royale (Coin Impasse du Parc).

M. A.M. Bégault, Mem. Act. Soc. Amer., *Hon. Secretary of the Association des Actuaires Belges. Membre correspondant de l'Institut des Actuaires Français,* 72 *Rue du Lac.*

M. Léon Duboisdenghien,
Directeur à la Caisse Générale d'Epargne et de Retraite, Rue Gérard, 96.

M. Léon Hamoir,
Directeur-Général de la Cie. des Propriétaires Réunis, 16 *Rue de Loxum.*

M. Fl. Hankar,
First Director of the Caisse Générale d'Epargne et de Retraite, 51 *Chaussée d'Haecht.*

M. Omer Lepreux, Mem. Act. Soc. Amer., *Directeur - Général de la Caisse Générale d'Epargne et de Retraite de Belgique. Président du Comité Permanent des Congrés Internationaux d'Actuaires. Président de l'Association des Actuaires Belges. Membre Correspondant de l'Institut des Actuaires Français. Membre de la Commission Permanente des Sociétés Mutualistes et de la Commission Centrale de Statistique. Membre de la Commission des Accidents du Travail. Ancien Capitaine du Génie Chargé de Cours à l'Ecole Militaire;* 48 *Rue du Fossé-aux-Loups.*

Denmark.

COPENHAGEN.

Prof. Thorvald Nicolai Thiele, Dr. Phil., *Astronomical Observatory.*

France.

PARIS.

M. Paul Guieysse, Mem. Act. Soc. Amer., *Président de l'Institut des Actuaires Français; Député du Morbihan;* 2 *Rue Dante.*

M. Hermann Laurent, Dr. Sc.,
Vice-President and Membre agrégé de l'Institut des Actuaires Francais, 11 *Rue du Val de Grâce.*

M. Léon Marie, Mem. Act. Soc., Amer., *Sub-Manager, Le Phénix Compagnie d'Assurances sur la vie; Membre Correspondant de l'Association des Actuaires Belges; Membre Agrégé de l'Institut des Actuaires Français;* 28 *Rue de Châteaudun IXe.*

M. Albert Quiquet,
Actuary, La Nationale Compagnie d'Assurances sur la vie; Secrétaire de l'Institut des Actuaires Français; 17 *Rue Laffitte IXe.*

M. Alfred Thomereau,
8 *Rue le Peletier.*

Germany.

BERLIN.

Dr. Grosse,
Hohenstaufenstrasse, 36, *W.,* 30.

GOTHA.

Dr. Johannes Karup,
Actuary of the Gotha Life Office.

M. Karl Samwer, Dr. Jur.,
Manager of the Gotha Life Assurance Company, Ohrdruferstrasse, 4.

Holland.

AMSTERDAM.

M. Corneille Louis Landré,
Actuary of the "Algemeene Maatschappij van Levensverzekering en Lijfrente", Damrak, 74; *Actuary of the "Nederlandsch Werkliedenfonds", the Hague; Vice-President of the Dutch Actuarial Society; Mitglied des Deutschen Vereins für Versicherungs-Wissenschaft; Membre Correspondant de l'Association des Actuaires Belges; Membre Correspondant de l'Institut des Actuaires Français; Membre du Conseil de direction du Comité Permanent des Congrès Internationaux d'Actuaires; Van Breestraat,* 185.

Hungary.

BUDAPEST.

M. Julius Altenburger,
Manager and Actuary of the Farmer's Insurance Association, viii. *Baross-utca,* 10.

Italy.

FLORENCE.

M. Guido Toja,
Actuary of "La Fondiaria" Life Assurance Company.

Russia.

ST. PETERSBURG.

M. Serge de Savitch,
Member of the Assurance Committee of the Ministry of the Interior, Nikolaewskaia, 35.

Spain.

MADRID.

Dr. José Maluquer y Salvador,
Insurance Counsellor (Ex-Actuary at the Home Office), Membre de l'Institut des Reformes Sociales, calle de Campomanes, 10.

Sweden.

STOCKHOLM.

M. Anders Lindstedt, Dr. Phil.,
Professor of Mathematics, and Director of the Polytechnicum; Counsellor of the Regency for Insurance Questions; Member of the Royal Swedish Academy of Science, etc.

Switzerland.

ZURICH.

Herr Dr. Gottfried Schaertlin,
Direktor der Schweizerischen Lebensversicherungs-und Rentenanstalt.

United States.

NEW YORK.

Mr. David Parks Fackler,
Ex-President of the Actuarial Society of America (1891-93); *Consulting Actuary,* 35 *Nassau-street.*

RULES

FOR THE

REGULATION OF THE LIBRARY.

1. The Library is open daily, from Ten to Five, from 1st of May to 30th September, and from Ten to Six from 1st of October to 30th April, except on Saturdays, when it is open from Ten to Two.

2. Members of the Institute are permitted to take out Two Books on making application in person, or by letter addressed to the Assistant Secretary; but no Member may keep any work longer than a Fortnight. If a Book be retained beyond that period, the borrower shall pay a fine of One Shilling per volume for each week, or part of a week, during which it is so retained, and shall not be permitted to obtain another from the Library until the missing book has been returned and the fine paid. When a Book is returned by a Member, it can be borrowed by him again, provided it has not been bespoken in the meantime by another Member.

3. Scientific Journals and Periodicals are not circulated until the volumes are completed and bound.

4. Cyclopædias and works of reference and certain other volumes are not circulated.

5. Any Member damaging, or losing, a work must pay an amount to be fixed by the Council as the equivalent of the damage or loss sustained.

6. Works taken from the shelves for reference are not to be replaced, but must be laid on the Library table.

7. A list of defaulters shall be submitted monthly at each meeting of the Council.

By Order of the Council.

November, 1899.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On Staff Pension Funds.

By George King, F.I.A., F.F.A., *one of the Vice-Presidents of the Institute of Actuaries; Consulting Actuary.*

[Read before the Institute, 30 January 1905.]

Introductory.

1. The present effort does not profess to be complete. It deals with the main problems that come before the Consulting Actuary in connection with Staff Pension Funds, but does not go into the details of what may be called the fancy benefits which are occasionally met with. The solutions of the general problems here given will, however, indicate the methods to be applied in attacking other, and sometimes more complex, questions, which, when the principles have been grasped, will not present serious difficulty.

2. At the outset, I should like to place on record the obligation I am under to our esteemed friend, Mr. Ralph Price Hardy. To him belongs the entire credit of raising, from the empirical to the scientific, the methods of dealing with these Funds. Some thirty years ago, he conceived the ideas, and elaborated the processes, by which the rates of contribution may be determined, and the valuations made, with something approaching to precision. His methods, in the course of years, may have been extended, and perhaps even improved upon. For instance, we are indebted to Mr. Henry William Manly for the plan, used throughout this paper, of involving the scale salaries in the denominators; a plan which marked a very great advance. Nevertheless, the development of the main principles is due entirely to Mr. Hardy. He was the pioneer, and we have followed in his footsteps.

3. It may perhaps be thought presumptuous that I should attempt this apparently intricate enquiry after the great achievements of Mr. Manly (*J.I.A.*, xxxvi, 209, and xxxvii, 193); but even he has not said the last word on the subject; and I venture to hope that what follows will be found useful to students, and to the practising actuary. The methods explained are many of them new, and differ, in some respects materially, from any previously published; and are often perhaps simpler, and occasionally more effective. The notation I have been accustomed to employ is very similar to that of Mr. Manly, and anyone acquainted with his will readily read mine. I therefore adhere to my own, and, in doing so, secure the advantage of distinguishing his functions from mine. These functions are similar in their nature, and are used for similar purposes, but they are not identical.

4. The Staff Pension Funds, which it is proposed at present to investigate, are those which, in consideration of periodical contributions calculated as a percentage of salary, promise retiring allowances to the employees of great industrial undertakings, these allowances being likewise calculated as a percentage, in some form, of salary; and which also provide for a return of contributions under various circumstances. It is assumed that membership is compulsory on all employees, or on certain classes of employees, and that none others are admitted: also that, should a member leave the service, his membership thereupon necessarily ceases. Were a voluntary element to be introduced, it would be requisite to resort to other methods.

5. To illustrate this paper we shall assume a typical Fund, supported by Contributions of 5 per-cent on the salaries, half payable by the members, and half by the company. These we shall call Ordinary Contributions. There will also be Extra Contributions, payable by members who enter above a certain age limit, these extras increasing with the age at entry. The Fund will be assumed to provide retiring pensions calculated in various ways according to salary, and to make certain returns of contributions, on withdrawal, or death, before retirement on pension.

Section I.

Statistics of the Fund.

6. The initial step is, to collect the data necessary, first, for investigating the experience of the Fund, and, second, for making

the valuation. It is desirable to obtain statistics for as long a period as possible, ten years preferably, but, in the case of a large Fund, five years might be sufficient. The information may best be supplied on cards. Formerly schedules were employed, but they are cumbrous and not so satisfactory. With schedules, exact dates cannot so easily be dealt with, and the ages therefore cannot be stated so conveniently. The following is a form of card which, with modifications to meet special circumstances, has been found to work very well in practice. On the back of the card the columns are continued, for use in future valuations.

ACTIVE LIST. No.........

Name ....

Date of Birth . . {Extra Contribution %}

" entry . . Age at entry

" exit . . Age at exit..........

Mode of exit {Amount paid on exit ...} £..........

CONTRIBUTIONS.

Year.	Member's 2½	Extra, if any.	Age.
Past Payments.			
1895			
1896			
1897			
1898			
1899			
1900			
1901			
1902			
1903			
1904			
TOTAL			

7. The register number and the name of the member are given for purposes of identification, and so that enquiries may be answered should mistakes be discovered, or doubtful points arise. It will be seen from the specimen card that a period of ten years is assumed to be covered by the investigation, and that the valuation is to be made as on 31 December 1904. The form of card speaks for itself. A card must be written for each employee who was on the active list at any time during the period; and the officials of the Fund will fill in all the particulars asked for, except the ages, which are to be left to the actuary to enter.

8. The space for *Past Payments* applies only to members who were on the active list at the commencement of the period, and in it the officials will insert the total payments made by members from the date of their entry to 31 December 1894. The *Total* at the foot of the card applies only to members on the active list on the valuation date, and must include the *Past Payments* at the top of the columns. This information is required for valuing the returns of contribution. The actual amount of salary is not asked for, because that can be at once ascertained by multiplying by 40 the *Member's Contributions at* $2\frac{1}{2}$ *per-cent.* One small practical point should be mentioned. For the year of entry and the year of exit the member in the majority of cases pays contributions for only part of the year. The fractional contributions should be written in in red ink, to distinguish them from the others, because multiplication by 40 does not give the rate of salary. Special enquiry should be made as to the rate of salary of each member on the active list on the valuation date who entered in the last year of the period, because this information is required for the purposes of the valuation, and is not supplied by the cards.

9. In determining the ages, it has been found most convenient to take for *Age at Entry* the age nearest birthday at the date of entry. This will be written in all cases in the space prepared for it at the top of the card. In the case of the members on the active list at the beginning of the period, the then age nearest birthday should be taken, and entered in the space above the heavy line in the column headed *Age.* Similarly, for members on the active list at the end of the period, the then age nearest birthday should be entered in the column headed *Age,* on the line marked *Total* at the bottom of the card.

10. For the ages at exit a different course must be followed. *Exit* may take place by Withdrawal, Death, or Retirement. For

the purpose of constructing an ordinary mortality table, we should adopt some formula which would give effect to the fact that members withdrawing or retiring are, so far as the Fund is concerned, at the risk of death for only a portion of the year of exit; but that would not suit the circumstances of a Staff Pension Fund. Here, in order to estimate the value of returns of contributions on death, we must have, not the ordinary rate of mortality, but the probability of dying in a year while on the active list; that is, the ratio between the number dying in a year, and the number entering on that year. In getting out the rate of mortality we must, therefore, treat the withdrawals and retirements as at risk for the whole year. Similarly, to obtain the rate of withdrawal, we must treat deaths and retirements as at risk for the whole year, and, to obtain the rate of retirement, we must treat deaths and withdrawals as at risk for the whole year. Thus, all these three functions have a common denominator, which is very convenient.

11. To obtain the *Age at Exit*, in the case of members who were on the active list at the beginning of the period, we add to the then age the subsequent curtate duration; and, in the case of members who entered during the period covered by the observations, we add to the age at entry the curtate duration since entry. The *Age at Exit* so found is to be entered in the space provided for the purpose.

12. The cards, so completed, are now ready for the extraction of the experience. They must be sorted, counted, and scheduled, in the usual way. As an example, Table 1 is given on page 185. It exhibits the statistics of a Fund recently valued.

13. From the statistics, so tabulated, we must now ascertain the rates of withdrawal, mortality, and retirement. The statistical table, besides the columns of age, Cols. 1 and 8, contains six columns, namely,

Col. 2. Survivors on the active list, brought under observation from antecedent entrants. These may be represented by the symbol b_x. The more usual symbol is s_x; but, in connection with Pension Funds, s_x is appropriated to represent the Scale Salary.

Col. 3. New entrants during the period, written n_x.

Col. 4. Withdrawals during the period, written w_x.

Col. 5. Deaths during the period, written d_x.

Col. 6. Retirements on pension during the period, written r_x.

Col. 7. Existing on the active list at the close of the period, written e_x.

14. From the Statistical Table, the function *Exposed to Risk*, or *At Risk*, as it may be called for brevity, is at once derived. If E_x represent that function, we have the formula

$$E_x = E_{x-1} + (b_x + n_x) - (w_{x-1} + d_{x-1} + r_{x-1} + e_x),$$

and for

$$\text{The rate of withdrawal, } q_x^w = \frac{w_x}{E_x}.$$

$$\text{The rate of mortality, } q_x^d = \frac{d_x}{E_x}.$$

$$\text{The rate of retirement, } q_x^r = \frac{r_x}{E_x}.$$

15. The rates so derived require to be graduated, and formerly I employed a graphic process. More recently, however, I have resorted to Woolhouse's graduation formula, which can be so easily applied by the summation method of the late J. A. Higham, *J.I.A.*, xxxi, 323. This, when used with discretion, gives results which are eminently satisfactory.

16. The various ratios, derived from Table 1, are given, after graduation, in Table 2, page 186.

17. The pensioners' list must be treated quite independently of the active list. Separate cards must be written for pensioners; but the form is so simple, that a specimen is not necessary. For those who were pensioners at the beginning of the period, the then age nearest birthday should be taken; and, for those who remain pensioners at the close, the then age nearest birthday. For those who became pensioners during the period, we insert the age nearest birthday at the date of retirement on pension; while we obtain the age at death by adding the curtate duration, in exactly the same way as in the case of deaths on the active list. There are usually very few commutations of pension; but, should there be any, we must take, for the age at exit, the age nearest birthday at the date of commutation. Symbolizing the commutations by w_x, we have for the number at risk,

$$E_x = E_{x-1} + (b_x + n_x) - (d_{x-1} + w_x + e_x).$$

18. In Table 2 the graduated rate of mortality among pensioners is included. It will be noticed that, from the earliest pensioning age down to age 50, the rate is uniform at 9 per-cent. This is in accordance with the actual experience, extending over 20 years, of the Fund used for illustration. Early pensions are taken only as a matter of necessity, when members are seriously failing in health, and the rate of mortality is very heavy. But there are curious discrepancies between different Funds in this respect. Another large Fund which, from the nature of the employment of its members, might have been expected to be very similar to our example, gave, at each of two investigations, extending altogether over the long term of 20 years, a rate of mortality for early pensioners of only about 6½ per-cent. No doubt, in the second case, retirement on pension was permitted more freely than in the first; but the fact emphasises the necessity of exercising the strictest caution in dealing with Staff Pension Funds. The statistics of one Fund cannot safely be used for another; and therefore, notwithstanding the excellent work done in this connection by Mr. Manly, I deprecate the publication of standard Pension Fund tables; and especially of voluminous monetary tables based upon them, such as those given to us by him. They are very apt to be misleading, unless indeed they are used with the most extreme discretion, and modified freely after collation with the actual experience of the Fund to be valued. It is better far, in the case of each Fund, to go to the comparatively small trouble of preparing special monetary tables suited to its own circumstances. Mr. Manly himself (*J.I.A.*, xxxvi, 258) has uttered a most emphatic word of warning, which should not be overlooked.

19. The statistical tables for the active list and for the pensioners' list, respectively, are of course closely related, because the retirements from the one are the new entrants in the other. Nevertheless, the recorded ages are not identical. They are arrived at in different ways for the two lists, because they are required for different purposes.

20. In connection with Pension Funds, the question of the scale of salaries to be adopted is very important, and is perhaps more difficult than any other that comes before us. It is easy to take out the present average salaries from the experience of the Fund itself, but these are only an uncertain indication which may have to be departed from widely. When a railway company, for instance, is rapidly extending, and when the members of the

Staff Pension Fund are consequently rapidly increasing in number, the increase takes place among members at comparatively small salaries, of whom the great majority will never reach the highly paid ranks. The number of General Managers, Chief Engineers, &c., will not increase, and, with the lapse of time, the tendency is for the average salaries at the older ages to fall, although every individual salary may increase. The principal officials are not the youngest men, and there are always coming up among them an ever-increasing number of officials at smaller salaries, so that the larger salaries cease to have so marked an effect on the average. In Funds that have passed through my hands, the salary experience down to about age 40 has remained fairly uniform from valuation to valuation; but at older ages there is a decided tendency in the average to fall. Effect may properly be given to this feature in determining on a scale of salaries for the valuation, but, obviously, the greatest caution must be exercised. To adopt a scale in which the salaries increase too slowly with age, would lead to an underestimate of the liabilities. In Table 2 a scale of salaries is inserted for purposes of illustration. It is taken from a Fund recently investigated, but it must not be considered to be applicable elsewhere.

21. From the ratios in Table 2, the Service Table, given in Table 3, page 187, is prepared. It is analagous to the ordinary mortality table, but has three columns of decrement, instead of only one. Also, it is more limited in extent. With the majority of Funds, it comes to a natural termination at some age between 65 and 70, because very few remain on the active list at more advanced ages.

Section II.

Particulars for the Valuation.

22. Having prepared the Service Table, the next step is to schedule the particulars for the valuation. As will be seen later on, it is generally desirable, and in many cases necessary, if real accuracy is to be secured, to group the facts by age at entry as well as by age attained; and, in any case, there is but little more trouble in summarizing the data by two stages instead of by only one. This is the course which we proceed to illustrate. On page 189 is given a suitable form of preliminary schedule, which we shall call Schedule I.

23. The cards relating to members on the active list on the valuation date are first sorted into groups according to the age

at entry, and again into sub-groups according to age attained; and the cards in the sub-groups are lastly sorted into numerical order, so that any particular card may be easily found if required. The particulars on the cards are then entered in Schedule I. Between the sub-groups, sufficient space must be left for the casting of the columns, and for certain further particulars, as will be seen presently.

24. The amounts of Past Contributions to be entered in columns 3 and 4, are those which appear at the foot of the cards. As regards column 5, $2\frac{1}{2}$ per-cent of Salary, that also is given at the foot of the cards, except in the case of members who entered during the last year of the period. For them we must take $2\frac{1}{2}$ per-cent of the salaries ascertained by enquiry, as explained in paragraph 8. Similarly for the extra contributions; and in order to arrive at the amount of these in the case of new members, we insert the percentage of the extra contributions which they pay. This is given at the top of the cards.

25. When all the cards have been scheduled, the columns are cast, and the totals entered in red ink to make them stand out distinct from the other figures.

26. The sum of the contributions in column 5 for each age attained, is $2\frac{1}{2}$ per-cent of the aggregate salaries which were drawn by the members in each group in the year immediately preceding the valuation; but, for valuation purposes, we require the salaries which, it is assumed, will be drawn in the year immediately following the valuation. We must therefore multiply the total at age attained, x, by the ratio $\frac{s_x}{s_{x-1}}$. The result may be called the *Adjusted $2\frac{1}{2}$ per-cent of Salary.* Similarly for the extra contributions in column 7. These adjusted amounts are entered in the space reserved in the respective columns at the foot of the groups for each age attained. The preliminary schedule is now complete.

27. For valuing prospective pensions, we usually require to know only the Adjusted $2\frac{1}{2}$ per-cent of Salary for each age attained under each age at entry, and we therefore transfer the adjusted totals of column 5 from Schedule I to Schedule II, of the form given on page 190. In Schedule II there will be a sheet for each age at entry, and on each sheet a line for each age attained. It may sometimes be necessary to include in Schedule II columns for past contributions (*see* paragraphs 162 and 165 to 182).

28. To value future contributions, and, generally also, to

value the various returns of contributions, we require the data arranged only according to age attained, discarding the age at entry. We must therefore summarize Schedule I again, by collecting together from the sheets for every age at entry the adjusted totals for each age attained. The process can be carried out conveniently by means of Schedule III, an intermediate schedule, the form of which is given on page 191. In Schedule III there will be a sheet for each age attained, and, on each sheet, a line for each age at entry.

29. When the columns of Schedule III have been cast, the totals are carried into the Grand Summary, in Schedule IV, of which the form is given on page 192. Schedule IV will consist of one sheet only, with a line for each age attained.

Section III.

Future Contributions.

30. Salaries are assumed to be payable uniformly throughout the year of age, an assumption that is sufficiently accurate for practical purposes. Also, s_x represents the salary which, one employee taken with another, each will receive during the year of age x to $x+1$. No assumption is made as to when increments of salary take place, and it is of no consequence whether they occur at the beginning of the year, or at any other period in the year. The function s_x has been formed on the basis of the general statistics of the Fund, in which are involved increments of salary accruing at all points of the year of age. In this respect s_x is like the function μ_x, which is essentially continuous, but which, for convenience, is expressed in yearly periods.

31. Salaries, then, being payable continuously throughout the year, so also are contributions; and these take the form of continuous increasing annuities. Mr. Manly has assumed that they are due at the beginning, but payable at the end, of the year; but this seems to me to introduce complications which can be easily avoided.

32. Contributions are payable by all on the active list, that is, by those represented by the l_x column of the Service Table. Now, that column corresponds to the l_x column of the ordinary Mortality Table, and differs therefrom in that it is subject to decrement, not only from death, but also from other causes. There are withdrawals from the active list, and there are retirements on pension. Mr. Manly throughout his papers has

assumed that deaths, withdrawals, and retirements, all take place at the end of the year of age; but here again I venture to depart from his methods. It is much nearer the truth to assume that all these causes of decrement are operative throughout the year, and that deaths, withdrawals, and retirements, are uniformly distributed over each year of age.

33. For a denominator, to be employed in some of the processes of the valuation, we form from the Service Table the function $D_x = v^x l_x$; but this is not in itself of immediate use in finding the value of future contributions. Just as we may, for the ordinary continuous annuity, write

$$\bar{a}_x = \frac{D_{x+\frac{1}{2}} + D_{x+1+\frac{1}{2}} + \&c.}{D_x},$$

so we may find the value of the special increasing continuous annuity which we are seeking, by preparing from the Service Table a column of the function $D_{x+\frac{1}{2}}$. This we can do by writing $D_{x+\frac{1}{2}} = \frac{1}{2}(D_x + D_{x+1})$; and, this being a continuous function, we may, by analogy, write $D_{x+\frac{1}{2}} = \bar{D}_x$. We therefore have

$$\bar{D}_x = \frac{D_x + D_{x+1}}{2}.$$

34. The salary for the year immediately following age x being s_x, the value at age x of 1 per-cent of such salary is

$$\frac{\bar{D}_x \times \dfrac{s_x}{100}}{D_x}.$$

Similarly, the value at age x of 1 per-cent of the salary to be received in the year of age $x+1$ to $x+2$, is

$$\frac{\bar{D}_{x+1} \times \dfrac{s_{x+1}}{100}}{D_x},$$

and so on. If, therefore, we form for each age the function

$$^s\bar{D}_x = \bar{D}_x \times \frac{s_x}{100},$$

and sum it, so that

$$^s\bar{N}_x = {}^s\bar{D}_x + {}^s\bar{D}_{x+1} + {}^s\bar{D}_{x+2} + \&c.,$$

we shall have

$$\frac{^s\bar{N}_x}{D_x}$$

to represent the value of 1 per-cent of the future salaries of an employee aged x, drawing at present a salary of s_x. If, therefore,

there be n employees, all aged x, whose aggregate salaries amount to ns_x, no matter what may be the individual salaries, then the value of 1 per-cent of the future salaries of all these members will be $n \times \frac{{}^s\overline{\mathrm{N}}_x}{\mathrm{D}_x}$.

35. Formulas based on these principles were formerly used in valuing future contributions, and they give accurate results when there is no disturbing element, and when the salary scale correctly represents, at all ages, the average salaries actually being drawn by the staff, and likely to be drawn in the future. Such would be the case if the Fund had reached a stationary state. But when the scale salaries have been adjusted, as discussed in paragraph 20, to represent the probable future average salaries of a growing Fund, as distinguished from present average salaries, the formula is no longer applicable. It is better to use a function which will give the value of 1 per-cent of future salaries, commencing at 100, and increasing according to scale. We have merely to divide the expression already found by s_x, and multiply by 100; or, what is the same thing, divide by $\frac{s_x}{100}$. The denominator of the expression will thus become $\mathrm{D}_x \times \frac{s_x}{100}$, which we may write ${}^s\mathrm{D}_x$; and writing ${}^f\mathrm{F}^c_x$ for the factor for valuing future contributions, we have

$$ {}^f\mathrm{F}^c_x = \frac{{}^s\overline{\mathrm{N}}_x}{{}^s\mathrm{D}_x}. $$

It will be seen that to construct a table of this factor, we must prepare tables of the functions ${}^s\mathrm{D}_x$, ${}^s\overline{\mathrm{D}}_x$, and ${}^s\overline{\mathrm{N}}_x$.

36. Thus, we have ${}^f\mathrm{F}^c_x$ as the value of 1 per-cent of future salaries, commencing at 100, and increasing according to scale. If there be an employee aged x, with present salary of s, the present value of a contribution of 1 per-cent of his future salary will be $\frac{\mathrm{s}}{100} \times {}^f\mathrm{F}^c_x$; and if there be n employees, all aged x, with aggregate salaries of S, the present value of a contribution of 1 per-cent of all their future salaries will be $\frac{\mathrm{S}}{100} \times {}^f\mathrm{F}^c_x$.

37. From another point of view, because 1 per-cent of a salary of 100 is unity, therefore ${}^f\mathrm{F}^c_x$ is the value of an annuity commencing at 1, and increasing according to the rate of increase in the scale salaries. Now, the contributions in columns 5 and 6

formula gives results as accurate as are attainable in the present state of our knowledge and experience.

38. As to the notation, the generic symbol F is used to represent valuation factors of every kind, and subsidiary symbols are annexed to distinguish one factor from another. Here, x indicates, as usual, the present age; c shows that contributions are in question; and, f that they are future contributions. It will be noted that no subsequent adjustment of any kind is required after the formula has been applied.

Section IV.

Returns of Contribution, without Interest.

Withdrawals.

39. The term *Withdrawal* is used to denote removal from the active list for any cause except death, or retirement on pension. Sometimes, on account of differences in the amount of the return, different causes of withdrawal may have to be distinguished from each other. This can be done by sub-dividing the column of w_x in the Service Table. For instance, the rules may provide for a return of only the member's own contributions in the event of his leaving the service voluntarily; and for a return of his own contributions, increased by the half or the whole of the contributions made by the Company on his behalf, in the event of his being compelled to leave the service, during the first ten years of his membership, on account of permanent and complete breakdown in health. We might represent this second category of withdrawals by the symbol h_x; and, instead of one column, headed w_x, in the Service Table, we should have two, headed respectively w_x and h_x. These two columns would be treated in the valuation separately, but on identical principles. Unless, however, the analysis of the experience of the Fund brings out features more than usually well marked, it is seldom necessary to go into such laborious details. In a recent case, where the total liabilities of the Fund were over £2,500,000, the liability in respect of health withdrawals amounted to only £3,900, and, had these been included with ordinary withdrawals, the difference in the estimate of liability would have been only about £1,300, a sum of scarcely sufficient importance to justify much extra work, and consequent expense, where the other figures involved are so large. Moreover, of Schedule IV are such annuities, and, if K_x be the contributions, whether ordinary or extra, at age x, their value is $K_x \times {}^{f}F^{c}_{x}$. This

if no distinction be made between ordinary and health withdrawals, the slight underestimate of liability so caused may well be set off against the slight overestimate due to including among the ordinary withdrawals those who are dismissed the service on account of gross misconduct, and who, under the rules of the majority of Funds, forfeit all their contributions.

40. We have spoken of health withdrawals as taking place only within the first ten years of membership, and this is the usual rule, because after ten years the incapacitated member would take a pension. Therefore to value health withdrawals strictly, tables in the "select" form would be necessary, but the complications thereby introduced would be prohibitive. Sufficient accuracy will be secured if, in obtaining the rate of health withdrawal, which may be written q^h_x, we ignore the limit of ten years, and include among the exposed to risk all members, no matter what may have been the duration of their membership. This will understate the value of q^h_x; but we can provide compensation for the understatement by treating all existing members as liable to health withdrawal, although many of them may have been more than ten years in the Fund.

(*a*) Past Contributions.

41. The withdrawals being assumed to be uniformly distributed over the year of age, the value of a unit to be returned if withdrawal take place in the first year following age x is $\frac{w_x . v^{x+\frac{1}{2}}}{D_x}$; if in the second year, $\frac{w_{x+1} . v^{x+1+\frac{1}{2}}}{D_x}$; and so on. Writing $w_x . v^{x+\frac{1}{2}} = \overline{C}^w_x$, and $\Sigma\overline{C}^w_x = \overline{M}^w_x$, we have for the value of a unit to be returned on withdrawal at any future time

$$^p F^w_x = \frac{\overline{M}^w_x}{D_x}.$$

42. If, now, we write S for the total amount of past contributions of all members at present aged x, we have $S \times {}^p F^w_x$ for the value of the return of these on withdrawal.

43. Here we see from the affixes to the symbol F, that we are dealing at age x with the return on withdrawal of past contributions.

44. The formula applies equally to past extra contributions if such be returnable.

(*b*) Future Contributions.

45. We had above, in paragraph 41, $\overline{M}^w_x = \Sigma\overline{C}^w_x$. If the return is to be 1 per-cent of salary, then the unadjusted function

for valuing the return on withdrawal at any future time in respect of the salary for the year x to $x+1$ is $\frac{s_x}{100} \times \overline{M}^w_x$, which we may write ${}^sM^w_x$. But this will give a value too large, because it assumes that, on withdrawal in the first year, a return will be made of 1 per-cent of salary for the whole year, instead of for only half a year, as is the case on the supposition of a uniform distribution of withdrawals. The necessary correction is made by deducting $\frac{1}{2}\cdot\frac{s_x}{100}\cdot\overline{C}^w_x$ from ${}^sM^w_x$. The result may be written ${}^s\overline{M}^w_x$, so that we have,

$$ {}^s\overline{M}^w_x = {}^sM^w_x - \frac{1}{2}\cdot\frac{s_x}{100}\cdot\overline{C}^w_x . $$

The arithmetical work of preparing this adjusted function may be shortened by constructing a column of $\overline{M}^w_x - \frac{1}{2}\overline{C}^w_x$, before involving the factor $\frac{s_x}{100}$ in forming the column of ${}^s\overline{M}^w_x$.

46. In order to obtain the value of a return on withdrawal at any future time of 1 per-cent of all future salaries, we must sum the column of ${}^s\overline{M}^w_x$, and write

$$ {}^s\overline{R}^w_x = {}^s\overline{M}^w_x + {}^s\overline{M}^w_{x+1} + \&c. $$

47. For the value of a return on withdrawal of 1 per-cent of all future salaries of an employee whose present salary is s_x, we have ${}^s\overline{R}^w_x \div D_x$; but, just as in dealing with the value of future contributions in paragraph 35, it is better to base our calculations on a present salary of 100, and write

$$ {}^fF^w_x = \frac{{}^s\overline{R}^w_x}{{}^sD_x} . $$

48. Thus we have the valuation factor ${}^fF^w_x$ for the value at age x of a return on withdrawal of 1 per-cent of future salary, commencing at 100, and increasing according to scale.

49. If there be an employee aged x, with present salary of s, the value of a return of 1 per-cent of his future salary will be $\frac{\text{s}}{100} \times {}^fF^w_x$; and if there be n employees, all aged x, with aggregate salaries of S, the value of a return on withdrawal of 1 per-cent of all their future salaries will be $\frac{\text{S}}{100} \times {}^fF^w_x$.

50. Here we see, from the affixes to the symbol F, that we are dealing at age x with the return on withdrawal of future contributions.

51. The return of 1 per-cent of a salary beginning at 100, may be looked upon as a return of all the payments of a continuous annuity, commencing at 1, and increasing according to the salary scale. Therefore, using K_x with the same meaning as in paragraph 37, the value of the return of the future contributions of all the members aged x is $K_x \times {}^{f}F_x^w$.

52. The adjustment effected by the deduction of $\frac{1}{2}\overline{C}_x^w$ explained in paragraph 45 is not of such slight importance as might, on first thought, be supposed. In the case of a large Fund recently valued, to have omitted the adjustment would have caused an overestimate, by fully $8\frac{1}{2}$ per-cent, of the value of the return of future contributions.

DEATHS.

53. The factors for death returns are calculated and employed on precisely the same principles as have been applied to those for returns on withdrawal. We have merely to use the column of d_x, instead of that of w_x, in the Service Table. We have

(a) Past Contributions.

$$\overline{C}_x^d = d_x \times v^{x+\frac{1}{2}}$$

$$\overline{M}_x^d = \Sigma \overline{C}_x^d$$

$${}^{p}F_x^d = \frac{\overline{M}_x^d}{D_x}$$

(b) Future Contributions.

$${}^{s}M_x^d = \frac{s_x}{100} \times \overline{M}_x^d$$

$${}^{s}\overline{M}_x^d = \frac{s_x}{100}(\overline{M}_x^d - \tfrac{1}{2}\overline{C}_x^d)$$

$${}^{s}\overline{R}_x^d = \Sigma\, {}^{s}\overline{M}_x^d$$

$${}^{f}F_x^d = \frac{{}^{s}\overline{R}_x^d}{{}^{s}D_x}$$

SPECIAL FORM OF RETURN.

54. One rather special form of return on death before pension is sometimes met with. It is provided that, if death occur

within ten years of entry, the whole of the member's own ordinary contributions are returnable, together with the whole of the contributions made by the Company on his behalf; but, if death occur after ten years, then the return is to be only one-half of the average salary from the date of entry. This is a troublesome benefit to value, and I know of no way of treating it satisfactorily without taking into account the age at entry as well as the age attained. A complete set of valuation factors must be prepared for each entry age, and about thirty sets in all will in many cases be required.

55. Let ${}_{y}z_{x}$ be the average of the scale salaries, from the commencement up to age x; so that, if y be the age at entry, and $x-y=t$, we shall have

$$ {}_{y}z_{x}=\frac{1}{t}(s_{y}+s_{y+1}+\ .\ .\ .\ +s_{x-1}). $$

56. The contributions returnable during the first ten years being 5 per-cent of actual salary received, if death occur in the first year, the return will be $2\cdot5\times\frac{s_y}{100}$;

if in the second year, $5\times\frac{s_y}{100}+2\cdot5\times\frac{s_{y+1}}{100}$;

if in the third year, $5\times\frac{s_y}{100}+5\times\frac{s_{y+1}}{100}+2\cdot5\times\frac{s_{y+2}}{100}$;

and so on for ten years. This is not a convenient form, because it does not fit in with the returns after ten years; but, with only slight loss of accuracy, another may be substituted. Instead of taking the salaries of the individual years, we may take the average salary, and assume that, if death occur in the first year, the return will be $2\cdot5\times\frac{{}_{y}z_{y+1}}{100}$; if in the second year, $7\cdot5\times\frac{{}_{y}z_{y+2}}{100}$; if in the third year, $12\cdot5\times\frac{{}_{y}z_{y+3}}{100}$; and so on until the tenth year, when the return will be $47\cdot5\times\frac{{}_{y}z_{y+10}}{100}$. After ten years the rate of return will be constant at 50 per-cent of the average salary; so that, when $t>10$, the return is $50\times\frac{{}_{y}z_{y+t}}{100}$, no matter what may be the value of t. By this form the value of the return will be very slightly overstated.

57. Special columns must be prepared. First of all, a table

of $\log \frac{y\bar{z}x}{100}$ is required for each age at entry, and for all values of x. This takes the form of successive columns, with the age at entry, y, at the top, and the duration of membership, t, at the side, t being $1+$curtate duration.

58. Next, we must have the values of $\log k_t$, where k is the percentage of average salary to be returned; and $k_1=2{\cdot}5$, $k_2=7{\cdot}5$, . . . $k_{10}=47{\cdot}5$; and $k_t=50$, when $t>10$. The values of $\log k$ should be written on a movable slip, log 50 being repeated on the slip down to the last possible duration of membership of the youngest entrant.

59. Lastly, we must have the mortality function, which is $v^{x+\frac{1}{2}}d_x$, symbolized, as before, by $\overline{C}^d_x$. A movable slip should be provided, with the values of $\log \overline{C}^d_x$ for all values of x, the age being given at the side.

60. The two movable slips are then placed side by side, in suitable positions, on the sheet of $\log \frac{y\bar{z}x}{100}$, and, by lateral addition of three numbers, the work is performed.

61. The function thus prepared is $\log {}^{z}_{y}\overline{C}^d_x$, where y is the age at entry, and x the age attained, and where

$$ {}^{z}_{y}\overline{C}^d_x = v^{x+\frac{1}{2}} \times d_x \times k_t \times \frac{y\bar{z}x+1}{100}\,. $$

Taking out the natural numbers, and summing, we have,

$$ {}^{z}_{y}\overline{M}^d_x = \Sigma\,{}^{z}_{y}\overline{C}^d_x $$

and, finally,

$$ {}_{y}F^d_x = \frac{{}^{z}_{y}\overline{M}^d_x}{{}^{s}D_x}\,. $$

62. This is the valuation factor, and the value of the return for n members aged x, who all entered at age y, and whose present total salaries are S, is $\frac{S}{100} \times {}_{y}F^d_x$. We have thus the whole return, of both past and future contributions, and not merely a return of 1 per-cent of salary.

63. In applying the formula, recourse must be had to Schedule II, and not to Schedule IV, which has been used in connection with the previous formulas. We multiply the numbers in column 3 of Schedule II by the appropriate valuation factors; and again multiply the grand total of the results by $\cdot 4$. This last step is taken because the contributions

in the Schedule must be multiplied by 40 to give the actual salaries, and the factor assumes a salary of 100.

64. The foregoing explanations may convey the impression that the process is intricate, but it is not so when practically applied. Only a certain amount of fairly simple arithmetical work is required. The principal trouble is to prepare the table of $\log \frac{z}{100}$, but, as we shall see later on (paragraphs 124 *et seq.*), that table will be useful in dealing with certain classes of prospective pensions.

65. The benefit here discussed is, so far as the Fund itself is concerned, financially equivalent to a return of the members' own contributions, no matter when death may occur. The cost in the aggregate is almost exactly the same. The more simple form of return, so far as the members individually are concerned, gives less during the first twenty years of membership, but more afterwards, and in this respect is the better. It is when death takes place after many years' contribution that a small return causes dissatisfaction to the family. The simpler form being more easily worked in practice, as we have seen in paragraph 53. it should be substituted for the other whenever possible.

Section V.

Returns with Compound Interest at the Valuation Rate.

66. Sometimes, in the case of death before pension, the contributions are returnable with compound interest at a rate guaranteed on the funds, and employed in the valuation. Under such a rule, the value of the return of future contributions, with interest, can be calculated readily, only one special commutation column being required; while, as regards past contributions, the mathematical theory is not more difficult, although the arithmetical application is more laborious.

67. When a sum is to accumulate at compound interest during a given status, and the present value is sought at the same rate of interest, the operations of interest and discount neutralize each other, and may be ignored. The value of the sum is simply the sum itself multiplied by the probability that it will be received. Thus, if a sum, S, is to accumulate at compound interest at rate i during the joint lives of x and y, and if it be receivable on the death of x if he die first, its present

L 2

value, also at rate i, is $Q^1_{xy} \times S$. This principle applies to the case before us, and the process can be followed without detailed explanations. We form a special column of *living* by summing the d_x column of the Service Table. This may be symbolized by λ^d_x. If a unit be receivable on death before pension, the probability that it will be received is $\frac{\lambda^d_x}{l_x}$; and this is also its present value at rate of interest i, when it is accumulating at the same rate.

(*a*) Future Contributions.

68. In a way analogous to that followed in finding the value of returns without interest in paragraphs 45 to 47, the value of a return with compound interest, on death before pension, of 1 per-cent of the first year's salary, which is assumed to be 100, is

$$\frac{v^{\frac{1}{2}}(\lambda^d_x - \frac{1}{2}d_x)\frac{s_x}{100}}{\frac{s_x}{100}l_x} = \frac{v^{x+\frac{1}{2}}(\lambda^d_x - \frac{1}{2}d_x)\frac{s_x}{100}}{{}^sD_x};$$

and of 1 per-cent of the second year's salary

$$\frac{v^{1+\frac{1}{2}}(\lambda^d_{x+1} - \frac{1}{2}d_{x+1})\frac{s_{x+1}}{100}}{\frac{s_x}{100}l_x} = \frac{v^{x+1+\frac{1}{2}}(\lambda^d_{x+1} - \frac{1}{2}d_{x+1})\frac{s_{x+1}}{100}}{{}^sD_x};$$

and so on. We may write

$$v^{x+\frac{1}{2}}(\lambda^d_x - \frac{1}{2}d_x)\frac{s_x}{100} = {}^s\overline{M}^{di}_x$$

and

$$\Sigma\,{}^s\overline{M}^{di}_x = {}^s\overline{R}^{di}_x$$

Then,

$${}^fF^{di}_x = \frac{{}^s\overline{R}^{di}_x}{{}^sD_x}.$$

69. To former symbols we add the affix, i, to indicate that returns with compound interest at the valuation rate are in question.

70. The factor ${}^fF^{di}_x$ gives the value of a return, with compound interest, on death before pension, of 1 per-cent of future salary, commencing at 100, and increasing according to scale. If there be n employees aged x, with aggregate salaries of S, the value of the return in respect of all of them is $\frac{S}{100} \times {}^fF^d_x$.

71. As before, we may look upon the 1 per-cent of a salary commencing at 100, as an increasing annuity commencing at unity; and, therefore, if K_x be the contributions at age x given in column 5 of Schedule IV, the value of the return is $K_x \times {}^{f}F^{di}_x$.

(*b*) Past Contributions.

72. When we seek the value of the return of past contributions, the question becomes more complex; because the amount to be returned depends very much on the time elapsed since entry, the accumulations being proportionately greater through the operation of interest when the time is long than when it is short. It is necessary, therefore, to treat each age at entry separately.

73. If the present age be x, and the age at entry y, then, in the case of a member whose present salary is s_x, the accumulated amount of the contributions of 1 per-cent of past salaries is,

$$\left\{\frac{s_y}{100}(1+i)^{x-y-\frac{1}{2}}+\frac{s_{y+1}}{100}(1+i)^{x-\overline{y+1}-\frac{1}{2}}+\ldots+\frac{s_{x-1}}{100}(1+i)^{x-\overline{x-1}-\frac{1}{2}}\right\}$$

$$=(1+i)^x\left\{\frac{s_y}{100}v^{y+\frac{1}{2}}+\frac{s_{y+1}}{100}v^{y+1+\frac{1}{2}}+\ldots+\frac{s_{x-1}}{100}v^{x-1+\frac{1}{2}}\right\}$$

$$=(1+i)^x \times {}_yZ^i_x,$$

when we write ${}_yZ^i_x$ for the summation within the brackets. This being the amount to be returned, with further accumulations at compound interest, on death before pension, its present value, by paragraph 67, is

$$\frac{\lambda^d_x}{l_x} \times (1+i)^x \times {}_yZ^i_x.$$

74. Thus we have the value of a return in respect of salary, at present s_x, enjoyed by a member now aged x who entered at age y; and, for a present salary of 100, we have for value,

$$\frac{100}{s_x} \times \frac{\lambda^d_x}{l_x} \times (1+i)^x \times {}_yZ^i_x$$

$$=\frac{\lambda^d_x}{v^x \cdot \frac{s_x}{100} \cdot l_x} \times {}_yZ^i_x = \frac{\lambda^d_x}{{}^sD_x} \times {}_yZ^i_x.$$

That is,

$${}^{p}_{y}F^{di}_x = \frac{\lambda^d_x}{{}^sD_x} \times {}_yZ^i_x.$$

75. It is not very laborious to construct a table of ${}_{y}Z^{i}_{x}$. It will contain a column for each possible age at entry. We must first take the table, consisting of one column, of the values of $\frac{s_x}{100} \times v^{x+\frac{1}{2}}$, already used in the construction of the factor ${}^{f}F^{di}_{x}$, and sum it continuedly for each age at entry, from above downwards, just as, but in reverse order, N_x is formed from D_x. Moreover, ${}_{y}Z^{i}_{x}$ corresponds to the terminal, and not the initial, form of N.

76. When the table has been completed, the logarithms are taken out, and entered in columns, the age at entry at the top, and the age attained at the side. A movable slip of $\log \frac{\lambda^{d}_{x}}{{}^{s}D_{x}}$ is also prepared; and then, by the lateral addition of two numbers, the table of $\log {}^{p}_{y}F^{di}_{x}$, and hence of ${}^{p}_{y}F^{di}_{x}$, is completed.

77. The factor ${}^{p}_{y}F^{di}_{x}$ gives, for a salary of 100, the value, with accumulations at compound interest at the valuation rate, of a return on death before pension of past contributions at the rate of one per-cent on salary. If there be n members, all aged x, who entered at age y, and whose present salaries aggregate S, the value of the return is $\frac{S}{100} \times {}^{p}_{y}F^{di}_{x}$.

78. Taking the contributions, K_x, from Schedule II, which are $2\frac{1}{2}$ per-cent of current salaries, the value of the return of the whole $2\frac{1}{2}$ per-cent is $K_x \times {}^{p}_{y}F^{di}_{x}$.

79. In this investigation we have assumed that all existing salaries have reached their present figure by increments according to scale; and I do not see what other course could be adopted with any advantage. We cannot make use of the actual past contributions tabulated in Schedule IV, because, there, both age at entry and accumulations at interest, are ignored; and, even were we in Schedule II to include columns for past contributions according to age at entry, that would not help us, because the interest element would be absent. To enable us to take account of actual past contributions, we should require to ask for the amount paid, in each year from entry, by every existing member; and that information would not be forthcoming. Moreover, were the information available, the labour of calculating the accumulations of each item at compound interest would be enormous, and no commensurate advantage would be secured by undertaking it. For practical purposes, the comparatively brief method discussed above is quite sufficient. It is true that we might attempt the method of paragraphs 98 and 99, where simple

interest is in question. That, however, would involve a very complicated double summation in the calculation of ${}_yZ^i_x$, and the gain in accuracy would be secured at great cost.

Section VI.

Returns with Compound Interest at other than the Valuation Rate.

80. In Section V we assumed compound interest at the valuation rate, but the limitation is not necessary. By suitably modifying the formulas, we may provide for the return of contributions with compound interest at other than the valuation rate.

81. If a unit, to be paid on death before pension, is to accumulate at rate j, and to be valued at rate i, j being, in practice, less than i, although in theory there is no need for this proviso, the value of the unit for the first year is $\frac{d_x}{l_x}\cdot\frac{(1+j)^{\frac{1}{2}}}{(1+i)^{\frac{1}{2}}}$; and for the second year $\frac{d_{x+1}}{l_x}\cdot\frac{(1+j)^{1+\frac{1}{2}}}{(1+i)^{1+\frac{1}{2}}}$; and so on. We may assume a rate of interest, J, such that $\frac{1}{1+J}=\frac{(1+j)}{(1+i)}$, which gives $J=\frac{i-j}{1+j}$, and prepare commutation tables based upon it. Writing $\overline{C}^{dJ}_x$ for $d_x\times(1+J)^{-x-\frac{1}{2}}$, and D^J_x for $l_x\times(1+J)^{-x}$, we have for the value of the unit, with compound interest at rate j, to be received on death before pension,

$$\frac{\overline{C}^{dJ}_x+\overline{C}^{dJ}_{x+1}+\&c.}{D^J_x}=\frac{M^{dJ}_x}{D^J_x},$$

when we write M^{dJ}_x for $\Sigma\overline{C}^{dJ}_x$.

(*a*) Future Contributions.

82. Taking the return as 1 per-cent of future salary, we have for the value of the return in respect of the salary for the first year following age x,

$$\frac{1}{l_x}\cdot\frac{s_x}{100}\cdot u^{\frac{1}{2}}\cdot\{\tfrac{1}{2}d_x(1+J)^{-\frac{1}{2}}+d_{x+1}(1+J)^{-1-\frac{1}{2}}+\&c.\}$$

$$=\frac{1}{l_x}\cdot u^{\frac{1}{2}}\cdot\frac{(1+j)^{-x}}{(1+i)^{-x}}\cdot\frac{s_x}{100}\{\tfrac{1}{2}d_x(1+J)^{-x-\frac{1}{2}}+d_{x+1}(1+J)^{-x-1-\frac{1}{2}}+\&c.$$

$$=\frac{1}{D_x}\cdot u^{\frac{1}{2}}\cdot(1+j)^{-x}\cdot\frac{s_x}{100}\{M^{dJ}_x-\tfrac{1}{2}\overline{C}^{dJ}_x\}$$

$$=\frac{1}{D_x}\cdot u^{\frac{1}{2}}\cdot u^x\cdot\frac{s_x}{100}\cdot\overline{M}^{dJ}_x,$$

when we write $(1+j)^{-x}=u^x$, and $M_x^{dJ}-\frac{1}{2}\overline{C}_x^{dJ}=\overline{M}_x^{dJ}$. We shall find that, similarly, the value of the return in respect of the salary

$$\text{for the second year is, } \frac{1}{D_x}\cdot u^{\frac{1}{2}}\cdot u^{x+1}\cdot\frac{s_{x+1}}{100}\cdot\overline{M}_{x+1}^{dJ},$$

$$\text{for the third year is, } \frac{1}{D_x}\cdot u^{\frac{1}{2}}\cdot u^{x+2}\cdot\frac{s_{x+2}}{100}\cdot\overline{M}_{x+2}^{dJ},$$

and so on. If now we write

$${}^s\overline{R}_x^{dj} \text{ for } \Sigma u^{\frac{1}{2}}u^x\frac{s_x}{100}\overline{M}_x^{dJ},$$

and pass from a salary of s_x to one of 100, we finally have the valuation factor

$${}^fF_x^{dj}=\frac{{}^s\overline{R}_x^{dj}}{{}^sD_x}.$$

Here we add the affix j, to denote that the return is to include compound interest at other than the valuation rate. Correction has been made for the fractional contribution paid for the year of death. The multiplication by $u^{\frac{1}{2}}$ may be performed at the end of the operations.

83. This gives the value of the return, with compound interest at rate j, on death before pension, of 1 per-cent of future salaries, commencing at 100, and increasing according to scale. If there be n members aged x, with aggregate salaries of S, the value of the return is $\frac{S}{100}\times{}^fF_x^{dj}$.

84. As before, we may look upon the 1 per-cent of salary commencing at 100, and increasing according to scale, as an increasing annuity, commencing at unity; and, therefore, if K_x be the contributions at age x, given in column 5 of Schedule IV, the value of the return of the whole $2\frac{1}{2}$ per-cent is $K_x\times{}^fF_x^{dj}$.

85. The investigation here is similar in principle to that in paragraphs 45 to 52 for returns without interest; but the factor, u^x, appears, and commutation columns are used in the numerator, based upon the rate of interest J, which is equal to $\frac{i-j}{1+j}$, instead of upon i. The rate J is special, but there is no difficulty in involving it in the commutation columns. When the return

is to include compound interest at $2\frac{1}{2}$ per-cent, and the valuation rate is 4 per-cent, then

$$J = \frac{\cdot 04 - \cdot 025}{1\cdot 025} = \cdot 01463.$$

If j be greater than i, J is negative, and $(1+J)^{-x}$ becomes a function of accumulation, instead of discount.

(*b*) Past Contributions.

86. To find the value of the return of past contributions, we may follow the method of paragraph 73, where we were considering accumulations at the valuation rate of interest. But, seeing that now accumulations are to be at rate j, that is the rate to be used in calculating the summation within the brackets. The summation will therefore be $(1+j)^x \times {}_yZ_x^j$, when we write

$${}_yZ_x^j = \left\{\frac{s_y}{100}\cdot\frac{1}{(1+j)^{y+\frac{1}{2}}} + \frac{s_{y+1}}{100}\cdot\frac{1}{(1+j)^{y+1+\frac{1}{2}}} + \ldots + \frac{s_{x-1}}{100}\cdot\frac{1}{(1+j)^{x-1+\frac{1}{2}}}\right\}.$$

This being the amount to be returned with further accumulations at rate j, its present value, by paragraph 81, is

$$\frac{M_x^{dJ}}{D_x^J} \times (1+j)^x \times {}_yZ_x^j.$$

$$= \frac{M_x^{dJ}}{D_x} \times {}_yZ_x^j.$$

87. Thus we have the value of the return, in respect of past contributions, of one per-cent of a salary, at present s_x, enjoyed by a member now aged x, who entered at age y. For a salary of 100 we therefore have for the value of the return,

$$\frac{100}{s_x}\cdot\frac{M_x^{dJ}}{D_x}\cdot{}_yZ_x^j,$$

which is equal to $\qquad \dfrac{M_x^{dJ}}{{}^sD_x}\cdot{}_yZ_x^j.$

That is $\qquad {}_y^pF_x^{dj} = \dfrac{M_x^{dJ}}{{}^sD_x}\cdot{}_yZ_x^j.$

88. In constructing the table of the values of this factor, we follow the course explained in paragraphs 75 and 76; only we have special rates of interest, namely j and J.

89. The factor ${}_y^pF_x^{dj}$ gives, for a salary of 100, the value, with accumulations at compound interest at rate j, of a return on

death before pension of past contributions at the rate of one per-cent on salary. If there be n members aged x, who all entered at age y, and whose present salaries aggregate S, the value of the return is $\frac{S}{100} \times {}^{p}_{y}F^{dj}_{x}$: or, as in paragraph 78, taking the contributions, K_x from Schedule II, the value of the return of the whole $2\frac{1}{2}$ per-cent is $K_x \times {}^{p}_{y}F^{dj}_{x}$.

90. Mr. Ernest C. Thomas (*J.I.A.*, xxxviii, 276), has investigated this problem. As regards future contributions, his method is very similar to the above; but he assumes that contributions are payable at the beginning of the year. As regards past contributions, however, he takes for granted that we know the total amount of the accumulations at interest for all members at present aged x, and, as already remarked (paragraph 79), I do not see how that information is to be obtained. Should it, however, be in our possession, of course all the labour of forming the factor ${}^{p}_{y}F^{dj}_{x}$, would be avoided. Writing B for the total accumulations in respect of all members aged x, the value of the return, by paragraph 81, is simply $B \times \frac{M^{dJ}_{x}}{D^{J}_{x}}$.

SECTION VII.

Returns with Simple Interest.

91. In former times it was not uncommon to find in the rules of Pension Funds a provision for simple interest on contributions returned on death before pension; but the custom seems to be dropping into desuetude. This is fortunate, because, at any rate as regards past contributions, the benefit is laborious to value with anything approaching to accuracy. Nevertheless, it may be well to suggest methods which can be followed.

92. The value of a sum to be paid with simple interest on death before pension is easy to calculate. Omitting the denominator, the value in respect of the first year is

$$v^{x+\frac{1}{2}}.d_x\left(1+\frac{i}{2}\right);$$

and of the second year

$$v^{x+1+\frac{1}{2}}.d_{x+1}\left(1+\frac{3i}{2}\right);$$

and so on. After summing, this becomes

$$(\overline{C}^d_x + \overline{C}^d_{x+1} + \&c.) + \left(\overline{C}^d_x \cdot \frac{i}{2} + \overline{C}^d_{x+1} \cdot \frac{3i}{2} + \overline{C}^d_{x+2}\frac{5i}{2} + \&c.\right)$$

$$= \overline{M}^d_x + i(\overline{C}^d_x + 2\overline{C}^d_{x+1} + 3\overline{C}^d_{x+2} + \&c.) - \frac{i}{2}\overline{M}^d_x$$

$$= \overline{M}^d_x + i(\overline{R}^d_x - \tfrac{1}{2}\overline{M}^d_x).$$

Therefore, inserting the denominator, the value sought is

$$\frac{\overline{M}^d_x + i(\overline{R}^d_x - \frac{1}{2}\overline{M}^d_x)}{D_x}.$$

The first portion of the formula gives the value of the sum itself, and the second, that of the interest.

(*a*) Future Contributions.

93. Omitting the denominator, the value of the return in respect of the salary of the first year is

$$\frac{s_x}{100}(\overline{M}^d_x - \tfrac{1}{2}\overline{C}^d_x) + \frac{s_x}{100}\left(\overline{C}^d_x \cdot \frac{i}{4} + \overline{C}^d_{x+1} \cdot \frac{3i}{2} + \overline{C}^d_{x+2} \cdot \frac{5i}{2} + \&c.\right)$$

$$= {}^s\overline{M}^d_x + i \cdot \frac{s_x}{100}(\overline{C}^d_x + 2\overline{C}^d_{x+1} + 3\overline{C}^d_{x+2} + \&c.) - \frac{i}{2} \cdot \frac{s_x}{100}(\overline{M}^d_x + \tfrac{1}{2}\overline{C}^d_x)$$

$$= {}^s\overline{M}^d_x + i \cdot \frac{s_x}{100} \cdot \overline{R}^d_x,$$

when we omit the last term $\frac{i}{2} \cdot \frac{s_x}{100}(\overline{M}^d_x + \frac{1}{2}\overline{C}^d_x)$ as insignificant. Finding now similar expressions for the salaries of the second and subsequent years, and summing, we shall have

$${}^s\overline{R}^d_x + i\left(\frac{s_x}{100}\overline{R}^d_x + \frac{s_{x+1}}{100}\overline{R}^d_{x+1} + \&c.\right)$$

$$= {}^s\overline{R}^d_x + i \times {}^s\Sigma\overline{R}^d_x,$$

when we write ${}^s\Sigma\overline{R}^d_x$ for the sum of $\frac{s_x}{100}\overline{R}^d_x$ for all values of x.

94. To obtain the value of the return in respect of a salary of s_x, we divide by D_x, or, in respect of a salary of 100, by ${}^sD_x$. Finally we therefore have,

$${}^sF^{di}_x = \frac{{}^s\overline{R}^d_x + i \times {}^s\Sigma\overline{R}^d_x}{{}^sD_x}.$$

95. By omitting the term $\frac{i}{2} \cdot \frac{s_x}{100} (\overline{\mathrm{M}}^d_x + \frac{1}{2}\overline{\mathrm{C}}^d_x)$ in paragraph 93, we assume that interest for a whole year is allowed for the year of death, and thus we cause a slight overestimate of liability.

96. The factor ${}^{sf}\mathrm{F}^{di}_x$ gives the value of the return, with simple interest, of a contribution of 1 per-cent of future salaries, commencing at 100. In respect of all members now aged x, we take the contributions, K_x, from column 5 of Schedule IV, and the value of the return of the whole $2\frac{1}{2}$ per-cent is $\mathrm{K}_x \times {}^{sf}\mathrm{F}^{di}_x$.

97. The factor is not difficult to prepare. It consists of two portions. The first, $\frac{{}^s\overline{\mathrm{R}}^d_x}{{}^s\mathrm{D}_x}$, is simply the factor of paragraph 47, only for deaths instead of withdrawals, and gives the value of the return of future contributions irrespective of interest. The second portion, $\frac{i \times {}^s\Sigma\overline{\mathrm{R}}^d_x}{{}^s\mathrm{D}_x}$, gives the value of the return in respect of the interest alone. To obtain ${}^s\Sigma\overline{\mathrm{R}}^d_x$, we sum $\overline{\mathrm{M}}^d_x$, so producing $\overline{\mathrm{R}}^d_x$. We then form the products $\frac{s_x}{100}\overline{\mathrm{R}}^d_x$; and, lastly, we sum again. The work is simple, and not lengthy.

(*b*) PAST CONTRIBUTIONS.

98. The total past contributions are given in column 3 of Schedule IV, and to obtain the value, in respect of them, of the return with future simple interest, we multiply by the factor given in paragraph 92,

$$\frac{\overline{\mathrm{M}}^d_x + i(\overline{\mathrm{R}}^d_x - \frac{1}{2}\overline{\mathrm{M}}^d_x)}{\mathrm{D}_x}.$$

99. We have now only to find the value of future returns of past interest. Assuming age at entry y, and age attained x, and a contribution of 1 per-cent of salaries, the total past interest is,

$$i\left\{\frac{s_y}{100}(x-y-\tfrac{1}{2}) + \frac{s_{y+1}}{100}(x-\overline{y+1}-\tfrac{1}{2}) + \ldots + \frac{s_{x-1}}{100}(x-\overline{x-1}-\tfrac{1}{2})\right\}$$

$$= i\left[x\left(\frac{s_y}{100} + \frac{s_{y+1}}{100} + \ldots + \frac{s_{x-1}}{100}\right) - \left\{\frac{s_y}{100}(y+\tfrac{1}{2}) + \frac{s_{y+1}}{100}(y+1+\tfrac{1}{2}) + \ldots + \frac{s_{x-1}}{100}(x-1+\tfrac{1}{2})\right\}\right]$$

This is an expression somewhat similar to that in paragraph 73; only there are two summations, of which we must take the difference. Hence the arithmetical process is laborious.

100. When we have thus found, for each age at entry, the total accumulations of past interest at each attained age, in respect of a present salary of s_x, we obtain the value of the return of this interest for a salary of 100 by multiplying by $\frac{\overline{M}_x^d}{{}^sD_x}$, and so arrive at the factor which will give us the value, for age at entry y, at age attained x, of a return of past simple interest on contributions of 1 per-cent of salary. This factor must be multiplied into the corresponding number in column 3 of Schedule II in order to obtain the total value of the return of past simple interest.

Section VIII.

Pensions.

101. The benefits so far considered have been of a subsidiary character. Returns of contributions are excrescences, devised to meet the grievances which the employees think exist when everything is forfeited on death or withdrawal before entering on pension. These grievances are more imaginary than real, because, if no return be made, more funds will be available for the pensions; and, in the prospect of higher pensions should they maintain their membership, the employees will receive compensation for all they pay in.

102. We shall now pass to the main, but not the most intricate, branch of the enquiry, and investigate the methods of dealing with the pensions themselves.

103. The simplest form of Pension Fund is that where there is an absolutely fixed pension age, at which every survivor must retire, and where no earlier pensions are allowed under any circumstances. Here, the pensions are merely deferred annuities, of amount varying with salary, and with duration of membership; and they present but little difficulty. They seldom occur in practice, and we shall not discuss them further. Should occasion arise, they would be treated on the principles that apply to the more complex cases.

104. There may be a fixed pension age, but with the proviso that, on complete and permanent breakdown in health, earlier pensions are to be allowed, proportionate to years of service, and to salary. Pensions taken at the fixed pension age have been

called Normal Pensions, and those taken on previous breakdown in health, Early Pensions. In former days no direct account was taken of early pensions, but an average pension age was adopted, at which all pensions were assumed to emerge. This course is not altogether satisfactory. It is not easy to determine a suitable average pension age; and results based upon even the most careful analysis must always be untrustworthy. Later on an improvement was introduced. The liability in respect of normal pensions was calculated by itself; and a separate process was applied to ascertain the liability in respect of early pensions. An average age was assumed, after careful investigation, at which all the members were presumed to have entered; and the value of the early pensions was estimated accordingly. This is the method which, so far as my knowledge goes, is frequently followed at the present day. Although much better than the first, it seems to me to be still open to grave objections. It is really impossible to determine an average age at entry which may be relied on to give accurate results; and it is much more satisfactory, if we can, to avoid all assumptions, and to deal with the facts as we actually find them. By the processes hereinafter explained, I think this is possible; and in several valuations which have recently fallen to my lot, these processes have been followed with success. We discard the distinction between normal and early pensions, and treat all alike; and, whenever advantageous, we involve the actual age at entry, as well as the age at retirement; and thus, at the cost of a little more arithmetical work, we arrive at conclusions which are as certain as present knowledge permits.

105. Moreover, this plan fits well into the conditions which frequently prevail in practice. There is no absolutely fixed pension age, but an optional age, at or after which the member may elect voluntarily to retire on pension, or may be superannuated by the Company, entirely independently of the state of his health. When such a rule exists, a few members sometimes remain on the active list up to a really advanced period of life, as may be seen by reference to Table 1.

106. The Service Table gives the number of retirements at each age. Below age 25, in the example selected to illustrate this paper, that number is always zero; and, at age 68, all who do not die before 69 are supposed to retire within the year. Therefore, age 68 might, in this particular case, be called the normal pension age, but there is no distinction in principle

between r_{68}, and r_x for any other value of x. We shall, therefore, make no difference in practice. Retirements, no matter at what age, shall be treated alike.

107. The first step in dealing with pensions is to prepare a table of annuities applicable to pensioners. Very seldom will the experience of a Fund itself be of sufficient extent to supply throughout the requisite data. It is necessary at the older ages to adopt some standard table. The English Life Table, No. 3, Males, is often employed, but sometimes the H^{M}, or the $\mathrm{H}^{\mathrm{M}(5)}$; and I have known of the circumstances being such as to indicate the use of the Government Annuity Table. The experience of the Fund to be valued must be our guide in the selection of a suitable table. There can be no hard and fast rule. At the younger ages, and generally down to some age between 60 and 70, the actual experience of the Fund can be more or less incorporated, and the annuity table completed by working backwards from the standard table by the usual formula, $a_x = vp_x(1+a_{x+1})$. In our example, p_x, from age 50 to the youngest pension age, is uniformly ·91.

108. It is proper to assume that pensions, taken one with another, are payable continuously throughout the year; and, seeing that they are presumed to be entered on in the middle of the year of retirement, the annuity-value we require is $\bar{a}_{x+\frac{1}{2}}$. It is sufficiently accurate to write,

$$\bar{a}_{x+\frac{1}{2}} = \tfrac{1}{2} + \tfrac{1}{2}(a_x + a_{x+1}).$$

109. As a preliminary, we may find the value of an annuity of 1 per annum to be entered upon on retirement whenever that may take place.

110. The value in respect of the first year following age x is evidently $\dfrac{1}{\mathrm{D}_x}.r_x.v^{x+\frac{1}{2}}.\bar{a}_{x+\frac{1}{2}}$, and, in respect of the second year, $\dfrac{1}{\mathrm{D}_x}.r_{x+1}.v^{x+1+\frac{1}{2}}.\bar{a}_{x+1+\frac{1}{2}}$, and so on. Writing $r_x.v^{x+\frac{1}{2}}\bar{a}_{x+\frac{1}{2}} = \mathrm{C}_x^{ra}$, and $\Sigma\mathrm{C}_x^{ra} = \mathrm{M}_x^{ra}$, we have $\dfrac{\mathrm{M}_x^{ra}}{\mathrm{D}_x}$ as the value of the annuity sought. In the case of our illustration, r_x for all ages less than 25, and hence also C_x^{ra}, is always zero; and, therefore, for these younger ages, M_x^{ra} is constant. The constant value should, however, for the sake of convenience, be tabulated, because the denominator, D_x, varies.

A.—PENSIONS BASED UPON AVERAGE SALARIES.

111. We shall first consider pensions calculated as a uniform percentage of average salary from the commencement of membership; and, to begin with, we shall assume that there is no limit to the pension, that a pension is claimable on retirement from ill-health, even should breakdown occur in the first year of membership, and that fractional parts of a year are allowed to count towards pension.

112. For the purpose of the valuation we must distinguish between past and future salaries.

(*a*) PAST SALARIES.

113. It may be troublesome to find the average past salary, the number of years of membership not being directly tabulated; but it is not necessary to do so, because evidently it is the same thing whether we take κ per-cent of average salary for each of the t years of membership, or κ per-cent of the total of the salaries received during the t years of membership.

114. Column 3 of Schedule IV gives, for each age attained, the total past contributions of $2\frac{1}{2}$ per-cent of the salaries; and, multiplying by ·4, we have 1 per-cent of such salaries. This is the annual amount, at 1 per-cent, of the prospective pensions due to past salaries; and, multiplying by $\frac{M_x^{ra}}{D_x}$, we arrive at the present value. This multiplier is the valuation factor used in dealing with past salaries, and we may therefore write,

$$^{p}F_x^{ra} = \frac{M_x^{ra}}{D_x}.$$

(*b*) FUTURE SALARIES.

115. We must find the value of a pension of 1 per-cent of the salary of each future year. In respect of the first year following age x, the value evidently is,

$$\frac{1}{D_x} \cdot \frac{s_x}{100} (M_x^{ra} - \tfrac{1}{2} C_x^{ra}),$$

because this provides, no matter when retirement may take place, a pension of 1 per-cent of the salary of the year x to $x+1$, and includes the correction, already discussed, for the broken period

should retirement take place within one year. Similarly for the year $x+1$ to $x+2$, we have,

$$\frac{1}{D_x}\cdot\frac{s_{x+1}}{100}\left(M^{ra}_{x+1}-\tfrac{1}{2}C^{ra}_{x+1}\right)$$

and so on. We may write,

$$\frac{s_x}{100}\left(M^{ra}_x-\tfrac{1}{2}C^{ra}_x\right)={}^s\overline{M}{}^{ra}_x$$

and

$$\Sigma\,{}^s\overline{M}{}^{ra}_x={}^s\overline{R}{}^{ra}_x$$

and we have for prospective pensions in respect of all future salaries $\dfrac{{}^s\overline{R}{}^{ra}_x}{D_x}$.

116. Here present salary is assumed to be s_x; but, as before, it is more convenient to deal with a salary of 100. This is effected by substituting ${}^sD_x$ for D_x in the denominator. Thus we have,

$${}^fF^{ra}_x=\frac{{}^s\overline{R}{}^{ra}_x}{{}^sD_x}.$$

117. This factor gives, in respect of a salary of 100, increasing according to scale, the value, due to future salaries, of a pension of 1 per-cent of average salary for each year of membership. If there be n members, all aged x, with salaries aggregating S, then the value for all of them is $\dfrac{S}{100}\times{}^fF^{ra}_x$.

118. To effect the valuation, we must multiply the numbers in column 5 of Schedule IV by the corresponding values of ${}^fF^{ra}_x$, and again multiply the grand total by ·4.

119. Both as regards past and future salaries, we have found the value of a pension of 1 per-cent, and from this we can pass to any other percentage. Thus, if the rules provide for a pension of one-sixtieth of average salary for each year of membership, we have merely to multiply the final result by 1·667.

120. The valuation factor in paragraph 116 provides for proportionate pensions in respect of broken years of membership, but frequently the rules allow only completed years to count. It is easy so to construct the formula as to fit into this regulation. In paragraph 115 we must omit the term $\frac{1}{2}C^{ra}_x$, and write $\dfrac{s_{x-1}}{100}M^{ra}_x={}^{s-1}M^{ra}_x$, and $\Sigma^{s-1}M^{ra}_x={}^{s-1}R^{ra}_x$. Then the valuation factor to be multiplied into the salaries at age x is $\dfrac{{}^{s-1}R^{ra}_{x+1}}{{}^sD_x}$.

121. From the foregoing it is seen that pensions calculated on the plan so far discussed can be dealt with in a very simple manner; but rarely are such pensions met with in practice. Usually there is a maximum limit to the pension, which is often two-thirds of the average salary; and almost always a member is not eligible for pension if he retire during the first few years of membership, generally ten. Also, occasionally, the percentage of average salary given as pension is not uniform for each year of membership. For instance, in a recent case, the following were the figures:—

Specimen Pension Scale.

Completed Years of Membership	Pension in percentage of Average Salary	Completed Years of Membership	Pension in percentage of Average Salary
10	25	25	43
11	26	26	44
12	27	27	45
13	28	28	46
14	29	29	47
15	30	30	48
16	32	31	50
17	34	32	52
18	35	33	54
19	36	34	56
20	37	35	58
21	38	36	60
22	39	37	62
23	40	38	64
24	42	39	66
		40 and over	$66\frac{2}{3}$

122. When these complications exist, a different method of valuation must be followed, and account must be taken, not only of age attained, but also of age at entry. The actuarial theory is not more difficult, but the amount of arithmetical work entailed is greater.

123. We shall now assume a Fund providing pensions on the following plan:—

(*a*) No pension if retirement take place within a fixed number of years, say ten, from entry.

(*b*) The proportion of average salary allowed as pension to increase with duration of membership, either regularly, say one sixtieth, or one fiftieth, for each completed year; or irregularly, as in the specimen scale given in paragraph 121.

(c) A maximum proportion, say two-thirds, of average salary, to be allowed as pension after a given term of membership.

124. With such conditions we cannot deal with past and future salaries separately, because the percentage of each year's salary allowed as pension does not remain uniform.* Thus, if the rule give one-fiftieth for each year, with a maximum of two-thirds, the maximum will be reached on completion of thirty-four years. In the case of a member aged 40 who entered at 20, should he retire at any age below 54 he will take one-fiftieth, or two per-cent, of each year's salary as pension; but should he retire at age 60 he will take only 1·667 per-cent of each year's salary, or, should he retire at 65, only 1·481 per-cent. Therefore, seeing that the percentages change for past as well as for future salaries, we cannot treat past salaries as in paragraphs 113 and 114, or future salaries as in paragraphs 115 and 116. We must proceed in a way somewhat similar to that explained in paragraphs 57 to 61 when we were discussing a special form of death return.

125. Let x be the attained age, and y the age at entry. As in paragraph 55, let ${}_yz_x$ be the average scale salary from the commencement up to age x, so that, if $x-y=t$,

$$ {}_yz_x = \frac{1}{t}(s_y + s_{y+1} + \ldots + s_{x-1}). $$

Also let κ_t be the scale percentage of average salary which will be paid as pension on retirement in the year of age x to $x+1$. Finally, as before, let $C^{ra}_x = r_x . v^{x+\frac{1}{2}} . \bar{a}_{x+\frac{1}{2}}$.

126. The commutation function for the value of the prospective pension in respect of the first year following age x is

$$ C^{ra}_x \times \kappa_t \times \frac{{}_yz_x}{100}, $$

which we may write ${}^z_yC^{ra}_x$; and, in respect of the second year,

$$ C^{ra}_{x+1} \times \kappa_{t+1} \times \frac{{}_yz_{x+1}}{100}, \text{ or } {}^z_yC^{ra}_{x+1}; $$

and so on. Writing now ${}^z_yM^{ra}_x$ for $\Sigma {}^z_yC^{ra}_x$, and inserting the denominator for a present salary of 100, we have the total value of prospective pension for every possible year of retirement

$$ {}_yF^{ra}_x = \frac{{}^z_yM^{ra}_x}{{}^sD_x}. $$

* *See*, however, the Addendum, page 176, and the Postscript, page 179, where methods are explained by which past and future salaries can be treated separately.

127. This is the valuation factor, and gives, for an entrant at age y, who is now aged x, and is enjoying a present salary of 100, the value of future pension according to scale, and it includes full pension, and not merely 1 per-cent, for both past and future salary. If there be n members, who all entered at age y, and are now aged x, and who are drawing salaries to the aggregate of S, the value of all their prospective pensions is ${}_yF^{ra}_x \times \frac{S}{100}$. In making the valuation we multiply the numbers in column 3 of Schedule II, by the appropriate valuation factors, and multiply the grand total of the results by ·4.

128. The calculation of the factors is to a certain extent laborious, but not difficult. We prepare a complete table of $\log \frac{{}^y z_x}{100}$, as explained in paragraph 57. The values of $\log C^{ra}_x$ must be written on a slip, and at the younger ages, where C^{ra}_x is zero, blanks must be left, so as to guide us in placing the slip in position. The values of $\log \kappa_t$ must also be written on a slip, again with blanks at the early years where κ is zero. The two slips are then placed in position on the table of $\log \frac{{}^y z_x}{100}$, and, by lateral addition of three numbers, the values of $\log {}^z_yC^{ra}_x$ are formed. This column in the process of formation is written on the left of a sheet, with age at entry at the top and age attained at the side; and from it, by successive easy operations, the values of ${}_yF^{ra}_x$ are formed. In passing from $\log {}^z_yM^{ra}_x$ to $\log {}_yF^{ra}_x$, we employ another movable slip, giving the values of colog ${}^sD_x$. A specimen of the whole work, taken from a recent case, is given in Table 4, page 188, but the actual figures do not relate to the Fund of which the statistics have been so far used by way of illustration.

129. Where there is a maximum limit to the pension, injustice is committed in certain cases, when the pension is based on average salary during the whole of membership. For instance, take two members, A and B, A entering at age 20, and B at age 30, and each drawing salary according to scale throughout membership; and let the scale of pensions be 2 per-cent of average salary for each year, with a maximum of two-thirds. The maximum is reached in 34 years. Now let both members retire at age 64. A will take two-thirds of average salary calculated from 30 to 64, and B two-thirds calculated from 20 to 64, and thus the pension of A will be greater than that

of B, although his term of service has been ten years shorter, and he has contributed less to the Fund. To remove the anomaly the pension should be calculated as a percentage of the average salary, of the last 34 years only, of membership; but in that case the formula of paragraph 126 would not apply. The valuation would have to be made on the principles set forth in the following paragraphs.

B.—Pensions based upon Terminal Salaries.

130. Instead of being calculated on average salaries throughout membership, pensions frequently depend on the salary of the last completed year, or on the average salary of the last three, or five, or seven, completed years, before retirement. The salary thus assumed as the basis of pension we shall style terminal salary, and denote by z_x. Thus, if the average salary of the last three completed years is to be taken,

$$z_x = \frac{s_{x-3}+s_{x-2}+s_{x-1}}{3}.$$

131. We shall now assume a Fund providing pensions on the following plan—

(*a*) No pension if retirement take place within a fixed number of years, say 10, from entry.

(*b*) The pension to be calculated on terminal salary, that is, on the average salary of the last m completed years of membership, m being any number from unity upwards; and the proportion to increase with duration of membership, either regularly, say one-sixtieth for each completed year, or irregularly as in the specimen scale given in paragraph 121.

(*c*) A maximum proportion, say two-thirds, of terminal salary, to be allowed as pension after a given term of membership.

132. Here, as was the case when we were dealing, in paragraphs 121 to 128, with limited pensions based upon average salary, we must treat each age at entry separately; but the work is a good deal easier, because, with terminal salaries, z_x is the same for all ages at entry, and only one column of the values of $\log \frac{z_x}{100}$ is required, instead of a separate column for each age at

entry. Consequently, there is also required only one column of $\log {}^{z}C_x^{ra}$ for all ages at entry, ${}^{z}C_x^{ra}$ being $r_x . v^{x+\frac{1}{2}} . \bar{a}_{x+\frac{1}{2}} . \frac{z_x}{100}$. There is but one exception, namely, when pensions are of the class discussed in paragraph 129, and when therefore m is a large number. In such case, a separate column of $\log \frac{z_x}{100}$ is required for each.

133. As before, let x be the attained age, and y the entry age; and let κ_t (where $t = x - y$), be the scale percentage of terminal salary, z_x, which will be paid as pension on retirement in the year of age x to $x+1$.

134. The function for the value of the prospective pension in respect of the first year following age x is ${}^{z}C_x^{ra} \times \kappa_t$, which we may write ${}^{z}_{y}C_x^{ra}$; and, in respect of the second year, ${}^{z}C_{x+1}^{ra} \times \kappa_{t+1}$, or ${}^{z}_{y}C_{x+1}^{ra}$; and so on. Writing now ${}^{z}_{y}M_x^{ra} = \Sigma {}^{z}_{y}C_x^{ra}$, and inserting the denominator, ${}^{s}D_x$, for a present salary of 100, we have the total value of prospective pension for every possible year of retirement,

$$ {}_{y}F_x^{ra} = \frac{{}^{z}_{y}M_x^{ra}}{{}^{s}D_x}. $$

135. This is the valuation factor, and gives, for an entrant at age y, who is now aged x, and is enjoying a present salary of 100, the value of future pension according to scale. It includes pension for both past and future salaries. If there be n members, who all entered at age y, and are now aged x, and who are drawing salaries aggregating S, the value of all their prospective pensions is $\frac{S}{100} \times {}_{y}F_x^{ra}$. For the valuation we use column 3 of Schedule II, as in paragraph 127.

136. The calculation of the factors is effected very much in the same way as explained in paragraph 128, for the similar function relating to average salaries. The difference is that, for the present function, we first form a column of the values of $\log {}^{z}C_x^{ra}$, which is the same for all ages at entry unless m be very large; and that, in passing from $\log {}^{z}C_x^{ra}$ to $\log {}^{z}_{y}C_x^{ra}$, we have a lateral addition of only two numbers instead of three. Thereafter, the two processes are identical, and the specimen working sheet given in Table 4, page 60, applies to both.

137. When pensions based on average salaries are taken on the conditions discussed in paragraphs 129, they virtually, as we have seen, come under the terminal salary plan; only that the m

of paragraph 131, subsection (*b*), is to a certain extent a variable, with an upper limit determined by the number of years of membership which must elapse before the maximum rate of pension is reached.

138. Mr. Manly (*J.I.A.*, xxxvi, 249) remarks that, "it may "appear strange, but it is a fact, that the valuation, when the "pension is based on last received salary and number of years' "service, is more laborious than when the annuity is based on "average salary and number of years' service." This is true when the pensions are of the uncomplicated character defined in paragraph 111, that is, when they are free from limitations of every kind. But the sweet simplicity disappears the moment limiting conditions are introduced. Then, pensions based on terminal salaries are more easy to manipulate than those based on average salaries (*see* paragraph 136). The limitations mentioned in paragraph 123, are almost universally met with in practice.

139. When the maximum pension is reached only after a long term of service, say 40 or 45 years, it may be proper in some cases to ignore it and employ the easy formulas for unlimited pensions investigated in paragraphs 112 to 120. The upper limit will affect, and that but slightly, only those who enter younger than about age 25, and with some Funds these are very few. In using the simpler formulas we ignore also the absence of pension when retirement from ill-health takes place within the first few years of service, but the liability to be assigned to such very early pensions is not great. The pensions would be few in number; they would of necessity be small; and, on account of the high rate of mortality, their value would be at a minimum. By adopting the somewhat indolent course of employing the simpler formulas, we overstate to a certain extent the liability, and perhaps that is not a disadvantage. Nevertheless, in the majority of instances, it is preferable to be enterprising, and to employ the more accurate methods, even should they involve a little extra trouble.

140. We have throughout assumed that, even at the oldest age in the Service Table, retirements are uniformly distributed over the year of age, and thus we have rejected the idea of an absolutely fixed pension age. This, so far as my own experience extends, is in accordance with general custom. Should there be, however, an absolute pension age, say u, beyond which none can remain on the active list, the formulas and methods are still

applicable, with only one slight modification. For C_u^{ra} we have merely to write $l_u . v^u . \bar{a}_u$, instead of $r_u . v^{u+\frac{1}{2}} . a_{u+\frac{1}{2}}$, for the term at which the summations commence, and all the other processes remain unchanged.

Section IX.

Returns on Death after Superannuation.

141. A provision is frequently found in the rules of Pension Funds that, if death occur after the pension has been entered on, but before the amount of pension received has equalled the total contributions paid, then the balance of the contributions shall be returned to the representatives of the deceased annuitant.

142. To find the value of such return is a problem that does not admit of a mathematically exact solution. When we are concerned only with ordinary deferred annuities of fixed amount, and at fixed premiums, Mr. Manly has shown that formulas may be devised to meet the purpose. But in Staff Pension Funds, both annuities and contributions are of variable amount, depending on variable salaries; and, moreover, the amount of the annuity also depends on the duration of membership. Returns, also, are made on withdrawal or death before pension, and, at any rate as regards withdrawals, part of the contributions paid in respect of departing members, is left in the Fund to supplement the pensions of those who remain. We must therefore discard mathematical formulas, and resort to empirical methods, treating each case on its merits according to circumstances. We shall arrive at a solution sufficiently exact for practical purposes; and, if we take care that such error as there may be shall be on the safe side, nothing more is required.

143. It would be endless to attempt to deal with all the possible conditions of this problem, but, by taking one example, every useful purpose will be served. We shall assume a Fund, with contributions of 5 per-cent of salary, half payable by the members, and half by the Company, but the whole to be taken into account in estimating the value of the return. We shall also assume that pension is to be at the rate of 2 per-cent of average salary for each completed year of membership, with a maximum of two-thirds of such average salary; and it will be convenient to employ the scale salaries given in Table 3.

144. The following statement supplies the necessary

particulars for decennial ages at entry, and decennial terms of service, and it is sufficient for the present purpose.

Age at Entry	Years' Service	Total Salary	Average Salary	Pension	Total Contributions	Number of Years' Purchase
15	10	473	47·3	9·5	23·7	2·50
	20	1,409	70·5	28·2	70·5	2·50
	30	2,777	92·6	55·6	138·9	2·50
	40	4,387	109·7	73·1	219·4	3·00
	50	6,197	123·9	82·6	309·9	3·75
25	10	936	93·6	18·7	46·8	2·50
	20	2,304	115·2	46·1	115·2	2·50
	30	3,914	130·5	78·3	195·7	2·50
	40	5,724	143·1	95·4	286·2	3·00
35	10	1,368	136·8	27·4	68·4	2·50
	20	2,978	148·9	59·6	148·9	2·50
	30	4,788	159·6	95·8	239·4	2·50
45	10	1,610	161·0	32·2	80·5	2·50
	20	3,420	171·0	68·4	171·0	2·50

145. The number of years' purchase is obtained by dividing the total contributions by the pension, and in this particular example it is always 2·50, except when the maximum rate of pension is reached, when it is greater. Seeing that the number of years' purchase is greatest with the largest pensions, we must not assume an average value too small, but here we shall be safe if we take three years.

146. It therefore appears that, for three years after entering on pension, a decreasing assurance is required. Assuming an annuity of unity payable quarterly, then in the first year,

3·00 will be at risk for three months,
2·75 for a second three months,
2·50 for a third three months,
and 2·25 for a fourth three months,

making, on the average, 2·625 for a year. Similarly, in the second year, 1·625 will, on the average, be at risk for a year; and, in the third year, 0·625.

147. With our specimen Fund the rate of mortality is 9 per-cent down to age 50, and less at the higher ages, so that at 65 it becomes only 7·3 per-cent. The higher the rate of mortality, the greater is the value of the return we are seeking; and we must not assume an average value too low; but, seeing

that the majority of pensions, both as regards number and amount, are taken at the higher ages, we shall be safe in this instance in assuming an average rate of mortality of 8 per-cent. We shall also take interest at 4 per-cent, a very usual rate with Pension Funds.

148. The single premium for the risk is therefore,

For the first year	$v^{\frac{1}{2}} \times \cdot 08 \times 2\cdot 625$	$= \cdot 206$
,, second year	$v^{1+\frac{1}{2}} \times \cdot 92 \times \cdot 08 \times 1\cdot 625$	$= \cdot 113$
,, third year	$v^{2+\frac{1}{2}} \times (\cdot 92)^2 \times \cdot 08 \times 0\cdot 625$	$= \cdot 038$
	Total single premium for the risk	$= \cdot 357$

The value at pension age of an annuity of unity is, throughout, owing to the peculiar rate of mortality, about 7·7; and, the single premium for the risk being ·357, it is equal to 4·67, or, say 4·75 per-cent of the value of the annuity. We shall therefore make ample allowance, in this instance, for the special return, if we increase the estimate of liability for prospective pensions by $4\frac{3}{4}$ per-cent. Were pensions calculated on last, instead of average, salary, we should find that an increase of about 3 per-cent of the reserve would be sufficient. The pensions would be larger, while the contributions would remain the same. Therefore the return would be less.

SECTION X.

Calculation of Rates of Contribution.

149. When a Fund is to be started, and the intending members have formulated the benefits they desire, the actuary is sometimes asked to quote the percentage of salary necessary to provide them. Theoretically the question is not difficult, but in practice it scarcely admits of trustworthy solution. Ample latitude should therefore be given in the rules for revision, when the experience of the Fund itself has had time to accumulate. As we have already seen, it is not safe to apply to one Fund the data derived from another, which, notwithstanding a superficial appearance of similarity, may really differ widely. Moreover, even if the statistics be extracted of the particular staff itself, for a period of years, that is only an uncertain guide; because the introduction of a defined system of pensions may greatly affect the rates of withdrawal and retirement. Therefore the following enquiry applies more to the adjustment of the rates

of contribution for new entrants into a Fund long established, than to the preparation of the rules of one about to be started.

150. In the foregoing Sections all the valuation factors that have to do with the future, and only these now concern us, relate to a salary of 100; and, therefore, in using them to determine the contributions required to provide certain definite benefits, we obtain the percentage of contributions for salaries of any other amount, and no further adjustment is necessary. It is generally the case that at a few of the younger ages at entry the ordinary contribution of 5 per-cent is sufficient, or even a little more than sufficient, to provide the benefits; but that, from perhaps age 25 at entry, an extra is required. Should the members themselves pay the whole of the contributions, then approximately accurate graduation according to age is important, because it would not be fair to one set of members if we were to make them contribute to the benefits of another set. But when the employer pays a substantial proportion, it is necessary only to see that the contributions as a whole are sufficient, because, under such circumstances, none of the employees subscribe sufficient to provide their own pensions and returns, and the employer is perfectly entitled to say that he pays his quota for the good of the service as a whole, and not merely for that of individuals.

151. The rules of Funds being of almost infinite diversity, it would not be practicable to give formulas for every possible scale of benefits; nor is that necessary. If a few typical examples be selected, the principles adopted in their solution will be equally applicable to others. We shall therefore assume a Fund, granting pensions according to a definite scale, with returns on death or withdrawal before superannuation, and charging ordinary contributions of five per-cent on salaries, and such extra contributions as may be required to maintain solvency.

152. *Example* 1. A Fund grants unlimited pensions as discussed in paragraphs 111 to 120, and is to charge such a rate of contribution as may be required, half payable by the members and half by the Company. Let c be the percentage of contribution, and κ the percentage of pension calculated on average salary. Also, let the whole of the contributions be returnable on death before pension, and half only, that is the member's portion, on withdrawal. Then, equating the value of the contributions to the value of the benefits, we have,

$$c \times {}^{f}\mathrm{F}^{c}_{x} = \kappa \times {}^{f}\mathrm{F}^{ca}_{x} + c({}^{f}\mathrm{F}^{d}_{x} + \tfrac{1}{2}\,{}^{f}\mathrm{F}^{w}_{x}),$$

whence $$c = \frac{\kappa \times {}^{f}\mathrm{F}^{ra}_{x}}{{}^{f}\mathrm{F}^{c}_{x} - ({}^{f}\mathrm{F}^{d}_{x} + \frac{1}{2}{}^{f}\mathrm{F}^{w}_{x})}.$$

If there be a return on death after pension, as discussed in paragraphs 141 to 148, F^{ra}_{x} must be multiplied by the appropriate ratio. This remark applies to all the examples.

153. *Example* 2. Instead of taking the factor for unlimited pensions, we may substitute one suitable to other conditions. Let the pensions be as discussed in paragraphs 131 to 134, while the returns remain as in Example 1. Then,

$$c \times {}^{f}\mathrm{F}^{c}_{x} = {}_{x}\mathrm{F}^{ra}_{x} + c({}^{f}\mathrm{F}^{d}_{x} + \tfrac{1}{2}{}^{f}\mathrm{F}^{w}_{x}),$$

whence $$c = \frac{{}_{x}\mathrm{F}^{ra}_{x}}{{}^{f}\mathrm{F}^{c}_{x} - ({}^{f}\mathrm{F}^{d}_{x} + \frac{1}{2}{}^{f}\mathrm{F}^{w}_{x})}.$$

Here the pension factor ${}_{x}\mathrm{F}^{ra}_{x}$ includes within itself the full pension, and not merely 1 per-cent of salary, and therefore the coefficient, κ, is not required. This form is general, and includes pensions of all kinds discussed in the present paper, and therefore, without further explanation, we shall adhere to it in the remaining examples.

154. *Example* 3. Let the Company contribute $2\frac{1}{2}$ per-cent on the salaries, and the members the balance of whatever total contributions may be required; and let the whole of the contributions be returnable on death before pension, but only the member's portion on withdrawal. Then,

$$c \times {}^{f}\mathrm{F}^{c}_{x} = {}_{x}\mathrm{F}^{ra}_{x} + c \times {}^{f}\mathrm{F}^{d}_{x} + (c - 2{\cdot}5)\,{}^{f}\mathrm{F}^{w}_{x}\,;$$

whence $$c = \frac{{}_{x}\mathrm{F}^{ra}_{x} - 2{\cdot}5 \times {}^{f}\mathrm{F}^{w}_{x}}{{}^{f}\mathrm{F}^{c}_{x} - ({}^{f}\mathrm{F}^{d}_{x} + {}^{f}\mathrm{F}^{w}_{x})}.$$

Here we may very possibly find that at some of the younger ages at entry c comes out less than 5 per-cent. In such case we adopt 5 per-cent, and the Fund will make a much needed small gain from the young new entrants. Only by repeated trial can we ascertain the age at which the turn takes place, and c begins to exceed 5 per-cent.

155. *Example* 4. Let the conditions be as in Example 3, only let the extra contributions be non-returnable; that is, let only 5 per-cent be returnable on death, and $2\frac{1}{2}$ per-cent on withdrawal. Then,

$$c \times {}^{f}\mathrm{F}^{c}_{x} = {}_{x}\mathrm{F}^{ra}_{x} + 5 \times {}^{f}\mathrm{F}^{d}_{x} + 2{\cdot}5 \times {}^{f}\mathrm{F}^{w}_{x}\,;$$

whence $$c = \frac{{}_{x}\mathrm{F}^{ra}_{x} + 5 \times {}^{f}\mathrm{F}^{d}_{x} + 2{\cdot}5 \times {}^{f}\mathrm{F}^{w}_{x}}{{}^{f}\mathrm{F}^{c}_{x}}.$$

Again, at the younger ages c may come out less than 5 per-cent.

156. *Example* 5. Let the return on death be of the special form discussed in paragraphs 54 to 65, and let the member's ordinary contributions at $2\frac{1}{2}$ per cent, but not the extra, be returnable on withdrawal. Then,

$$c \times {}^{f}\mathrm{F}^{c}_{x} = {}_{x}\mathrm{F}^{ra}_{x} + {}_{x}\mathrm{F}^{d}_{x} + 2\cdot5 \times {}^{f}\mathrm{F}^{w}_{x} ;$$

whence
$$c = \frac{{}_{x}\mathrm{F}^{ra}_{x} + {}_{x}\mathrm{F}^{d}_{x} + 2\cdot5 \times {}^{f}\mathrm{F}^{w}_{x}}{{}^{f}\mathrm{F}^{c}_{x}} .$$

Here, the valuation factor ${}_{x}\mathrm{F}^{d}_{x}$ includes the whole death return, and does not call for a numerical coefficient.

157. In all the foregoing examples we have assumed that the returns are to be without interest; but if, on death, interest is to be allowed, we have merely to employ the valuation factor ${}^{f}\mathrm{F}^{di}_{x}$, or ${}^{f}\mathrm{F}^{dj}_{x}$, instead of ${}^{f}\mathrm{F}^{d}_{x}$.

158. *Example* 6. Sometimes the question of paragraph 149 is reversed, and, the rates of contribution and the returns having been fixed, the actuary is asked what scale of pensions can be allowed. If the pensions are to be of the unlimited description of paragraphs 111 to 120, the answer is simple. From the equation in Example 1 we have merely to find κ instead of c. The equation becomes—

$$\kappa \times {}^{f}\mathrm{F}^{ra}_{x} = c \times {}^{f}\mathrm{F}^{c}_{x} - c({}^{f}\mathrm{F}^{d}_{x} + \tfrac{1}{2}{}^{f}\mathrm{F}^{w}_{x}) ;$$

whence
$$\kappa = \frac{c \times {}^{f}\mathrm{F}^{c}_{x} - c({}^{f}\mathrm{F}^{d}_{x} + \frac{1}{2}{}^{f}\mathrm{F}^{w}_{x})}{{}^{f}\mathrm{F}^{ra}_{x}} .$$

159. When, however, pensions are of any other description the question cannot be answered so easily. For instance, if the present rules provide pensions at the rate of one-fiftieth of average salary for each year of membership, with a maximum of two-thirds, and if there be a deficiency, we cannot by solving an equation say what reduction in the percentage of pension will bring about solvency, because, by reducing the percentage, we prolong the term of service which must elapse before the maximum rate of pension is reached, and so disturb the proportion. Only a fresh valuation will disclose the effect which a change in the scale of pensions will produce, and this is a lengthy process. By comparing two valuations, however, an actuary of mature experience may be able to judge what change in the scale of pensions should be adopted.

Concluding Remarks.

160. In bringing this paper to a close, it may be well to emphasize the fact that it is not based merely on theoretical investigation. The majority of the formulas discussed have been practically applied, or rather, were worked out in detail to meet the circumstances of actual cases that presented themselves. Perhaps the most salient feature is the method of valuing prospective pensions, and certain other benefits, by taking account, not only of the age attained, but also of the age at entry. It may be objected that that method involves a considerable amount of scheduling and arithmetical work; but that is more in appearance than in reality. The method, on the other hand, secures simplicity and certainty, and so amply compensates for the small additional labour incurred. No complicated adjustments are required, and all confusion is avoided. No one who is averse to trouble should undertake the valuation of Staff Pension Funds. But, after all, when the operations have been reduced to a well devised system, carefully formulated in advance, they can be carried out speedily and accurately, by assistants who have had no special technical training; and I can testify, from an experience which has not been of very limited extent, that difficulties vanish when they are boldly faced. A few extra days devoted by the clerical staff to the valuation of a large Fund, are well spent when the results come out clearly and intelligibly, without having to be put through further processes.

161. So far from thinking that the principle of involving the age at entry has been carried to extreme, I believe that it will have to be still further extended in the not distant future. We shall be driven, in dealing with retirements, to construct what may be called "select" tables. When there is an optional age, which is often 60, for retirement, it is not until a Fund has been long established that the full effect of the force of retirement is felt. At the end of twenty years, only those who entered at the outset at age 40 or over will be entitled to claim pension; and, unless pressure be brought to bear on them, they will prefer to remain longer on the active list, because the pensions to which they would be entitled would be small. Also, the directors will hesitate to superannuate compulsorily the employee, if to do so would consign him to comparative penury. But when a Fund has been established for forty years or more, the same considerations will not prevail. An ever-increasing number of members

will have become entitled to the maximum rate of pension, and neither employer nor employed will have much scruple in taking advantage of the benefits of the Fund. We may therefore be certain that the rate of retirement at and over the optional age will prove to be heavier among those who enter early in life than among those who enter late; but this feature is masked when we rely upon only "aggregate" tables. None of the Funds which have passed through my hands have existed long enough to render feasible the application of the select method to the rate of retirement, but the time is rapidly approaching when efforts in that direction will have to be made.

162. In conclusion, a final point should be mentioned. There is one limitation in the benefits which has not so far been discussed, mainly because it has been absent from the rules of the Funds which have been the occasion of this paper. Sometimes returns of contribution on withdrawal or death are not permitted until the member has contributed for a certain number of years, often ten; but, after the preliminary term, returns date back to the commencement of membership. When this provision exists, the formulas of the paper overstate the value of the returns. Nevertheless, it will be the shortest and simplest process to employ these formulas, which are used in conjunction with Schedule IV, and then to apply an adjustment by means of an extension of Schedule II. To Schedule II should be added columns for Past Contributions, and these columns should be filled in, if not fully, at any rate in respect of all those who have not completed the stipulated term of membership. A brief calculation will then give, for each age at entry, the value of the return during the preliminary period; and the total of such value for all ages at entry will have to be deducted from the gross value found from Schedule IV. This method is not more laborious, and is much more satisfactory, than to assume an average age at entry. In fact, it gives absolutely accurate results.

163. The valuations of Staff Pension Funds afford a remarkable illustration of the powers of the Commutation Column method, and of the many directions in which it may be extended. To the genius of Mr. R. P. Hardy the conception of the development is due; and I end, as I began, by saying that he has worthily earned, and will certainly receive, the gratitude of the profession. It has not been possible to discriminate, as the paper proceeded, the formulas which, as they stand, were actually derived from him,

because the majority of them have undergone a process of evolution. I beg of him, therefore, to accept this general acknowledgment.

164. Mr. Manly's services, also, in giving us the effective weapon of the ratios of the salaries, to take the place of the salaries themselves, must not be forgotten.

ADDENDUM.

165. In paragraphs 123 to 128, when we discussed pensions based on average salaries, but with limitations, and increasing sometimes irregularly, we ignored the actual past salaries, and formed, by what may be called method No. 1, valuation factors based on the assumption that all current salaries had reached their present amount by increments according to scale. This assumption is not entirely satisfactory, and should be avoided if possible; because past salaries are actual realized facts, to be brought into account whenever practicable, if the utmost attainable accuracy is to be secured. I had not been able to evolve a formula by means of which this could be done; but since the paper was set up in type the question has had further consideration, and I have succeeded in devising what may be called method No. 2, which is a plan whereby, with a certain amount of clerical labour, the desired result will be produced. The difficulties in the way are set forth in paragraph 124, and these had to be overcome.

166. In solving the problem, we divide the investigation into two sections, and find separately the value of prospective pensions due to past and to future salaries respectively, just as we did in paragraphs 111 to 120, when considering pensions without complications. The method will apply to all pension scales where the pensions are in any way limited or irregular. The pensions, after a given term of membership, may increase by regular gradations, say one-sixtieth of average salary for each completed year of membership, with a maximum percentage; or they may increase irregularly, as in the illustrative scale given in paragraph 121. The form of the scale must, however, be altered, so as to give the ratio of the pension to total salary, instead of the percentage to average salary. If t be the completed years of membership, and κ_t the corresponding percentage of average salary, the function to be involved in the formulas is therefore $\frac{\kappa_t}{100\,t}$, which we may write κ'_t.

Past Salaries.

167. Three ages have to be dealt with, namely—the original age at entry, y; the age at valuation, x; and the age which will be completed just before the pension is entered on. This last we may call $x+m$. We shall take t as the total completed years of membership from the original age at entry, y, to the age at retirement, $x+m$, so that $x+m=y+t$.

168. Representing first the total past salaries by unity, the value of the prospective pension to be entered on in the year following age $x+m$ is $\frac{C^{ra}_{x+m}\times\kappa'_t}{D_x}$. We may write $C^{ra}_{x+m}\times\kappa'_t={}_yC^{ra}_{x+m}$; and when we take the sum for all values of m, which we may write ${}_yM^{ra}_x$, we have $\frac{{}_yM^{ra}_x}{D_x}$ for the value of the prospective pension to be entered on at any future time. This is the factor for valuing prospective pensions due to past salaries, so that we have

$$ {}^p_yF^{ra}_x=\frac{{}_yM^{ra}_x}{D_x}. $$

169. If there be n members, now aged x, who all entered at age y, and whose aggregate past salaries amount to S, the value of all their prospective pensions due to past salaries is $S\times{}^p_yF^{ra}_x$.

170. To use the formula, we must in Schedule II include a column giving the total past payments at $2\frac{1}{2}$ per-cent. To effect the valuation we multiply these past payments at each age by the corresponding factor, and then multiply by 40 the grand total of the results.

171. To form the table of ${}^p_yF^{ra}_x$, we prepare a movable slip giving the values of $\log\kappa'_t$, and another giving those of $\log C^{ra}_x$; and then, placing the slips side by side into position regulated according to age at entry, y, we calculate, by lateral addition of two numbers, the column of $\log{}_yC^{ra}_x$. Table No. 4 supplies a type of the remaining calculations.

Future Salaries.

172. The pension due to future salaries, to emerge in the year following age $x+m$, is based upon the salaries $s_x+s_{x+1}+\ldots+s_{x+m-1}$, which we may write $\Sigma^{m-1}_x s$. Omitting the denominator, its value is $C^{ra}_{x+m}\times\Sigma^{m-1}_x s\times\kappa'_t$. For this we may adopt the symbol ${}^{\Sigma}_yC^{ra}_{x+m}$, and, for the sum for all values of m from unity upwards, ${}^{\Sigma}_yM^{ra}_x$. This summation is like that adopted

for the terminal form of the usual N column, and does not at age x include the term involving C^{ra}_x. That is because fractional years do not count, and no pension accrues in respect of the salary of the last broken year.

173. Inserting the denominator, the value of the pension, due to future salaries, of a member who entered at age y, and who is at present aged x, and is drawing a salary of s_x, is $\frac{{}^{\Sigma}_{y}M^{ra}_x}{D_x}$. Passing, as usual, to a salary of 100, we have the valuation factor,

$$ {}^{f}_{y}F^{ra}_x = \frac{{}^{\Sigma}_{y}M^{ra}_x}{{}^{s}D_x}. $$

174. If there be n members aged x, who all entered at age y, and who are at present drawing salaries aggregating S, the value of all their prospective pensions due to future salaries is $\frac{S}{100} \times {}^{f}_{y}F^{ra}_x$. To effect the valuation we multiply the numbers in column 3 of Schedule II by the corresponding valuation factors, and multiply the grand total of the results by ·4.

175. To form ${}^{f}_{y}F^{ra}_x$, we construct a table of $\Sigma^{m-1}_x s$, which will contain a column for each present age x. This is done by starting from age x, and summing the column of scale salaries successively from above downwards, setting s_x against age $x+1$; s_x+s_{x+1} against age $x+2$; and so on. The logarithms of these numbers are taken and entered in columns on a sheet, the age x at the top, and the age $x+m$ at the side. Then, by means of the two movable slips giving respectively $\log C^{ra}_x$ and $\log \kappa'_t$, we form, by the lateral addition of three numbers, the values of $\log {}^{\Sigma}_{y}C^{ra}_x$. In the process, the position of the slip of $\log C^{ra}_x$ must be regulated by the valuation age, x, and that of the slip of $\log \kappa'_t$ by the age at entry, y. The operation of passing from $\log {}^{\Sigma}_{y}C^{ra}_x$ to ${}^{f}_{y}F^{ra}_x$ will take the form of Table 4.

176. In constructing the table of ${}^{p}_{y}F^{ra}_x$ we have already formed the values of $\log C^{ra}_x + \log \kappa'_t$, but these cannot be used conveniently in the present connection, because to do so would necessitate the lateral addition of numbers appearing in columns contained in two complete tables. It is more easy in practice to do the work by means of movable slips.

177. The method of valuation No. 2 involves nearly double the labour of No. 1, because two sets of factors have to be prepared, and two subsequent sets of multiplications effected,

but by No. 2 we have somewhat greater probability of accuracy. It would be interesting to value a large Fund in both ways, and compare the results. I doubt very much whether the possible gain in accuracy would be found to be sufficient to compensate for the additional work and expense. It will be noticed that the effects of the methods are identical as regards future salaries, only that in No. 1 these are involved in one formula with past salaries. It is only as regards the measure of liability in respect of past salaries that the methods differ.

POSTSCRIPT.

178. Fortunately, I showed a proof of this Addendum to my friend, Mr. E. C. Thomas, and he has brought to my notice a shorter and more elegant way, due to Mr. Manly, of finding the value of ${}^{f}_{y}\mathrm{F}^{ra}_{x}$.

179. Let

$$ {}^{s-1}_{\ y}\mathrm{M}^{ra}_{x} = s_{x-1} \times {}^{1}_{y}\mathrm{M}^{ra}_{x}, $$

and

$$ {}^{s-1}_{\ y}\mathrm{R}^{ra}_{x} = \Sigma\, {}^{s-1}_{\ y}\mathrm{M}^{ra}_{x}. $$

Then

$$ {}^{f}_{y}\mathrm{F}^{ra}_{x} = \frac{{}^{s-1}_{\ y}\mathrm{R}^{ra}_{x+1}}{{}^{s}\mathrm{D}_{x}}. $$

180. This value of the factor is identical with that given in paragraph 173, but is arrived at by a much briefer process. We can use the tables already prepared in calculating ${}^{p}_{y}\mathrm{F}^{ra}_{x}$, and we are spared all the trouble of computing the table of $\log \Sigma^{m-1}_{x} s$. Moreover, the formulas are brought into strict analogy with those for pensions without limitations, discussed in paragraphs 113–114 and 120; only that now we give effect to the age at entry, whereas in the other case that was not necessary. This harmony is in itself a decided advantage.

181. Method No. 1, paragraphs 123 to 128, involved the troublesome construction of a table of $\log \frac{{}_{y}z_{x}}{100}$, but by this improvement of Method No. 2 that is no longer required. The labour of preparing the two sets of factors for the improved Method No. 2 cannot, therefore, be greater than that of preparing the one set for Method No. 1. Thus, there can be no doubt but that, for the future, the improved Method No. 2 will hold the field.

182. I understand that the improved method No. 2 was employed by Mr. Manly and Mr. Thomas in a recent valuation, but it has never heretofore been published. I therefore retain the demonstrations in paragraphs 123 to 128 of the Paper, and in the Addendum, as a matter, perhaps, of historical interest, and to vindicate my claim to originality, but not to priority. Messrs. Manly and Thomas, I am told, used the formula with an average age at entry, which was perfectly appropriate under the circumstances with which they had to deal. But with the cases which have fallen to my lot, as illustrated in Table I, where the range of ages at entry was so extended, an average age at entry was impossible; and therefore it was imperative that each age at entry should be treated separately according to the methods emphasized throughout this Paper. Here, again, we have an excellent example of the fact that each Fund must be dealt with on its own merits.

KEY TO THE NOTATION.

Service Table.

l_x = Number remaining on the Active List at age x.
w_x = ,, withdrawing between ages x and $x+1$.
d_x = ,, dying ,, ,, ,,
r_x = ,, retiring on Pension between ages x and $x+1$.
$\lambda^d_x = \Sigma d_x$.
$\lambda^w_x = \Sigma w_x$.
q^w_x = Rate of Withdrawal.
q^d_x = Rate of Mortality.
q^r_x = Rate of Retirement on Pension.

Salaries.

s_x = Scale Salary drawn in year of age x to $x+1$.
${}_yz_x$ = Average of Scale Salaries from age y to age x, where y is entry age, and x age attained.
z_x = Terminal Salary. (*See* paragraph 130.)
$\Sigma_x^{m-1} s = s_x + s_{x+1} + \&c. + s_{x+m-1}$.
κ_t = Percentage of Salary taken as Pension on Retirement in year t to $t+1$.

$\kappa'_t = \dfrac{\kappa_t}{100t}$.

Interest.

i = Valuation Rate of Interest.

j = Rate of Interest other than the Valuation Rate.

$J = \dfrac{i-j}{1+j}$ = Special Rate of Interest, when $(1+J)^{-1} = \dfrac{1+j}{1+i}$.

$v^x = (1+i)^{-x}$.

$u^x = (1+j)^{-x}$.

General Commutation Symbols.

$D_x = v^x l_x$.

$\overline{D}_x = D_{x+\frac{1}{2}} = v^{x+\frac{1}{2}} l_{x+\frac{1}{2}} = \frac{1}{2}(D_x + D_{x+1})$, approximately.

${}^sD_x = D_x \times s_x \times {\cdot}01$.

${}^s\overline{D}_x = \overline{D}_x \times s_x \times {\cdot}01$.

${}^s\overline{N}_x = \Sigma\, {}^s\overline{D}_x$.

Future Contributions.

${}^fF^c_x = {}^s\overline{N}_x \div {}^sD_x$ = Factor for valuing Future Contributions.

Returns on Withdrawal, without Interest.

$\overline{C}^w_x = v^{x+\frac{1}{2}} \times w_x$.

$\overline{M}^w_x = \Sigma \overline{C}^w_x$.

${}^pF^w_x = \overline{M}^w_x \div D_x$ = Factor for valuing Withdrawal Returns of Past Contributions.

${}^s\overline{C}^w_x = \overline{C}^w_x \times s_x \times {\cdot}01$.

${}^sM^w_x = \overline{M}^w_x \times s_x \times {\cdot}01$.

${}^s\overline{M}^w_x = {}^sM^w_x - \frac{1}{2}{}^s\overline{C}^w$.

${}^s\overline{R}^w_x = \Sigma\, {}^s\overline{M}^w_x$.

${}^fF^w_x = {}^s\overline{R}^w_x \div {}^sD_x$ = Factor for valuing Withdrawal Returns of Future Contributions.

Returns on Death, without Interest.

$\overline{C}^d_x = v^{x+\frac{1}{2}} \times d_x$.

$\overline{M}^d_x = \Sigma \overline{C}^d_x$.

${}^pF^d_x = \overline{M}^d_x \div D_x$ = Factor for valuing Death Returns of Past Contributions.

${}^s\overline{C}^d_x = \overline{C}^d_x \times s_x \times {\cdot}01$.

${}^sM^d_x = \overline{M}^d_x \times s_x \times {\cdot}01$.

${}^s\overline{M}^d_x = {}^sM^d_x - \frac{1}{2}{}^s\overline{C}^d_x$.

${}^s\overline{R}^d_x = \Sigma\, {}^s\overline{M}^d_x$.

$F^d_x = {}^s\overline{R}^d_x \div {}^sD_x$ = Factor for valuing Death Returns of Future Contributions.

Special Death Return. (*See* paragraphs 54 to 65.)

k_t = Proportion of average salary to be returned on death in the tth year from entry.

$^{z}_{y}\overline{C}^{d}_{x} = v^{x+\frac{1}{2}} \times d_x \times k_t \times {}_y z_{x+1} \times \cdot 01.$

$^{z}_{y}\overline{M}^{d}_{x} = \Sigma\, ^{z}_{y}\overline{C}^{d}_{x}.$

${}_yF^{d}_{x} = {}^{z}_{y}\overline{M}^{d}_{x} \div {}^{s}D_x$ = Factor for valuing Special Death Return. Here y represents the age at entry, and x the age attained.

Returns on Death, with Compound Interest at Rate i.

${}^{s}\overline{M}^{di}_{x} = v^{x+\frac{1}{2}} \times (\lambda^{d}_{x} - \frac{1}{2}d_x) \times s_x \times \cdot 01.$

${}^{s}\overline{R}^{di}_{x} = \Sigma\, {}^{s}\overline{M}^{di}_{x}.$

${}^{f}F^{di}_{x} = {}^{s}\overline{R}^{di}_{x} \div {}^{s}D_x$ = Factor for valuing Death Returns of Future Contributions, with Compound Interest at Rate i.

${}_yZ^{i}_{x} = (v^{y+\frac{1}{2}} \times s_y \times \cdot 01 + v^{y+1+\frac{1}{2}} \times s_{y+1} \times \cdot 01 + \&c.,$
$+ v^{x-1+\frac{1}{2}} \times s_{x-1} \times \cdot 01).$

$^{p}_{y}F^{di}_{x} = \dfrac{\lambda^{d}_{x} \times {}_yZ^{i}_{x}}{{}^{s}D_x}$ = Factor for valuing Death Returns of Past Contributions, with Compound Interest at Rate i.

Returns on Death, with Compound Interest at Rate j.

$D^{J}_{x} = l_x \times (1+J)^{-x}.$

$\overline{C}^{dJ}_{x} = d_x \times (1+J)^{-(x+\frac{1}{2})}.$

$M^{dJ}_{x} = \Sigma\overline{C}^{dJ}_{x}.$

$\overline{M}^{dJ}_{x} = M^{dJ}_{x} - \frac{1}{2}\overline{C}^{dJ}_{x}.$

${}^{s}\overline{R}^{dj}_{x} = \Sigma u^{\frac{1}{2}} u^{x} \times \overline{M}^{dJ}_{x} \times s_x \times \cdot 01.$

${}^{f}F^{dj}_{x} = {}^{s}\overline{R}^{dj}_{x} \div {}^{s}D_x$ = Factor for valuing Death Returns of Future Contributions, with compound interest at Rate j.

${}_yZ^{j}_{x} = (u^{y+\frac{1}{2}} \times s_y \times \cdot 01 + u^{y+1+\frac{1}{2}} \times s_{y+1} \times \cdot 01 + \&c.,$
$+ u^{x-1+\frac{1}{2}} \times s_{x-1} \times \cdot 01).$

$^{p}_{y}F^{dj}_{x} = \dfrac{M^{dJ}_{x} \times {}_yZ^{j}_{x}}{{}^{s}D_x}$ = Factor for valuing Death Returns of Past Contributions, with Compound Interest at Rate j.

Returns on Death, with Simple Interest.

$$\overline{\mathrm{R}}_x^d = \Sigma \overline{\mathrm{M}}_x^d.$$

$$^s\Sigma\overline{\mathrm{R}}_x^d = \overline{\mathrm{R}}_x^d \times s_x \times \cdot 01 + \overline{\mathrm{R}}_{x+1}^d \times s_{x+1} \times \cdot 01 + \&c.$$

$$^f\mathrm{F}_x^{li} = \frac{{}^s\overline{\mathrm{R}}_x^d + i \times {}^s\Sigma\overline{\mathrm{R}}_x^d}{{}^s\mathrm{D}_x^l}$$ = Factor for valuing Death Returns of Future Contributions, with Simple Interest. Here the Factor has the same symbol as with Compound Interest, but confusion cannot result, because Compound and Simple Interest will not both occur in the same Fund.

Pensions.

$$\mathrm{C}_x^{ra} = r_x \times v^{x+\frac{1}{2}} \times \bar{a}_{x+\frac{1}{2}}.$$

$$\mathrm{M}_x^{ra} = \Sigma \mathrm{C}_x^{ra}.$$

$$^s\overline{\mathrm{M}}_x^{ra} = (\mathrm{M}_x^{ra} - \tfrac{1}{2}\mathrm{C}_x^{ra}) \times s_x \times \cdot 01.$$

$$^s\overline{\mathrm{R}}_x^{ra} = \Sigma {}^s\overline{\mathrm{M}}_x^{ra}.$$

$$^{s-1}\mathrm{M}_x^{ra} = \mathrm{M}_x^{ra} \times s_{x-1} \times \cdot 01.$$

$$^{s-1}\mathrm{R}_x^{ra} = \Sigma^{s-1}\mathrm{M}_x^{ra}.$$

κ_t = Percentage of Salary to be paid as Pension.

Average Salaries. Unlimited Pensions.

$^p\mathrm{F}_x^{ra} = \mathrm{M}_x^{ra} \div \mathrm{D}_x$ = Factor for valuing Prospective Pensions due to Past Salaries.

$^f\mathrm{F}_x^{ra} = {}^s\overline{\mathrm{R}}_x^{ra} \div {}^s\mathrm{D}_x$ = Factor for valuing Prospective Pensions due to Future Salaries, when fractional years count.

$^f\mathrm{F}_x^{ra} = {}^{s-1}\mathrm{R}_{x+1}^{ra} \div {}^s\mathrm{D}_x$ = Factor for valuing Prospective Pensions due to Future Salaries, when fractional years do not count.

Average Salaries. Pensions with Limitations.

$${}_y^z\mathrm{C}_x^{ra} = \mathrm{C}_x^{ra} \times \kappa_t \times {}_yz_x \times \cdot 01.$$

$${}_y^z\mathrm{M}_x^{ra} = \Sigma {}_y^z\mathrm{C}_x^{ra}.$$

${}_y\mathrm{F}_x^{ra} = {}_y^z\mathrm{M}_x^{ra} \div {}^s\mathrm{D}_x$ = Factor for valuing Prospective Pensions due to both Past and Future Salaries, in respect of Members now aged x, who entered at age y.

$${}_{y}C^{ra}_{x+m} = C^{ra}_{x+m} \times \kappa'_{t}.$$

$${}_{y}M^{ra}_{x} = \Sigma_{y}C^{ra}_{x+m} \text{ for all values of } m.$$

${}^{p}_{y}F^{ra}_{x} = {}_{y}M^{ra}_{x} \div D_{x} =$ Factor for valuing Prospective Pensions due to Past Salaries, in respect of Members now aged x, who entered at age y.

$${}^{\Sigma}_{y}C^{ra}_{x+m} = C^{ra}_{x+m} \times \Sigma^{m-1}_{x}s \times \kappa'_{t}.$$

$${}^{\Sigma}_{y}M^{ra}_{x} = \Sigma{}^{\Sigma}_{y}C^{ra}_{x+m} \text{ for all values of } m.$$

${}^{f}_{y}F^{ra}_{x} = {}_{y}M^{ra}_{x} \div {}^{s}D_{x} =$ Factor for valuing Prospective Pensions due to Future Salaries, in respect of Members now aged x, who entered at age y.

Or

$${}^{s-1}_{y}M^{ra}_{x} = s_{x-1} \times {}_{y}M^{ra}_{x}.$$

$${}^{s-1}_{y}R^{ra}_{x} = \Sigma{}^{s-1}_{y}M^{ra}_{x}.$$

${}^{f}_{y}F^{ra}_{x} = {}^{s-1}_{y}R^{ra}_{x+1} \div {}^{s}D_{x} =$ Factor, as above.

Terminal Salaries. Pensions with Limitations.

$${}^{z}C^{ra}_{x} = r_{x} \times v^{x+\frac{1}{2}} \times \bar{a}_{x+\frac{1}{2}} \times z_{x} \times \cdot 01.$$

$${}^{z}_{y}C^{ra}_{x} = {}^{z}C^{ra}_{x} \times \kappa_{t}.$$

$${}^{z}_{y}M^{ra}_{x} = \Sigma{}^{z}_{y}C^{ra}_{x}.$$

${}_{y}F^{ra}_{x} = {}^{z}_{y}M^{ra}_{x} \div {}^{s}D_{x} =$ Factor for valuing Prospective Pensions due to both Past and Future Salaries, in respect of Members now aged x, who entered at age y.

TABLE 1.—*Actual Experience.*

(1) Age	(2) Survivors b_x	(3) Entrants n_x	(4) Withdrawals w_x	(5) Deaths d_x	(6) Retirements r_x	(7) Existing e_x	(8) Age
11	..	1	..	..	..	..	11
2	..	9	1	..	..	1	2
3	6	147	21	..	..	2	3
4	36	876	94	4	..	45	4
15	102	1,488	244	4	..	175	15
6	193	1,137	298	5	..	311	6
7	240	710	333	8	..	379	7
8	249	437	332	11	..	425	8
9	215	282	287	13	..	407	9
20	187	211	246	14	..	342	20
1	172	196	222	10	..	330	1
2	150	161	183	9		303	2
3	132	135	148	11	..	258	3
4	137	136	121	11	..	247	4
25	147	101	98	10	1	229	25
6	142	97	82	6	1	235	6
7	136	84	64	5	1	237	7
8	116	81	52	9	3	222	8
9	113	67	57	4	..	198	9
30	109	68	35	6	1	169	30
1	104	69	38	7	2	181	1
2	107	76	38	7	2	163	2
3	89	61	29	6	..	136	3
4	98	48	30	8	3	165	4
35	88	54	31	5	1	161	35
6	73	57	22	4	1	165	6
7	76	48	23	6	1	159	7
8	73	35	28	6	..	140	8
9	63	45	18	6	1	131	9
40	60	40	16	3	3	126	40
1	55	28	11	3	2	131	1
2	56	35	15	4	1	117	2
3	48	30	11	5	3	109	3
4	50	23	9	7	3	107	4
45	59	21	4	4	1	97	45
6	40	16	8	7	..	87	6
7	43	8	5	5	1	86	7
8	47	7	4	7	4	78	8
9	45	5	7	4	3	77	9
50	30	6	6	6	2	62	50
1	29	..	1	7	3	57	1
2	30	..	3	7	2	53	2
3	23	..	3	5	5	48	3
4	22	..	1	6	2	48	4
55	22	..	1	4	3	52	55
6	13	..	1	5	4	35	6
7	16	..	1	6	4	35	7
8	13	..	1	4	4	34	8
9	14	..	..	4	8	29	9
60	12	..	..	1	23	21	60
1	16	..	..	4	11	17	1
2	13	..	..	3	12	16	2
3	4	..	..	4	8	7	3
4	8	..	..	2	8	10	4
65	4	..	..	2	11	7	65
6	8	..	..	2	15	3	6
7	7	.	..	1	8	4	7
8	6	..	..	2	7	1	8
9	5	..	..	1	7	2	9
70	3	..	..	1	11	1	70
1	2	..	..	2	6	1	1
2	1	..	..	..	2	1	2
3	1	..	..	2	3	..	3
4	3	..	..	..	3	1	4
75	1	..	..	1	2	..	76
Totals	4,162	7,136	3,283	326	213	7,476	Totals

TABLE 2.—*Rates of Withdrawal, Mortality, and Superannuation of Members on the Active List; and Rates of Mortality of Pensioners. Also Scale Salaries.*

Age	Active List: Rate of Withdrawal per-cent	Active List: Rate of Superannuation per-cent	Active List: Rate of Mortality per-cent	Pensioners' Rate of Mortality per-cent	Scale Salary	Age
10	19·62	...	·18	...	14	10
11	17·90	...	·20	...	16	11
12	16·12	...	·22	...	18	12
13	14·99	...	·24	...	20	13
14	13·70	...	·26	...	22	14
15	12·55	...	·28	...	24	15
16	11·55	...	·30	...	27	16
17	10·62	...	·32	...	32	17
18	9·80	...	·34	...	37	18
19	9·17	...	·38	...	44	19
20	8·58	...	·41	...	51	20
21	8·05	...	·44	...	57	21
22	7·45	...	·46	...	62	22
23	6·79	...	·46	...	67	23
24	6·07	...	·46	...	72	24
25	5·35	·03	·44	9·00	76	25
26	4·64	·05	·43	9·00	80	26
27	4·02	·06	·42	9·00	84	27
28	3·53	·07	·41	9·00	87	28
29	3·15	·08	·41	9·00	91	29
30	2·85	·10	·42	9·00	95	30
31	2·64	·10	·44	9·00	99	31
32	2·49	·11	·45	9·00	103	32
33	2·38	·11	·46	9·00	108	33
34	2·30	·11	·46	9·00	113	34
35	2·24	·11	·46	9·00	118	35
36	2·18	·11	·46	9·00	123	36
37	2·11	·12	·46	9·00	128	37
38	2·02	·13	·46	9·00	133	38
39	1·90	·15	·46	9·00	137	39
40	1·77	·18	·48	9·00	141	40
41	1·62	·21	·51	9·00	144	41
42	1·47	·22	·56	9·00	146	42
43	1·31	·23	·63	9·00	148	43
44	1·18	·24	·70	9·00	150	44
45	1·06	·25	·79	9·00	152	45
46	1·00	·26	·88	9·00	154	46
47	·95	·31	·97	9·00	156	47
48	·91	·38	1·07	9·00	158	48
49	·87	·48	1·16	9·00	160	49
50	·81	·55	1·26	9·00	162	50
51	·74	·64	1·38	8·89	164	51
52	·66	·67	1·48	8·77	166	52
53	·59	·76	1·58	8·66	168	53
54	·51	·86	1·67	8·55	170	54
55	·44	·98	1·74	8·43	172	55
56	·35	1·60	1·82	8·32	174	56
57	·25	2·64	1·91	8·21	176	57
58	·15	4·50	2·02	8·09	178	58
59	·05	5·74	2·17	7·98	180	59
60	...	14·00	2·33	7·86	182	60
61	...	8·79	2·50	7·75	184	61
62	...	10·15	2·64	7·64	186	62
63	...	10·88	2·75	7·52	188	63
64	...	13·05	2·79	7·41	190	64
65	...	24·00	2·83	7·30	192	65
66	...	30·00	2·91	7·18	194	66
67	...	42·00	3·07	7·07	196	67
68	...	96·64	3·36	6·96*	198	68

* At higher ages of Pensioners, the rate of mortality is that shown by the English Life Table No. 3, Males.

TABLE 3.—*Service Table.*

Age	Remaining on Active List	Withdrawals	Deaths	Pensioned	Total Decrement	Age
10	100,000	19,620	180	...	19,800	10
11	80,200	14,356	161	...	14,517	11
12	65,683	10,785	145	...	10,930	12
13	54,753	8,207	132	...	8,339	13
14	46,414	6,358	121	...	6,479	14
15	39,935	5,012	112	...	5,124	15
16	34,811	4,021	104	...	4,125	16
17	30,686	3,259	98	...	3,357	17
18	27,329	2,678	93	...	2,771	18
19	24,558	2,252	93	...	2,345	19
20	22,213	1,906	91	...	1,997	20
21	20,216	1,627	89	...	1,716	21
22	18,500	1,379	85	...	1,464	22
23	17,036	1,157	78	...	1,235	23
24	15,801	959	73	...	1,032	24
25	14,769	791	65	4	860	25
26	13,909	645	60	7	712	26
27	13,197	531	55	8	594	27
28	12,603	444	52	9	505	28
29	12,098	380	50	10	440	29
30	11,658	332	49	12	393	30
31	11,265	297	49	12	358	31
32	10,907	272	49	12	333	32
33	10,574	252	48	12	312	33
34	10,262	236	47	11	294	34
35	9,968	223	46	11	280	35
36	9,688	211	45	11	267	36
37	9,421	198	44	11	253	37
38	9,168	185	43	12	240	38
39	8,928	170	41	13	224	39
40	8,704	154	42	15	211	40
41	8,493	138	43	18	199	41
42	8,294	122	46	18	186	42
43	8,108	106	51	19	176	43
44	7,932	94	56	19	169	44
45	7,763	82	62	19	163	45
46	7,600	75	67	20	162	46
47	7,438	71	72	23	166	47
48	7,272	65	78	29	172	48
49	7,100	62	82	34	178	49
50	6,922	56	87	38	181	50
51	6,741	50	93	43	186	51
52	6,555	43	97	44	184	52
53	6,371	38	101	48	187	53
54	6,184	32	103	53	188	54
55	5,996	26	105	59	190	55
56	5,806	20	106	92	218	56
57	5,588	14	107	148	269	57
58	5,319	8	107	239	354	58
59	4,965	3	108	285	396	59
60	4,569	...	106	640	746	60
61	3,823	...	95	336	431	61
62	3,392	...	90	344	434	62
63	2,958	...	81	322	403	63
64	2,555	...	72	333	405	64
65	2,150	...	61	516	577	65
66	1,573	...	46	472	518	66
67	1,055	...	32	443	475	67
68	580	...	19	561	580	68

TABLE 4.—*Pension Factors.*

Age attained x	Age at Entry 25: $\log {}^{z}_{25}C^{ra}_{x}$	${}^{z}_{25}C^{ra}_{x}$	${}^{z}_{25}M^{ra}_{x}$	$\log {}^{z}_{25}M^{ra}_{x}$	$\log {}_{25}F^{ra}_{x}$	${}_{25}F^{ra}_{x}$	Age attained x
25	...	...	377,090	5·57645	1·95212	89·56	25
26	...	...	377,090	5·57645	1·97293	93·96	26
27	..	...	377,090	5·57645	1·99159	98·08	27
28	...	...	377,090	5·57645	2·01339	103·13	28
29	...	...	377,090	5·57645	2·02867	106·82	29
30	...	...	377,090	5·57645	2·04311	110·44	30
31	...	...	377,090	5·57645	2·05711	114·05	31
32	...	...	377,090	5·57645	2·07097	117·75	32
33	...	...	377,090	5·57645	2·08089	120·47	33
34	...	...	377,090	5·57645	2·09127	123·39	34
35	2·5534	358	377,090	5·57645	2·10213	126·51	35
36	2·5944	393	376,732	5·57603	2·11309	129·74	36
37	2·6352	432	376,339	5·57558	2·12450	133·20	37
38	2·7101	513	375,907	5·57508	2·13623	136·85	38
39	2·7786	601	375,394	5·57449	2·15132	141·68	39
40	2·8716	744	374,793	5·57379	2·16618	146·62	40
41	2·9759	946	374,049	5·57293	2·18388	152·71	41
42	2·9990	998	373,103	5·57183	2·20412	160·00	42
43	3·0438	1,106	372,105	5·57067	2·22393	167·47	43
44	3·0606	1,150	370,999	5·56937	2·24337	175·13	44
45	3·0731	1,183	369,849	5·56802	2·26265	183·08	45
46	3·1099	1,288	368,666	5·56663	2·28183	191·35	46
47	3·1812	1,518	367,378	5·56511	2·30110	200·03	47
48	3·2916	1,957	365,860	5·56331	2·32060	209·22	48
49	3·3700	2,344	363,903	5·56099	2·34024	218·90	49
50	3·4269	2,672	361,559	5·55818	2·36010	229·14	50
51	3·4885	3,080	358,887	5·55496	2·38010	239·94	51
52	3·5055	3,203	355,807	5·55121	2·40026	251·34	52
53	3·5493	3,542	352,604	5·54729	2·42054	263·35	53
54	3·5974	3,957	349,062	5·54290	2·44098	276·05	54
55	3·6482	4,448	345,105	5·53795	2·46139	289·33	55
56	3·8443	6,987	340,657	5·53232	2·48176	303·22	56
57	4·0530	11,300	333,670	5·52332	2·50145	317·29	57
58	4·2623	18,290	322,370	5·50835	2·52003	331·15	58
59	4·3388	21,820	304,080	5·48299	2·53677	344·17	59
60	4·6892	48,890	282,260	5·45065	2·55276	357·08	60
61	4·4072	25,540	233,370	5·36805	2·55986	362·96	61
62	4·4140	25,140	207,830	5·31771	2·57381	374·81	62
63	4·3804	24,010	181,890	5·25081	2·58775	387·03	63
64	4·3887	24,470	157,880	5·19833	2·60232	400·24	64
65	4·5710	37,240	133,410	5·12519	2·61662	413·64	65
66	4·5118	32,490	96,170	4·98304	2·62271	419·48	66
67	4·4621	28,980	63,680	4·80400	2·62972	426·30	67
68	4·5403	34,700	34,700	4·54033	2·63850	435·01	68

Schedule I.

Detailed Particulars for Valuation. Age at Entry

Age Attained	Register Number	Past Contributions		2½% of Salary	Percentage of Extra	Amount of Extra
		Members' 2½	Extra, if any			
(1)	(2)	(3)	(4)	(5)	(6)	(7)

Schedule II.

Prospective Pensions.

Particulars for Valuation. Age at Entry..................

Age Attained	Number of Members	Adjusted $2\frac{1}{2}$% of Salaries		Valuation Factor		Product	
(1)	(2)	(3)		(4)		(5)	

N.B.—It may sometimes be necessary to include in this Schedule columns for Past Contributions. *See* paragraphs 162 and 165 to 182.

Schedule III.

Intermediate Schedule. Contributions and Returns.

Particulars for Valuation. Age Attained..................

Age at Entry	Number of Members	Past Contributions		Adjusted Contributions	
		Members' 2½ %	Extra	2½ % of Salaries	Extra
(1)	(2)	(3)	(4)	(5)	(6)

SCHEDULE IV.

Contributions and Returns. Particulars for Valuation.

Age Attained	Number of Members	Past Contributions		Adjusted Contributions	
		Members' 2½ %	Extra	2½ % of Salaries	Extra
(1)	(2)	(3)	(4)	(5)	(6)

Abstract of the Discussion.

Mr. G. J. Lidstone said he felt sure he was only expressing the opinion of the meeting when he said that the paper added to their already great indebtedness to Mr. King, and that it would certainly take high rank, even when judged by that very exacting standard, his own previous work. Until four years ago, when Mr. Manly's paper was read, the *Journal* contained no reference to this very interesting and important subject. Those of them who had the privilege of knowing Mr. R. P. Hardy—to whom Mr. King paid a generous tribute, in which they would all wish to join—were well aware that the literature of the subject was contained in his reports and notebooks, and many were the efforts made to induce him to publish his methods for the benefit of others, but without success. The Institute was fortunate in having the subject dealt with by two actuaries of the eminence of Mr. Manly and Mr. King, who had left the marks of their own individuality upon it. He had almost said, dealt with in duplicate, for their papers covered very much the same ground, although each took up problems which were not dealt with by the other. Probably students would have to read these papers together, because even when the same subject was dealt with by both it was frequently treated in a different manner. Mr. King had introduced a new development of an interesting character, in the use of select tables dealing with the different ages at entry; and this, while it increased the numerical work, frequently simplified the results. He had also simplified the formulas by dealing with functions in the middle of the year, and thus obviated troublesome adjustments. Further, he had simplified many of the demonstrations by using—although he had not expressly stated—a principle which was of very general application. When they had to apply the commutation method to benefits involving average salary or total contributions, there were two courses open. (1) They could deal with each future year, and consider for that year the amount of benefit which would be payable. This would involve a preliminary summation of the salary or the contributions from the age at entry y to the valuation age x. (2) They might fix their attention on the benefit arising from the salary or contributions of a given year, and consider the present value of that benefit for all the future years in which it might emerge. This would involve a preliminary summation of commutation functions from the valuation age x to the end of life. It would be found that, in the case of future benefits arising from *past* salaries or contributions, the first method was the simpler, but the second was better for benefits in respect of *future* salaries or contributions. Mr. Thomas's improved solution of the problem referred to in the postcript, was a very good example of the advantages of adhering to this principle.

Passing to points of detail, he would refer to the problem of articles 54 to 65. There Mr. King adopted an approximate solution, because the returns for the first ten years took an inconvenient form. It might be worth while to point out that by a simple transformation the accurate solution could be obtained with little additional labour. The return for the nth year could be expressed as n times 5 per-cent of the average salary, less $2\frac{1}{2}$ per-cent

of the final salary $[(n\times\cdot05)_y z_{y+n}-\cdot025s_{y+n}]$, and this formula fitted in perfectly with the return after ten years, the only differences being that after that period there was no deductive term, and the variable coefficient $(n\times\cdot05)$ was replaced by the constant $\cdot5$.

In sections V and VI, Mr. King, in dealing with the returns of past contributions plus compound interest thereon, ignored the *actual* past contributions, and assumed that they had all grown according to scale, and he stated that to deal with the problem in the same way as with simple interest (*vide* section VII) would involve a very complicated double summation; but he (Mr. Lidstone) thought that by approaching the subject from a different standpoint a comparatively simple solution could be obtained. The actual past contributions were known, and the value of the return, plus *future* compound interest, could be found by the formula of article 81. The value of the past interest according to scale, plus future interest thereon, could be found as follows. In article 86, Mr. King gave an expression for the value of the return in respect of past contributions according to scale, plus past and future compound interest at rate j. If they put $j=0$ in calculating Z in that article, they would have this simple expression for the value of the return of past scale-contributions plus *future* interest only, viz.,

$$\frac{\mathrm{M}_x^{d\mathrm{J}}}{\mathrm{D}_x^{\mathrm{J}}}\times{}_y\mathrm{Z}_x^0,\text{ where }{}_y\mathrm{Z}_x^0=\cdot01\overset{x-1}{\Sigma_y}(s_y).$$

The difference between this and the original expression in article 86 gave

$$\frac{\mathrm{M}_x^{d\mathrm{J}}}{\mathrm{D}_x^{\mathrm{J}}}[{}_y\mathrm{Z}_x^j\times(1+j)^x-{}_y\mathrm{Z}_x^0]\text{ or say }\phi$$

for the value of the return of past interest plus future interest thereon, but disentangled from the past contributions themselves. This expression was based on the assumption that past contributions had been according to scale. If they divided the expression ϕ by the *assumed* past contributions $\overset{x-1}{\Sigma_y}(s_y)$, they would have $\dfrac{\phi}{\overset{x-1}{\Sigma_y}(s_y)}$, an abstract factor which, when multiplied by the *actual* past contributions, would give the value of the return of past interest plus future interest thereon. There would then be no approximation to the actual amount of past contributions, but only as to the average interest factor, or, in other words, the average term for which past interest had accrued. This was in accordance with the general principle that estimates should be corrected by facts, as far as and so soon as the facts became known. Where so much was necessarily uncertain, as it was in respect to these funds, it was desirable to follow the principle to the fullest extent, even at the cost of some increased arithmetical work. A similar principle could be applied to the formula for simple interest returns in article 99; and it might be worth while to point out that that formula itself could be put into a more convenient form for calculation. If they transferred the co-efficient $y-\frac{1}{2}$ to the first

member inside the square brackets they would have this result:

$$\cdot 01i[(x-y+\tfrac{1}{2})(s_y+\ \dots\ +s_{x-1}) \\ \qquad -(s_y+2s_{y+1}+\ \dots\ +(x-y)s_{x-1})]$$

$$=\cdot 01i[(t+\tfrac{1}{2})\Sigma s_y-\Sigma\Sigma s_y]$$

where $t=x-y$, and the summation was taken from entry-age to the year preceding the valuation-age. In this way they would get rid of the troublesome multiplication in the form $(y+\frac{1}{2})s_y$. and reduce it to a simple double summation.

He had referred to these points of detail in the hope that they might be of interest to those who had to follow the paper closely. But, of course, all these small questions would shrink into insignificance in comparison with the question—at once the most difficult and the most important—how to fix the scale of average salaries. Mr. King said that "The present average salaries give only an uncertain indication, which may have to be departed from widely"; but, unfortunately, he gave no help as to how the departure should be arrived at. Mr. Manly, in dealing with the same subject, said "We proceed to make an intelligent graduation, using our discretion, particularly in the case of a young institution, to modify the high salaries by the assumption that a certain number of the young members of the staff will be in receipt of much lower salaries when they arrive at the higher ages." He was sure they would feel grateful to Mr. Manly and Mr. King if they would give further information as to how this should be done. To him (the speaker) it seemed that when a process produced a result which was anomalous and unfit for practical use, it did not point so much to the necessity of introducing adjustment, as of seeking another method which would produce results free from the anomalies. Mr. Manly had pointed out that they were not concerned with the actual scale of salaries, but only with the rate of increase from year to year. With a body of changing constitution, they had no right to expect that that rate of increase could be found by comparing the average salaries enjoyed on a given date by one set of men aged x, with the average salaries enjoyed at the same time by another set of men aged $x+1$, any more than they would expect to find the probability of living a year, by comparing the number of persons aged x with the number aged $x+1$ on a given day in a fluctuating population. What they required was a direct comparison between the salaries of the men aged x to-day, and the salaries of the same men a year hence, when they would be of age $x+1$. With properly arranged books, there should be no difficulty in getting the rates of increase in this way from year to year, with the additional advantage that any change in the rates of increase could be directly watched. He believed that by this means they would avoid many of the anomalies and the more or less arbitrary adjustments which Mr. King and Mr. Manly had suggested, but unfortunately, without affording much information as to how they should be applied in practice.

Mr. Archibald Hewat: The subject which Mr. King has brought before us is much too large for us to attempt to discuss

it in any detail this evening. I am glad to see that he refers to "fancy benefits" and "excrescences." That is the trouble to the actuary in dealing with funds of this kind, either in framing a scale of contributions or in making a valuation. If those who have to do with the framing of these funds would content themselves with dealing merely with a pension fund, it would be better. But there are those who do not wish wholly to part with the money contributed by them, and so the savings-bank element is introduced in addition. Then somebody wants the contributions paid back to their families when they die, thus introducing the further element of life assurance. Then they want also something for the widow, thereby adding yet another item, namely, the widow's fund. In a recent case I observed a very peculiar element: "No pension is to be paid except out of the profits of the company." As I do not think Mr. Hardy, or Mr. Manly, or Mr. King, will be able to supply functions or factors for that element, I would commend that to the students, or those who have more recently passed through the hands of the examiners, to try and work out a formula. With regard to withdrawals or secessions, one requires to be careful in making estimates for the future. If we look back on the past history of the employees, that may be a poor guide, because, if there has been no pension fund in the past, they may have been withdrawing in larger numbers than they will do in future, when they have a pension fund to look to. There is another troublesome item I have had to deal with in practical work of this kind, and that is the element of female lives. They introduce reasons for withdrawal which are obvious, but which do not exist in the case of males. With regard to the salary scale, that also is extremely difficult to arrive at, for various reasons. My practice is to find out the actual experience of the past, as far as possible, to come to certain conclusions, and then call in the Accountant, or the Secretary, or some one who has the practical work of dealing with these things and knows about them; and then between us we adjust a scale. With regard to that point, there is another thing that the practical actuary has to be extremely careful about, and that is to see that the exceptionally large salaries are not too numerous, in proportion to the number of employees, to disturb what might be considered a normal average. With regard to scale of salary, of course we have to keep in view that such has to be considered, not only in relation to contributions, but also to the benefits. Mr. King points out that to adopt a scale of salaries which increases too slowly would under-estimate liabilities; but it also under-estimates the assets, in accumulating and valuing the contributions. He further points out that at older ages there is a tendency for the average salary to fall. That is so, because Directors are not likely to increase an employee's salary within a few years of his retiring on pension, for that would give him a much larger pension. The formulas are most interesting and informing. They are so clear as to be almost self-interpreting. They are, however, something like the snowball, which gathers as it grows, for I find, as one goes through the paper, there are innocent-looking formulas, which take up very small space, one of which, for example, employs fifteen letters, nine of them being different. This contribution to

this difficult and increasingly important subject will be a most valuable text book for students of all ages. In article 149, Mr. King says: "When a fund is to be started, and the intending members have formulated the benefits they desire, the actuary is sometimes asked to quote the percentage of salary necessary to provide them." But that is too late. What would be thought of a man building a house without first having an architect? The same holds with regard to the actuary; in framing a scheme, the actuary should be called in at the beginning. I am not here to tell you of many disastrous things which have happened in connection with pension funds by mere arithmeticians and book-keepers formulating schemes, and working away, until a day arrived when they found they had much more than the actuary's modest initial fee to pay for. With regard to all these questions—scales of contribution and of benefit—we can, neither in pension funds, nor widows' funds, nor life assurance work, get at exact figures, therefore it is, in a sense, somewhat of the nature of trial and error. And that is the great advantage—in this as in life assurance—of the periods of valuation or testing times. Seven or eight years ago, I had to frame a pension scheme, the females to retire at 55 and the males at 60, and was asked to quote a scale of contribution. I declined to do so, remarking that at these ages I could only quote what would be prohibitive rates. I preferred to give rates for pensions to be entered upon at 60 and 65 years of age respectively, and leave over till valuation periods to see if the success of the fund would warrant a gradual reduction of the pension age, from time to time; or, if then preferred, a reduction in the scale of contributions. The valuation means a periodical revision of the simple equation, Contribution = Benefit. By the results then brought out it is seen whether the contribution should be raised, or the benefit be reduced, or *vice versa*. The fund to which I referred had some money available with which to start it, and assist in levelling inequalities in years of age and of service. I found out the maximum amount available, and urged that it should all be put in as a nucleus. When I came to make the first valuation, I found that larger sum useful, and it made the report more palatable, especially as a good deal of that nucleus still remains. There is one thing about the paper which is specially noteworthy, and that is a demonstration of the adaptability of the commutation method. To those who understand that, it is easy to follow these interesting formulas. I observe that Mr. King makes elaborate preparations and formulas for withdrawals, and has introduced the h_x unhealthy pensioners, and refers to pensioners at age 25. I have not yet in my practice been introduced to so youthful a pensioner. I think that is much too early for a young man to contemplate being pensioned. It is sometimes—and indeed in a good many cases—much to the interest of the pension fund to give a man his money back, thus giving him what in many cases is an inadequate surrender value for what is really a deferred annuity. It is the same with the pensioner who comes on, because of ill health, earlier than was contemplated. He is what we know as "a bad life", and the fund will soon be relieved of what might otherwise prove to be a heavy burden.

Mr. Ralph P. Hardy: I feel considerable diffidence in joining in the debate on this paper, in which, by the generous friendship of Mr. King, I am alluded to in such specially handsome terms. But as I am conscious that I may rely upon the good feeling of my friends, I do just venture to intrude myself, because what we have to deal with is not any production of my own, but a highly-finished article, bearing throughout the distinct marks of an independently initiated and pursued line of thought and treatment, elaborated with all the technical art that only an actuary experienced in real work can command, and perfected with a skill born of a long and varied practice in his profession. If Mr. Hewat impeaches the general validity of these valuations, I take leave to say that they have passed far beyond the "trial and error" stage; and although they may not present that perfect solution which time may yet supply, they still present a reasonable anticipation of the state of things, and afford sufficient ground for practical procedure. We are all fully aware of the extreme danger of dealing with cessations, for the greater security you give to the future of a service, and the greater the inducements you supply to a staff to remain on, the more you steady the service, and to the same extent you destroy the asset in respect of secessions upon which you have been hitherto relying. I think I may say truly that, if I needed an illustration of what our training, when applied to a cultivated mind, will produce, or how the fluctuating elements of composite data can be steadied and held fast for observation, and how, under the powerful processes employed, they can be constrained to disclose the main current of the underlying truths they enfold, I would confidently appeal to such a paper as this, as being one which is eminently worthy of this Institute, and as affording another valuable sample of the notable work done by the author, whether educational, as in the Text Book, or, as in the present case, being a further contribution to our treasuries of professional knowledge. Subsequent experience has abundantly confirmed me in the views I ventured to express in 1894, in the debate which took place on my friend Mr. Hewat's and Mr. Chatham's paper (see *J.I.A.*, vol. xxxi, 471), that these particular investigations exercise an extraordinary fascination upon minds which are not content with the usual perfunctory explanations of the mere appearances which things present, but, with a laudable curiosity, are desirous of probing into the inner working of our social phenomena, and of noting how outside causes deflect, and too often control, the expected tendency of human action, and disarrange the ordinary development of the anticipated sequences. To such—and I believe they are a growing number within our walls—this paper will be found invaluable; while those still in the student age will find here a full repertory of scientifically adjusted and practically tested processes—though still to be employed with intelligent discrimination—calculated to ensure, not only that due weight should be given to the amount and quality of the observations, but that a proper measurement should be applied to those complicated social phenomena which are exhibited in the various Service and other pension funds.

Mr. T. G. Ackland: I should like to join with other speakers in an expression of my appreciation of the great value of this paper,

and how eminently it appears to be deserving of prolonged and careful study. I propose to limit my present remarks to one or two matters of comparatively minor detail, arising on a cursory reading of the paper. I have been somewhat puzzled to understand why Mr. King goes apparently out of his way to introduce the function $s_x \div 100$, in deducing his special commutation tables. No doubt that has some convenience in this respect, that it reduces the extent of the figures involved, by shifting the decimal point two places to the left; but as the $s_x \div 100$ is introduced both in the numerator and the denominator, it appears to me that the 100 must disappear, and that we are simply left with the s_x entering into the numerator for ascending values of x, whilst the value of s_x enters also into the D_x, which usually figures in the denominator. This is in itself a small matter, but seems rather calculated to bewilder the student, who may be uncertain whether the author is dealing with 1 per-cent upon a salary of £100 subject to increase, or with a simple variable or increasing annuity, starting with unity. It seems to me that the latter assumption is far preferable, and that the introduction of the 1 per-cent of 100 only tends a little to confuse matters. We have nothing to do with a contribution or pension of 1 per-cent upon 100, but the expressions deduced by Mr. King, as it seems to me, simply represent the increasing ratio of the salary, considered as unity at the outset, combined with the elements of mortality, interest, withdrawal, and the like. In the matter of adjustments, I am glad that Mr. King has heroically dealt with the matter by computing the continuous functions, and so avoiding the little awkwardness which some of us felt in some of Mr. Manly's problems and their solutions, where the adjustment had to be dealt with separately and subsequently. I suppose we shall agree that it is better to deduce the continuous function directly, and so do away with the necessity for all later adjustments. In the matter of notation, it would sometimes seem as if Mr. King's symbols were varied from Mr. Manly's, a little without full justification. The insertion of perhaps some small additional letter varies the function with which we have got to be more or less familiar, as Mr. Manly laid it before us; and it may be a little confusing in the *Journal* to have two or three distinct symbols expressing practically the same function. Again, in articles 110 and 125, and I think in other places in the paper, Mr. King rather unexpectedly introduces the symbols C and M for what appears not to be in the nature of an assurance benefit, but rather of an annuity benefit, and it may be questioned whether these symbols are the most appropriate in the circumstances. I quite think that Mr. King, in dealing with the matter of pension by introducing a rate of retirement at every age, is on the right lines. Mr. Hewat has spoken of the practical impossibility of a man retiring at age 25, but I suppose there is nothing impossible in the contemplation of a young man being paralysed, or suffering from some serious disease which would incapacitate him and render him, so long as he lived, eligible for a pension. At any rate, it seems to be a more satisfactory way, theoretically, to assume a rate of retirement at every age, which would no doubt rise to *maxima*, as shown in the tables appended, at

ages 60 or 65, but which would be in operation at all other ages. I entirely agree with the remarks of Mr. Lidstone on the question of the ratio of salary at different ages. I think there is a great deal of importance in what he said in that matter, and indeed in his remarks throughout. It has sometimes given me anxiety that, in dealing with these pension funds, we seem to be making to some extent a false comparison, in comparing the average salary as deduced, say at age 20, with the average salary of different people, as deduced at age 21. In one case at least of such a fund, I elaborately laid out the increasing salary of the same people as they passed through successive ages, and got, I think, more satisfactory results, although, of course, with a considerable increase of labour. That suggestion is, I submit, worthy of being more fully considered.

Mr. E. C. Thomas: Although the subject of pension funds has come before the Institute on several occasions recently, it is one which so teems with possibilities, not to say pitfalls, that the thanks of this meeting will be heartily awarded to Mr. King for the interesting contribution he has submitted to us this evening. Many of the problems which he has dealt with have been already discussed by Mr. Manly, but it is interesting, I think, to have the point of view of another master of the subject; and it must be a satisfaction to Mr. Manly to find his conclusions and formulas so amply confirmed by Mr. King. There are slight differences in method between Mr. King and Mr. Manly, mainly with regard to the adjustments. Mr. Ackland touched upon this question of the adjustments, but I think we are possibly in danger of doing Mr. Manly some injustice on the point. It must be remembered that, when he was writing his paper, he wished, as I understand it, to make his formulas applicable to any case which might arise, and at the same time to free his demonstrations from any unnecessary difficulty. There may not be any difficulty about using $v^{x+\frac{1}{2}}$ instead of v^{x+1}, but even here, when we remember that Mr. Manly was illustrating with a number of different funds, with different rules, and different methods of procedure, I think we must admit his method was convenient, for the purpose he had in view. By relegating contributions to the end of the year, he was able, by one simple multiplying factor applied only to the totals, to allow for the exact method of payment of contributions, whether monthly, quarterly, half-yearly, or weekly. As to the correction for the fractional payment in the year of exit, I think those who remember their student days will admit that it presents the matter in a more simple manner, if in the first instance, such corrections be ignored. There are additional conveniences; for instance, the correction in the case of a return at death is the same, whether the return is simply the contributions alone, or the contributions with simple interest, or with compound interest at the valuation rate, or any other rate. Sometimes a correction is required on both sides of the account, to the value of the contributions, and to the value of the return benefit; and if the whole of the contributions are returned in any given event, the correction is the same on both sides; and can therefore be dispensed with altogether. In article 90 Mr. King refers to a formula which was given in the *Journal* recently (see *J.I.A.*, xxxviii, 276) for deducing the value of the return of the

contributions, with compound interest at a rate differing from the valuation rate. It is gratifying to find that his formula entirely confirms that previously published. It is true that at the outset, in the former case, contributions were considered as payable at the beginning of the year, but that was a purely tentative assumption, and the necessary corrections were indicated, that for the final payment in the year of exit being identical with that previously published by Mr. Manly for the return without interest. There is a slight difference in Mr. King's methods. He first forms a column of log v^x at rate J, whereas in my formula the method indicated was to take the difference between log v^x and log u^x direct from "Jones on Annuities." I might mention that the symbol u^x, which is a very convenient distinction in cases like this, was actually suggested to me by Mr. King. There is one little inconvenience about Mr. King's formula, namely, that in calculating the value of the benefit in respect of the past contributions, assuming that the amount of those contributions, accumulated at the rate (j) is known, he requires a special commutation column D_x, at the rate J; that was not required by my formula. It was surprising to me to find that Mr. King thinks it would be necessary to calculate an assumed value for these past accumulations. Personally, I thought there would be no difficulty in obtaining the facts from the officials of the fund, and that has been borne out in practical experience in at least one instance. Mr. King refers to a slight apparent discrepancy between his formulas for the returns accumulated at rates i and j respectively. In article 68, we have $v^{\frac{1}{2}}$ $(\lambda_x^d - \frac{1}{2}d_x)$, &c., while in article 82 it is $u^{\frac{1}{2}}$, and not $v^{\frac{1}{2}}$, that is outside the bracket. I think the apparent contradiction vanishes when we remember that, in the first case the contributions have been accumulated and discounted at the same rate (i), and that it is really the accumulation factor which has been over-estimated, and not the discounting factor under-estimated. $v^{\frac{1}{2}}$ in this case is really $u^{\frac{1}{2}}$, but the two symbols being identical, there is no necessity to use u at all. That is my explanation of the apparent discrepancy. After all, however, these are minor points, and the real object of the paper is undoubtedly, as I gather, to introduce to us the necessity and the method of bringing into account the age at entry, as well as the age attained. In this Mr. King has rendered the Institute signal service. His methods are not mere theory, but are the outcome of laborious work performed in practical valuations, and he has shown in certain—it may be hoped rare—cases, that these methods are imperative, and he has also shown from his own experience that the labour involved is not prohibitive. A combination of two somewhat unusual circumstances is required in order to make these methods really necessary. The first condition is, that the scale of pensions, when expressed as a percentage of total salary, should vary within wide limits; and the second, that the new comers should enter in large numbers at the older ages, and so render the use of an average entry age inadmissible. With regard to the first condition, I think the more usual experience is that the scale is represented by n times a constant percentage, either of the average salary, or the last salary. But in either of these cases, Mr. Manly's more simple formulas can be used. Sometimes the scale appears to

be irregular, but when it is put in the form of a function of the total salary, it is found to vary within narrow limits. I have in mind a scale which appeared to be intractable, but when put into that form it varied irregularly between 2·20 and 2·40 per-cent of the total salary. As to the second condition, I think it will be admitted that the deviation from the mean age must be considerable, for the use of an average age to be inadmissible. I should be sorry to think that Mr. King's prediction as to the necessity for the use of select tables is likely to be fulfilled in the future. Nobody knows better than Mr. King the practical difficulties involved. It is difficult enough at any time to get sufficient data to form an ordinary table, but the difficulties if select tables were required would be immensely increased.

Mr. H. W. Manly: I should like to join in the general expression of congratulation to Mr. King for having brought this paper before us. There is, however, a little personal matter which perhaps ought to be explained. In the first part of the paper it is stated that to Mr. R. P. Hardy belongs the entire credit of raising from the empirical to the scientific the methods of dealing with these funds, and the author goes on to say that he was the pioneer, and we have followed in his footsteps. Well, when I read that, I thought it looked something like a charge that I had used somebody else's formulas without due acknowledgment. Now I have never done that, and perhaps Mr. King did not mean it. I have never had the good fortune to be associated with Mr. Hardy in any work which has involved any of these problems, and I have never seen his formulas. At the same time I have no doubt that Mr. Hardy has had all these formulas in his note books for many years, but he has never published them. I approached the subject, curiously, not with the intention at first of writing a paper upon it, but it grew out of another subject. There was no treatise on pension funds, and it occurred to me that possibly it would be of great interest to the students to know how many of these problems were solved. With that object in view, I treated each benefit as a separate problem, in order that students might be able to solve them in future. It would not be at all surprising, if two or three actuaries, or a dozen actuaries, were set down to solve the same problem, that they should arrive at very similar, or indeed the same, results; and I do not take to myself the credit of having first solved these problems. Mr. King gives me credit for introducing the ratio of salaries. I adopted that principle 25 years ago, and in the few cases I have had to deal with since, I have always adopted it. I did not know that it was peculiar to myself at all, but Mr. King thinks it is. I am surprised somebody else had not done that before. But there was one thing in that paper which I did think was absolutely original, and that was the notation. Mr. King says he has used a similar notation for a long time, which only shows how two great minds, working independently at the same subject, make the same discovery. I said I had purposely arranged my work in the form of problems for the education of the student, and then I thought that anybody who had mastered those problems, and wanted to bring them into practical use, would not find it difficult to do so. Mr. King has explained how they can be

brought into practical use by adopting $v^{x+\frac{1}{2}}$ instead of v^{x+1}, and introducing the adjustment for that difficult part of the question, the $\frac{1}{2}s_x$, in the first part of his commutation tables, instead of afterwards, as I have done. It is an improvement, and, personally, I always use it myself. But for the student, I did not think it was necessary. I think Mr. King raises some difficulties which are unnecessary. For instance, in article 26, he says: "But, for valuation purposes, we require the salaries which, it is assumed, will be drawn in the year immediately following the valuation." I always ask for the salary at the date of the valuation, and I always get it; there is no difficulty about it. Similarly with regard to past contributions and the accumulations at compound interest. Curiously, that problem has come into most of the funds I have had to do with. I have always asked to be supplied with the past contributions, and the accumulated interest to date, and I always got it. In article 54, with reference to a special form of return, Mr. King says, "A complete set of valuation factors must be prepared for each entry age, and about thirty sets in all will in many cases be required." The benefit there is so small that it is really not worth taking the trouble of making elaborate tables for it; you can easily make a very close approximation. The first part of this problem is a temporary increasing assurance of 5 per-cent of salary for 10 years, the value of which at entrance will be—

$$\frac{\cdot 05\left({}^d\mathrm{R}^s_x - {}^d\mathrm{R}^s_{x+10} - \sum_x^{x+9} s_x . {}^d\mathrm{M}^s_{x+10}\right)}{\mathrm{D}^s_x}$$

A table of these values can be constructed thus—

Age	Col. (1)	Col. (2) = Σ Col. (1)	Col. (3)	Col. (4) = Σ Col. (3)
x	${}^d\overline{\mathrm{C}}_x$	$={}^d\overline{\mathrm{M}}_x - {}^d\overline{\mathrm{M}}_{x+10}$	$=$ Col. $2 \times s_x$	$={}^d\overline{\mathrm{R}}^s_x - {}^d\overline{\mathrm{R}}^s_{x+10} - \sum_x^{x+9} s_x . {}^d\overline{\mathrm{M}}^s_{x+10}$
⋮	⋮			
$x+9$	${}^d\overline{\mathrm{C}}_{x+9}$	...	...	...

The second part of the problem is one which I have never met with. I have met it in another form, namely the payment at death of half the salary at date of death. In that case the factor after 10 years is $\frac{1}{2}{}^d\overline{\mathrm{M}}^{ls}_x \div \mathrm{D}^s_x$. It is a well-known approximation—and not infrequently used for larger benefits than this—that the present salary will on the average be the average salary at death. It gives too small a value at the younger ages, and too large a value at the older ages, but these, as a rule, balance each other. To approximate then to the second part of Mr. King's problem, it is only necessary, after the ten years, to multiply the salary by $\frac{1}{2}({}^d\overline{\mathrm{M}}_x \div \mathrm{D}_x)$. For this object, all the salaries of all the members of longer standing than ten years could be classified according to present age. For the first ten years

the factor for past salaries is $\cdot05$ Col. (2) $\div D_x$ and the factor for future salaries

$$\{\cdot05 \text{ Col. } (4) + \tfrac{1}{2}{}^d M^s_{y+10}\} \div D^s_x.$$

Now these values are microscopically small, and I should only consider it necessary to make three of such tables, say for ages 20, 30 and 40 at entrance, and I should add together the classified particulars for all ages at entrance, 15–24, 25–34, and 35–44. There is one thing on which I congratulate Mr. King, and in which I agree with him. That is, that in the case of pensions being based upon the last salary, or the average of the last three, or five, or seven years, there is a great advantage in classifying the members according to the age at entry. Whether you use Mr. King's tables, or whether you use my formula—I should prefer my own, because it would not be necessary to make so many tables, it is certainly a great advantage to have them classified in that way.

The PRESIDENT (Mr. H. Cockburn): Anything from the pen of our friend Mr. King deserves and attracts our careful attention, and we have to thank him once more for a very valuable contribution to our literature, on a subject which is, I think, happily of increasing importance, the modern development of pension funds, which points in itself to a prudent foresight, and likewise tends to the stability of the relations between employer and employed. Mr. King's papers are always practical, however scientific they may be at the same time, and what he has given us to-night will not rank as the least important amongst his many valuable contributions. I am sure you will wish to join with me in passing a hearty vote of thanks to him, and after doing that, I will ask him to make any observations which he would like on what has passed.

Mr. KING, in replying on the discussion, said: It was not without a considerable amount of trepidation that I ventured to send in a paper on this, which is really a difficult subject, and I expected much more serious criticisms than have been offered. In fact, there have been practically no criticisms. But you have received my effort most kindly, as you always do whenever anyone tries to serve the Institute. I should like to specially thank Mr. Lidstone for his remarks, which I look upon as most valuable additions to the paper, and I hope that Mr. Lidstone will extend them sufficiently, so that they may appear in that shape in the printed report of the discussion. One difficulty which has been referred to by several speakers, is that of the scale salaries; and it is a great difficulty. But it must not be thought that in my work I have only looked at the salaries at the valuation date; I have looked at them year after year, and I have compared the course of events and salaries ten years ago with the salaries at the present day. So we get men who were drawing salaries ten years ago, and we get the same men ten years older. I have found practically that, if we have a well-devised scale, with the lapse of time, the actual salaries tend to approximate to it. As to settling the scale, I agree with what Mr. Hewat said; we must call in the Accountant. As a matter of fact, to get the scale salaries we must take a great deal of trouble; we must ascertain the salaries paid, and the rates of increase for the

different grades in the service; we must get the maximum in each grade; we must get the probable promotions from one grade to another; we must get the probable proportionate numbers in each grade; and having done that, we must work out what will be the result as to average salary, when the fund reaches a stationary state. When the fund is rapidly growing, the scale salaries so arrived at will be almost necessarily less than the actual salaries at the higher ages, but when we involve the actual salaries, as is done in the formulas, that objection is overcome. Mr. Hewat is perfectly justified in saying that the rate of withdrawal is a decreasing quantity. I have always found it so where investigations have been made at successive dates. The rate of withdrawal has been found gradually to go down, but presently it will probably reach the limit. Meantime we have to be careful not to assume the rate of withdrawal too large. If the fund is a large one, considerable salaries do not in themselves affect the finances of the fund much, but I cannot agree that Directors do not sometimes raise salaries in view of retirement. They have a great idea of the immense wealth of these pension funds, and if by raising the salary they can secure for their official a better retiring sum, which does not come out of the pocket of the company, they think they are performing a good act. That has to be provided against, and it is well to fix a maximum for the pension. It is not necessary really where there is no selection of that kind, but it prevents selection. As to pensioners at so young an age as 25, if Mr. Hewat will look at Table 1 in my paper, he will find there actually is a pensioner who came in at age 25. There is another at age 26, another at age 27, and three others at age 28. Therefore they have to be provided for. It is a very small ratio at these ages, but there is no more trouble to start at age 25 than later. Mr. Ackland has spoken about my using the denominator 100, and it is true it disappears in the division. The main object was to reduce the number of figures in the commutation columns, and, though that may seem a trivial reason, to get the set of tables into smaller compass, and on a smaller sheet of paper. With regard to speaking sometimes of an annuity, and sometimes of 1 per-cent of salary, I think it is desirable to look at things from two or three different points of view. I have tried to do justice in my paper to predecessors, and if I have unwittingly not been quite successful with Mr. Manly, I beg his pardon. There is no intention of being unfair to anyone, and there is no insinuation to cast on anyone; I wish simply to give to each what, so far as my knowledge extends, is his due. As to Mr. Manly's notation, no doubt it is original, but mine, which is very similar, was not taken from his, because in connection with pension and similar funds it is the notation I have been accustomed to use for many years, and originally was largely derived from Mr. Hardy. In one or two cases I may have borrowed one of Mr. Manly's symbols, but the scheme, as a whole, is independent of his work, and was in use before I ever saw his paper.

I hope Mr. Manly will put in clear form his suggestion as to the special form of death return. That was carried out in full for my valuation, very much as an experiment to see how the method would work, and it did not give any trouble beyond a certain amount of

arithmetical work, but of course it would be well to have a shorter process. In this connection I should like to refer again to Table 1, because that table shows the extraordinary range of the ages at entry. The entrants come in at all ages from 11 to 50, and this illustrative fund is not the only one that has shown this marked characteristic. It is this long range of ages at entry that causes difficulty, by greatly increasing the arithmetical work, in that it necessitates so many separate sheets of factors, one for each age at entry. It may, however, frequently happen that there are only five or six entry ages, and in such cases the labour of the valuation would be very much reduced.

CORRESPONDENCE.

[We have received the following letters having relation to Mr. George King's paper.—ED. *J.I.A.*]

STAFF PENSION FUNDS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the discussion which followed the reading of Mr. King's paper on the 30th ultimo, I was prevented by want of time from mentioning the following formula, and should not now trouble you with the same but that it has been represented to me that, as an illustration of two important general principles, it might prove of interest to your readers.

The formula has reference to the special death benefit discussed by Mr. King in articles 54 to 65 of his paper. The benefit is as follows—

(*a*) The return of the whole of the contributions on death, if that event occur within the first 10 years of membership.

(*b*) A payment of one-half of the average annual salary, on death after 10 years.

The problem may be treated by means of an average entry age (if circumstances permit), or by taking groups of entry ages as advocated by Mr. Manly, or in the more thorough way, mentioned in the paper, of considering each entry age separately; but in each case the method of procedure would be the same.

It will be noticed that the benefit is a particular case of a payment assessed for the first n years as a function of the total salary, and thereafter as a function of the average salary. Now it is obvious that these forms of expression for the benefit are interchangeable; but it will be found, as was shown in Mr. Manly's earlier paper, that the form which will give the best working formula is usually, if not invariably, the one involving the total salary.

Further, as Mr. Lidstone pointed out in the discussion before mentioned, still bearing in mind the exigencies of our *practical* requirements, the more convenient course is to consider each year's contribution separately, and follow its course through the various years of assurance, rather than to fix the attention on the years of assurance, and value the benefit as it emerges in each successive year.

Proceeding therefore on these lines, we have (using Mr. King's notation)—

k'_t = the proportion of *total* salary returnable on death in the tth year

for first 10 years $k'_t = \cdot 05$

for the 11th year $k'_t = \frac{\cdot 500}{10 \cdot 5} = \cdot 0476$

for the 12th year $k'_t = \frac{\cdot 500}{11 \cdot 5} = \cdot 0435$

and so on.

Now in regard to the salary paid in the first year after entry, the value of the return, omitting the denominator, is as follows—

$$s_x\left\{(\tfrac{1}{2}\overline{C}_x \times k'_1) + (\overline{C}_{x+1} \times k'_2) + (\overline{C}_{x+2} \times k'_3) + \ldots\right\}$$

In respect of the 2nd year's salary,

$$s_{x+1}\left\{(\tfrac{1}{2}\overline{C}_{x+1} \times k'_2) + (\overline{C}_{x+2} \times k'_3) + (\overline{C}_{x+3} \times k'_4) + \ldots\right\}$$

and so on.

The first column required is therefore—

$$\overline{C}_x \times \text{appropriate } k'_t = (\text{say})\ C'_x.$$

We then require to sum the above column, and, in order to allow for the half payment over-valued in the year of death, we can use Mr. King's ingenious device, and deduct, from each figure in the resulting column, $\frac{1}{2}C'_x$, thus—

$$\sum_x^{\omega} C'_x - \tfrac{1}{2}C'_x = (\text{say})\ M'_x.$$

The further columns required are as follows—

$$M'_x \times s_x = {}^sM'_x$$

$$\Sigma\,{}^sM'_x = {}^sR'_x.$$

In regard to past year's service, the value of the return is—

$$(\text{Total past salary}) \times \frac{M'_x}{D_x}\ *$$

and in respect of future service,

$$\text{Present salary} \times \frac{{}^sR'_x}{{}^sD_x}\left\{\text{where } {}^sD_x = (D_x \times s_x)\right\}.$$

I have used the function s_x, in preference to $\frac{s_x}{100}$, in order to shorten the explanation; in practical work, however, I think the latter is much more convenient, and as it is brought into both the numerator and denominator it is obvious that no disturbance is caused in the results whether $\frac{s_x}{100}$ or any other fraction of s_x is employed.

It has been assumed, in assessing the values of k'_t, that the average salary would be calculated on the actual salary received up to

* Strictly speaking the column M'_x used in this connection should be simply $\Sigma C'_x$ without any deduction, and not the column M'_x already referred to.

the moment of death, but in practice it would probably be computed on the salary for each year entered upon; thus, for those dying in the 11th year, for instance, the average salary would be $\left(\sum_{x}^{x+10} s_x \div 11\right)$, and the value of k'_t would therefore be $\frac{\cdot 500}{11} = \cdot 04545$. If this condition held good, our formula would be slightly simplified, because for all years after 10 we should require no correction for the final payment, and M'_x for ages $(x+10)$ and onwards would be simply $\Sigma C'_x$.

The method proposed has the advantage of being completely in accord with that described in the postscript to Mr. King's paper (articles 178-182) for assessing the value of superannuation benefits, and is indeed obvious from a careful consideration of the explanations there given.

It seemed, however, desirable to elaborate the process a little, in order to illustrate that, with the aid of the two principles already mentioned, certain apparently complicated benefits can be reduced to a simple and orderly form.

I am, Sir,

Your obedient servant,

ERNEST C. THOMAS.

St. Mildred's House, E.C.
28 *February* 1905.

THE NOTATION OF PENSION FUND PROBLEMS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The different notations employed by Mr. King and Mr. Manly in their papers on Pension Funds are likely to be confusing to those who may find it desirable—as suggested in the recent discussion—to read those papers together, and it would seem that the time has arrived when the Council of the Institute might profitably consider the question of settling and prescribing an official notation. As a preliminary, I venture to ask Mr. King to reconsider his symbols (for which Mr. Manly has no equivalent) for the various valuation multipliers or "factors." For all these functions, though representing very various benefits, Mr. King uses the one generic symbol F, which acts merely as a scaffolding on which to build up a number of subsidiary symbols serving to distinguish one function from another. This sacrifices—as it would seem, needlessly—the great advantages secured by the fundamental principle of the Institute notation, namely, that the principal symbol, which first strikes the eye, determines the nature of the function, while the small letters denote particular ages, terms of years, etc. I suggest that separate principal letters be used for withdrawal, death and pension benefits, and that a *capital* letter denote benefits in respect of *past* contributions or salaries, and a *small italic* letter benefits in respect of *future* contributions or benefits. The particular letters are immaterial, but if Mr. King would substitute another symbol, say, w_x^h for his h_x (which is but rarely wanted and is not in fact used in any of his formulas), we should have the three

consecutive letters F, G and H available for withdrawals, deaths and pensions respectively. That this would introduce a considerable simplification will be seen by the following comparisons:

WITHDRAWAL BENEFITS: IN RESPECT OF—			
Past contributions	F_x	instead of	${}^pF^w_x$
Future "	f_x	"	${}^fF^w_x$
DEATH BENEFITS: IN RESPECT OF—			
Past contributions	G_x	"	${}^pF^d_x$
Future "	g_x	"	${}^fF^d_x$
Past contributions with interest . .	${}_yG^i_x$	"	${}^p_yF^{di}_x$
Future contributions with interest . .	g^i_x	"	${}^fF^{di}_x$
PENSION BENEFITS: IN RESPECT OF—			
Past salaries—Unlimited Pensions . .	H_x	"	${}^pF^{ra}_x$
" " Pensions with limitations .	${}_yH_x$	"	${}^p_yF^{ra}_x$
Future salaries—Unlimited Pensions .	h_x	"	${}^fF^{ra}_x$
" " Pensions with limitations	${}_yh_x$	"	${}^f_yF^{ra}_x$

The relatively few cases in which the benefit relates both to past and future contributions or salaries could conveniently be represented by a hollow capital letter.

That the conventional symbols F, f, G, g, H, and h would be readily learnt is conclusively proved by experience of the similar symbols in the Institute notation, and the suggested new notation would certainly be more easily written, printed, and read than that used by Mr. King.

It would trespass too much on your space to discuss in detail the symbols for commutation functions, but here also it would be desirable, if practicable, to introduce some simplification. In any case, much is gained by having a simple and easily-written notation for the factors which are actually used in a valuation.

I am, Sir, your obedient Servant,

6 *February* 1905. G. J. LIDSTONE.

Changes in Pure Premium Policy-Values consequent upon variations in the Rate of Interest or the Rate of Mortality, or upon the introduction of the Rate of Discontinuance. By GEORGE JAMES LIDSTONE, F.I.A., *Actuary of the Alliance Assurance Company, Limited.*

[Read before the Institute, 27 February 1905.]

I. INTRODUCTORY REMARKS.

1. THE volumes of the *Journal* contain numerous theoretical investigations dealing with the changes produced in Policy-values by alterations in the basis of valuation, and a number of interesting and important results have been obtained, of which

the most valuable have been embodied in the *Text-Book*, Part II, Chapter xviii, Articles 39–72. These investigations are almost entirely based upon analytical transformations of algebraical expressions for the Policy-value. Fruitful as this method has been, and elegant as are many of its processes and results, it yet labours under certain serious disadvantages. From the nature of the method it can be applied only to the limited class of benefits—practically confined to ordinary Whole-Life and Endowment Assurances, with uniform Premiums and Benefits—which admit of the Policy-values being expressed in an algebraical form involving only one class of function, *e.g.*, a_x, A_x or P_x: so that relations which are really of very wide generality remain undemonstrated, except for those particular classes of Assurance. Again, demonstrations based upon a particular mathematical formula for the Policy-value are not easily modified to meet changes of conditions which, though slight in themselves, may render the fundamental formula completely inapplicable: while abstract analytical demonstrations are both harder to grasp in the first instance, and harder to reproduce or adapt when subsequently required, than are demonstrations based upon more concrete considerations.

2. There are, therefore, considerable advantages attaching to a method which, instead of dealing with purely mathematical relationships of an abstract character, proceeds from a consideration of the concrete financial entities involved in the problem and exhibits as it were the actual building-up of the Policy-values, as well as the manner in which variations arise. Not only is the mental view of the problem in this light clearer and more sharply defined, and therefore more easily recalled at will: it is also of wider range, and more capable of extension to meet changing conditions.

3. The method which it is now proposed to develop is of so simple a character that, while of very wide application, it is capable of being reduced to the form generally known as "Verbal Explanation",* and it was, in fact, originally arrived at by that road. If algebraical symbols and equations are now introduced, it is chiefly because they enable the ideas involved in the "Verbal Explanation" to be more briefly expressed, and the results to be more easily compared with known relations.

* This phrase, which is perhaps not in itself a very accurate one, is used in accordance with long-established custom to denote a train of reasoning which deals directly with the *things* which form the subject matter of the problem, and not with abstract symbols or formulæ representing those things.

4. The principal symbols employed will be as follows:

p_n = Probability that if the status be in existence at the beginning of the nth year, it will survive to the end of the year.

q_n = Probability that if the status be in existence at the beginning of the nth year, the sum assured, S_n, will become payable at the end of the year.

N.B.—In the case of a contingent status, $p+q$ will not necessarily equal Unity.

P = Uniform Pure Premium payable so long as the status endures.

S_n = Sum Assured for the nth year.

i_n = Rate of Interest for the nth year.

v_n = Corresponding value of 1 due at the end of the year.

V_n = Policy-value after n years *before* payment of the Premium then due.

$\bar{V}_n = V_n + P =$ "the Initial Reserve" = Reserve after n years immediately *after* payment of Premium then due.

Accented symbols denote a special Basis of Valuation.

The subscript n is usually omitted, and the symbols V or $\bar{V}$ denote the values at the beginning, and the symbols V_1 and $\bar{V}_1$ values at the end, of the particular year in question.

There is no subscript on the right-hand side, to indicate the nature of the status, because the results obtained are very general, but it is important to note that the reasoning relates throughout to the successive values (as the duration increases) of a Policy effected on a particular status, *i.e.*, the durations are supposed to vary, but the *age at entry* of the life or lives remains constant.

In the case of a last survivor status, which may remain in existence though one or more of the constituent lives be dead, p, q, V and $\bar{V}$ must be given their mean values, having regard to the different possible combinations and their relative probabilities.*

* *See* Remarks on pp. 250, 251.

Summary of Special Terms.

5. In the course of this paper a number of special terms are used, as substitutes for somewhat lengthy phrases, with a view of abbreviating the verbal arguments and thus rendering them both more succinct and easier to follow. These special terms are all defined in the course of the paper, but it may be convenient to collect the definitions here in tabular form:

Special Term	Definition	Mathematical Equivalent
Normal Basis . .	The standard basis of valuation giving standard Policy-values with which other Policy-values are to be compared.	Values of functions denoted by unaccented symbols.
Special Basis . .	A basis of valuation differing from the Normal Basis and to be compared with it.	Values of functions denoted by accented symbols.
Normal Policy-values .	Calculated on Normal Basis . .	V or $\bar{\mathrm{V}}$
Special Policy-values .	Calculated on Special Basis . .	V' or $\bar{\mathrm{V}}'$
Initial Policy-value .	Policy-value at commencement of a year *including* the Premium then due.	$\overline{\mathrm{V}}$, $\overline{\mathrm{V}}'$, $\bar{\mathrm{V}}$, $\bar{\mathrm{V}}'$
Excess . . .	The positive or negative algebraical difference between any function on the Special Basis and the same function on the Normal Basis.	$q'-q$ $p'-p$ $i'-i$ $\mu'-\mu$ &c.
Accumulated Excess Premium.	The Excess Premium with Interest at the Special Rate.	$(\mathrm{P}'-\mathrm{P})(1+i')$ or $\bar{\mathrm{P}}'-\bar{\mathrm{P}}$
Excess Interest . .	Difference between Interest at Special rate and at Normal rate, calculated on Normal Initial Reserve.	$(i'-i)\overline{\mathrm{V}}$, or $(\delta'-\delta)\overline{\mathrm{V}}$
Excess Strain . .	[Excess Claims of the year] + [Excess Normal Reserve, for continuing Policies, at the close of the year.]	$(q'-q)\mathrm{S}+(p'-p)\mathrm{V}_1$ or $(\mu'-\mu)\mathrm{S}+\left[\frac{d}{dt}(p'-p)\mathrm{V}\right]$
Critical Function .	[Excess Strain] − [Excess Interest] .	See above.
Critical Function for discontinuances.	Expected Profit on Withdrawals taken negatively.	$-wp(\mathrm{V}_1-\mathrm{SV})$, or $-\bar{w}(\overline{\mathrm{V}}-\overline{\mathrm{SV}}$
Remainder . . .	Accumulated Excess Premium *less* the Critical Function.	$\mathbb{R}$
Variation Fund . .	Accumulated amount of differences between the Accumulated Excess Premium and the [Excess Strain − Excess Interest]. N.B.—The accumulation is with interest and benefit of survivorship.	...

II. General Principles of the New Method.

6. The fundamental principle on which the demonstrations proceed is the perfectly general relation

$$\mathbb{V}(1+i)=\mathrm{V}(1+i)+\mathrm{P}(1+i)=q\mathrm{S}+p\mathrm{V}_1 \quad . \quad . \quad . \quad (1)$$

That is to say, the Initial Reserve (which includes the Premium just paid), accumulated at interest to the end of the year, will then provide the Sum Assured for all cases in which the benefit emerges, and the Reserve at the end of the year for the continuing Policies. This relation is too obvious and too well known to need any comment here.

If a different basis of valuation, denoted by accented symbols, give identical Policy-values, the following equation will subsist—

$$\mathrm{V}(1+i')+\mathrm{P}'(1+i')=q'\mathrm{S}+p'\mathrm{V}_1 \quad . \quad . \quad . \quad . \quad (2)$$

and hence

$$\mathrm{V}(i'-i)+\mathrm{P}(i'-i)+(\mathrm{P}'-\mathrm{P})(1+i')=(q'-q)\mathrm{S}+(p'-p)\mathrm{V}_1 \quad (3)$$

or

$$\mathbb{V}(i'-i)+(\mathrm{P}'-\mathrm{P})(1+i')=(q'-q)\mathrm{S}+(p'-p)\mathrm{V}_1 \quad (4)$$

In what follows this will be called the "Equation of Equilibrium." It may be noted that for all absolute assurances on single or joint lives $p+q=p'+q'=1$, and the right-hand member of the last equation may then be put into the convenient form $(q'-q)(\mathrm{S}-\mathrm{V}_1)$.

7. The verbal explanation of the "Equation of Equilibrium" is simple. If two sets of Policies, securing identical benefits and giving equal Policy-values, be conceived to be worked out side by side on different bases (which may be distinguished as the "Normal" and the "Special"), the financial working of the year under consideration must differ as follows: The Special Basis Policies will receive the Excess* Premium increased by a year's Interest at the special rate, and will also earn Excess Interest on the Normal Initial Reserve. These additional receipts must be exactly sufficient to provide at the end of the year the Excess Claims and the Excess Reserves, the total of which may be termed the "Excess Strain."

* The "excess" may be either positive or negative, and the word is used as an abbreviated expression for the algebraical difference between the accented and non-accented functions.

8. The Equation of Equilibrium may be put into the form

$$(P'-P)(1+i')=[(q'-q)S+(p'-p)V_1]-[V(i'-i)] \quad . \quad (5)$$

that is, if the Special Policy-values are equal to the Normal Policy-values, the Accumulated Excess Premium must each year be equal to the [Excess Strain—the Excess Interest].

9. If, however, the Special Basis be such that the [Excess Strain—the Excess Interest] varies in comparison with the accumulated Excess Premium, instead of being always equal to that quantity, the Special Policy-values will differ from the Normal Policy-values. In that case $(P'-P)(1+i')$ may be considered as divided into two parts, in varying proportions from year to year; the first part being equal to the [Excess Strain—the Excess Interest], and the second representing the algebraical remainder, which may be called E. Now conceive the first portion only to be carried into the Assurance Fund, and the second portion to be accumulated separately (at interest and with benefit of survivorship), thus forming a distinct Fund, which may be termed the Variation Fund. The Equation of Equilibrium shows that the first portion of the Premium will just suffice to build up Policy-values exactly equal to the Normal Policy-values: hence the Variation Fund at any point will represent the difference between the Special and the Normal Policy-values.

10. It is evident that the constant quantity $(P'-P)(1+i')$ must represent the mean value of the variable quantity [Excess Strain *less* Excess Interest], in the sense that the present values of these two quantities (calculated at the date of entry and for the whole duration of the assurance, on the special basis), must be identical.* It follows from this that the present value of the variable *difference* between these two quantities, that is the present value of E, must be zero. Hence, unless each individual value of E be zero (in which case it has been seen that the Normal and

* This will be seen at once if it be considered that the [Excess Strain *less* Excess Interest] represents the loss that will be shewn if the Valuation be made on the Normal-Basis but the Experience be in accordance with the Special-Basis. The present value of such loss must be equal to the present value of the uniform difference between the Normal-Basis Premium and the Special-Basis Premium which will just work out the assurance, with neither profit nor loss, on the Special (Experience) basis. A purely mathematical demonstration of the relation is given in Appendix B to the Author's Paper "On the Distribution of the Divisible Surplus of a Life Assurance Company", *J.I.A.*, xxxii, 73, *et. seq.*: *vide* Equation (12) page 108, where it must be borne in mind that if R'' equals the Reserve on the Experience basis, represented in this Paper by V', we must have at the date of entry $R_0=R''_0$, or the present value of the loss equals the present value of the Excess Premium.

Special Reserves are identical), the values of $\mathbb{E}$ must be sometimes positive and sometimes negative, or, in other words, $(P'-P)(1+i')$ must lie between the greatest and least values of the [Excess Strain—the Excess Interest]; and the accumulations of the positive elements must, at the limiting age, be numerically equal to the accumulations of the negative elements, thus reducing the Variation Fund to zero.

11. The results of Articles 9 and 10 are so fundamental, and a clear understanding of them so essential, that it may be well, before proceeding further, to give a numerical example by way of illustration. Take the case of an ordinary Whole-Life Policy assuring unity on a life aged 85 at entry, the Normal Basis being H^M 3 per-cent; and let the Special Basis be H^M 4 per-cent with an addition of 5 years to the age. Then Table A (facing page 218) gives the necessary materials.

12. In this Table, Column 11 gives the present value of the [Excess Strain *less* Excess Interest], namely, ·2516, and this is precisely equal to the present value of $(P'-P)(1+i')$, namely, $\cdot 09676 \times (v\mathbf{a}_{90})^{H^M 4}$ or $\cdot 09676 \times \cdot 96154 \times 2\cdot 7039$. The total of Column 10 gives the present value of $\mathbb{E}$, which is seen to be zero, the positive and negative elements counterbalancing each other. Column 9 shews that the values of $\mathbb{E}$ itself are sometimes positive and sometimes negative; and Columns 12–15 shew that the difference between the Normal and Special Policy-value is equal to the Variation Fund, that is, the accumulated amount of $\mathbb{E}$.

13. Returning to the general argument, it has been seen that $\mathbb{E}$ is sometimes positive and sometimes negative. Suppose that it commences by being $\genfrac{}{}{0pt}{}{\text{negative}}{\text{positive}}$, passes through the value zero and thereafter becomes and remains $\genfrac{}{}{0pt}{}{\text{positive}}{\text{negative}}$. Then up to the zero point the Variation Fund must be $\genfrac{}{}{0pt}{}{\text{negative}}{\text{positive}}$ (because it consists entirely of $\genfrac{}{}{0pt}{}{\text{negative}}{\text{positive}}$ elements) and it must continually increase in numerical value: after the zero point the Variation Fund must eventually* diminish numerically (through the introduction of elements of the contrary sign), until it reaches zero at the limiting age. The Variation Fund cannot

* The diminution will not usually commence immediately after the zero point because the Variation Fund is accumulating with interest and benefit of survivorship, and for some years this may more than counterbalance the reduction due to elements of contrary sign.

in the given conditions become $\substack{\text{positive}\\ \text{negative}}$ after reaching zero, for if it did the $\substack{\text{positive}\\ \text{negative}}$ value would be increased by the subsequent $\substack{\text{positive}\\ \text{negative}}$ values of $\mathbb{R}$ and could not ultimately become zero, as it must in fact do. Therefore the Variation Fund must be $\substack{\text{negative}\\ \text{positive}}$ throughout, and hence the Special Policy-values must always be $\substack{\text{less}\\ \text{greater}}$ than the Normal Policy-values.

14. The above reasoning shows—

(i) That if $\mathbb{R}$ be *constant,* its value must be zero and the two sets of Policy-values will be equal throughout;

(ii) That if $\mathbb{R}$ be first *negative* and then change permanently to *positive* (which it will certainly do if its value be continually *increasing*), the Special Policy-values will be the *smaller* throughout;

(iii) That if $\mathbb{R}$ be first *positive* and then change permanently to *negative* (which it will certainly do if its value be continually *decreasing*), the Special Policy-values will be the *higher* throughout.

15. Now since $\mathbb{R}$ is equal to

$$[\text{Accumulated Excess Premium}] \textit{ minus } [\text{Excess Strain} - \text{Excess Interest}]$$

$$i.e., \quad [(P'-P)(1+i')]-[(q'-q)S+(p'-p)V_1-(i'-i)\mathbb{V}] \quad . \quad (6)$$

it will (provided i' be constant) be equal to zero throughout, or continually increasing, or continually decreasing, according as the [Excess Strain — Excess Interest] be constant, decreasing, or increasing. Thus, if the [Excess Strain—Excess Interest] be called the Critical Function, *the Special Policy-values will be* (1) *equal to, or* (2) *always less than, or* (3) *always greater than the Normal Policy-values, if the Critical Function be* (1) *constant,* (2) *continually decreasing, or* (3) *continually increasing; provided that the special rate of Interest be constant.*

16. These relations, which have been obtained by general reasoning, are the foundation of and the key to all that follows. The relations are in effect very simple ones. Just as a Reserve-value itself arises from the circumstance that a uniform Premium is paid for a varying risk, thus requiring the Surplus Premium in the early years (over and above the amount of current risk) to be accumulated

to form a Reserve for future years in which the current risk will be greater than the Premium: so when two different bases of valuation are compared a uniform Excess Premium is paid to cover a varying Excess Outgo, and the difference between the uniform Excess Premium and the current Excess Outgo has to be accumulated to form a Special Reserve Fund which will meet similar differences in the reverse direction in subsequent years. The varying Excess Outgo is the [Excess Strain—Excess Interest], and the Special Reserve Fund is what has been termed the Variation Fund, representing the difference between the Special and Normal Policy-values. If the Excess Premium be at first greater and afterwards less than the Excess Outgo, the Special Fund will be positive and the Special Basis Policy-value will therefore be *greater* than the Normal Basis Policy-value; while if the conditions are reversed the contrary will be the case.

17. It will be shown hereafter that if the Special Basis be connected with the Normal Basis by a definite law, the condition that the Critical Function shall be continually increasing or continually decreasing will be fulfilled in a number of important particular cases, and it follows that in those cases general relations between the Special and Normal Reserves can be found without calculating the Special Premiums P′. But when two Mortality Tables having no intimate connection are compared, the Critical Function may follow no definite law, and then, of course, the method will fail to disclose any universal law (even if any such exist) as to the relations of the Policy-values. If, however, the value of P′ is known, information as to the relative Policy-values may in some cases be obtained from mere inspection of the values of $\mathbb{E}_t$, (*i.e.*, the Accumulated Excess Premium—the Critical Function), though the Critical Function itself be neither continually increasing nor continually decreasing. For example, the Critical Function may vary irregularly in such a manner that, though sometimes increasing and at other times decreasing, yet $\mathbb{E}_t$ changes sign only once—that is, $\mathbb{E}_t$ commences by being $\substack{\text{positive}\\ \text{negative}}$ and changes to $\substack{\text{negative}\\ \text{positive}}$; and in these circumstances it has been shown that the Special Reserves will be throughout $\substack{\text{greater}\\ \text{less}}$ than the Normal.

18. Next, let the values of $\mathbb{E}_t$ take the following signs in succession, namely—

$$- \ldots 0 + \ldots 0 -,$$

then the previous results show that up to the first zero point the Special Values will be less than the Normal; at some point between the two zero points the Special Values will become greater than the Normal and they will thereafter remain greater until the end of life.

19. Again, if the values of $\mathbb{R}$ take the following signs in succession, namely—

$$+\ldots 0-\ldots 0+,$$

the previous results show that up to the first zero point the Special Values will be greater than the Normal; at some point between the two zero points the Special Values will become less than the Normal, and they will thereafter remain less until the end of life.

20. In the foregoing investigation it has been tacitly assumed that according to the Special Basis the existence of the status must necessarily cease *at or before* (but in no case after) the point at which it ceases according to the Normal Basis; for if this be not the case, the expression $[(q'-q)\mathrm{S}+(p'-p)\mathrm{V}_1]$ will have no meaning at any point after the cessation of the status on the Normal Basis. The condition in question does not affect in any way the application of the theory to variations in the rate of Interest, nor to the introduction of the Rate of Discontinuance; nor does it affect the general propositions given in Section IV of this paper as regards variations in the rate of Mortality where the Normal and Special rates have a direct functional relation. The point may, however, be of importance when the method is applied numerically (in the way described in Articles 17–19) to two Mortality Tables which have no intimate connection. In such a case let the nth year be the last during which the status remains in force according to the Normal Basis. Then if the Excess Strain of the nth year be put equal to $[(q'_n-q_n)\mathrm{S}_n+p'_n\mathrm{V}'_n]$ it may be shown (by means of the investigation referred to in the foot-note on page 214) that $\mathbb{R}$ will retain its fundamental property, namely, that its present value for the period from the date of entry to the end of the nth year will be zero, and the methods of Articles 17–19 can be applied without extending the investigation beyond the nth year. It will, however, in this case be necessary to know the value of P', since this enters into V'_n, which is involved in the formula for the Excess Strain of the nth year. A numerical example is given in the second half of Table A.

lation of the 1
and illustrati

Excess Strain	E. In
$(q'-q)(1-V_1)$	$(i'$
(6)	
·0665	
·0845	
·1030	
·1410	
·1862	
·2379	
·3002	
·3385	
−·0633	-
−·0760	-
−·0860	-
−·1089	·
−·1325	·
−·1556	
−·1862	
−·3385†	

$[q'-q+p'V'_1]$.

Table A

Giving examples of the Calculation of the Excess Strain, the Excess Interest, the Function [illegible] and the Variation Fund, and illustrating the relations given in Articles 9, 10, and 20

Normal Basis (unincreased symbols) HM [illegible], age at entry, 85; Special Basis (increased symbols) HM [illegible], age at entry, 90; (P′ − P)(1 + i′) = +.0068

Year n	V	V + P − V̄	1 − V₁	$i' - i$	Excess Strain $(i-q)(1-V_1)$	Excess Interest $(i'-i)V$	Excess Strain less Excess Interest	Accumulated Excess Premium less Ex. Str. − Ex. Int. R	Σ [illegible] $\frac{D_n}{D_x}$	Ex. Str. − Ex. Int. [illegible] $\frac{D_n}{D_x}$	Variation Fund at beginning of nth year (Col. 14)[illegible] $= V' - V$	(12) × (1 + i′)	(13) + (9)	[illegible] V	n
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
0	0000	2383	9550	0596	0665	0024	0641	0327	+ 0311	0646	0000	0000	0327	0000	0
1	0450	2833	9074	0931	0845	0028	0817	+ 0151	+ 0101	0511	0451	0172	0623	0901	1
2	0926	3309	8571	1201	1030	0033	0997	− 0029	− 0013	0439	0907	0943	0914	1833	2
3	1426	3809	7988	1765	1110	0038	1372	− 0101	− 0111	0377	1410	1466	1062	2836	3
4	2012	4395	7328	2511	1862	0044	1818	− 0850	− 0131	0280	1818	1891	1040	3830	4
5	2672	5055	6652	3576	2379	0051	2328	− 1360	− 0099	0170	2111	2195	0835	1783	5
6	3348	5731	5961	5036	3002	0057	2945	− 1977	− 0050	0073	2301	2393	0416	5619	6
7	4039	6422	5219	6487	3385	0061	3321	− 2353	− 0031	0015	2263			6302	7
									+ 0415	[illegible]					
									− 0415						

Normal Basis (unincreased symbols) HM [illegible], age at entry, 90; Special Basis (increased symbols) HM [illegible], age at entry 85; (P′ − P)(1 + i′) = −.0068

Year n	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	n
0	0000	3314	9096	− 0596	− 0635	− 0033	− 0600	− [illegible]	− 0348	− 0582	0000	0000	− [illegible]	0000	0
1	0901	4218	8167	− 0931	− 0760	− 0042	− 0718	− 0210	− 0179	− 0545	− 0451	− 0168	− 0708	0450	1
2	1833	5147	7161	− 1201	− 0860	− 0051	− 0809	− 0150	− 0081	0156	− 0907	− 0931	− 1081	0926	2
3	2836	6149	6170	− 1765	− 1089	− 0062	− 1027	+ 0061	+ 0029	− [illegible]	− 1110	1152	− 1183	1426	3
4	3830	7144	5217	− 2511	− 1325	− 0071	− 1254	0286	+ 0092	[illegible]	− 1818	− 1873	− 1777	2012	4
5	1783	8097	4351	− 3576	− 1556	− 0081	− 1177	+ 0517	+ 0116	[illegible]	− 2111	− 2175	− 1658	2672	5
6	5619	8962	3698	− 5036	− 1862	− 0089	− 1773	+ 0815	+ 0129	0280	− 2301	− 2370	− 1755	3348	6
7	6302	9615	5219*	− 6487	− 3385†	− 0086	− 3299	+ 2331	+ 0245	[illegible]	− 2263			4039	7
									− 0611	− [illegible] $(P'-P)(1+i)\ddot{a}$ [illegible]					
									+ 0611						

* 1 − V [illegible] † [illegible] $q - q + i'V$ [illegible]

21. The new method of investigation having now been expounded and fully discussed in its general form, it will be desirable to consider its application to important special cases, and this will be done in the succeeding Sections. To avoid misunderstanding, it may be well to state explicitly that the conditions obtained in the course of the investigation, while they are *sufficient* to ensure that the Policy-values shall bear the stated relations, are not *necessary* conditions—in other words, if the condition be fulfilled, the given relations *must* exist, but if the condition be *not* fulfilled, the given relations may or may not exist, according to the circumstances of each particular case.

III. Variations in Rate of *Interest*, the Rate of Mortality remaining unchanged.

22. Putting $q'=q$, $p'=p$, in the Equation of Equilibrium, there results

$$V(i'-i)+(P'-P)(1+i')=0 \quad . \quad . \quad . \quad . \quad (7)$$

or

$$V(i-i')=(P'-P)(1+i') \quad . \quad . \quad . \quad . \quad . \quad (8)$$

that is to say, the Excess Premium accumulated at the special rate must be numerically equal, but of opposite sign, to the Excess Interest on the Initial Normal Reserve, which includes the Normal Premium. If this equation hold good each year we shall have

$$(V_{n-1}+P)(i_n-i'_n)=(P'-P)(1+i'_n) \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

whence

$$i'_n=i_n-\frac{(P'-P)(1+i_n)}{V_{n-1}+P'} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and the Policy-values based on the special rates of Interest will, for all durations, be equal to those based on the normal rate or rates. This result is thought to be new, and it may be of interest to give an example of the varying rates of Interest equivalent to a fixed rate of, say, 3 per-cent.

TABLE B.

Rates of Interest, i_n, yielding Policy-values equal to those based on the uniform rate of ·03 [H^{M} Mortality, Ordinary Whole-Life Assurance].

n	P′ − P = + ·001			P′ − P = − ·001		
	Age at Entry			Age at Entry		
	20	30	40	20	30	40
1	− ·03745	− ·02202	− ·00830	+ ·10762	+ ·08787	+ ·07138
2	− ·01348	− ·00267	+ ·00628	·07749	·06488	·05487
3	− ·00216	+ ·00633	·01296	·06430	·05481	·04762
4	+ ·00461	·01153	·01678	·05670	·04916	·04356
5	·00919	·01492	·01925	·05169	·04553	·04098
10	·01961	·02236	·02453	·04060	·03776	·03553
15	·02329	·02507	·02640	·03679	·03498	·03362
20	·02518	·02644	·02735	·03486	·03358	·03267
25	·02633	·02725	·02789	·03369	·03276	·03211
30	·02709	·02778	·02824	·03293	·03223	·03176
35	·02761	·02813	·02848	·03240	·03187	·03152

23. Next, let the excess rate of Interest take such a value that $\mathbb{V}(i-i')$ varies in value in comparison with $(\text{P}'-\text{P})(1+i')$, the accumulated Excess Premium. Then the Special Policy-values will differ from the Normal Policy-values, and it is necessary to consider the Critical Function

$$[\text{Excess Strain} - \text{Excess Interest}]$$

which in this case is equal to

$$-\mathbb{V}(i'-i) \text{ or } \mathbb{V}(i-i'),$$

since the Excess Strain vanishes.

24. Consider the important case in which i and i' have fixed values, so that $i-i'$ is constant. If the constant value of $i-i'$ be $\genfrac{}{}{0pt}{}{\text{negative}}{\text{positive}}$ and if $\mathbb{V}$ be continually increasing algebraically, it follows that the critical function $\mathbb{V}\ (i-i')$ will be $\genfrac{}{}{0pt}{}{\text{decreasing}}{\text{increasing}}$ algebraically, and it has already been shown that under these conditions the Special Policy-values will be $\genfrac{}{}{0pt}{}{\text{less}}{\text{greater}}$ than the Normal

Policy-values. *Thus, whatever the form of benefit, a constant* $\begin{matrix}\textit{increase}\\ \textit{decrease}\end{matrix}$ *in the fixed rate of Interest will* $\begin{matrix}\textit{diminish}\\ \textit{increase}\end{matrix}$ *the Policy-values, provided that the Normal Policy-values are always increasing algebraically from year to year.*

25. In the case of an ordinary Whole-Life Policy the condition that V shall be continually increasing will be satisfied if a constantly diminish as x increases, which is in agreement with the result of the analytical demonstration given in the *Text-Book*, Part II, Chapter xviii, Article 70. Thus it is seen that the analytical condition there obtained for a particular kind of Policy is a special case of a much more general result. It may be useful to give a numerical illustration showing that if the condition is not fulfilled the results may be irregular. This will appear clearly from the following Table, based on the Mortality Table appended to the *Text-Book*, Part II.

TABLE C.

Showing the values of P_0, ${}_1V_0$, ${}_2V_0$, *and* ${}_3V_0$. *Text-Book Mortality Table.*

Rate of Interest	P_0	${}_1V_0$	${}_2V_0$	${}_3V_0$
3 per-cent	·01609	−·1086	−·1351	−·1476
4½ ,,	·01630	−·1079	−·1356	−·1497
5 ,,	·01656	−·1075	−·1354	−·1497
6 ,,	·01726	−·1065	−·1344	−·1493

26. Here it is seen that the Policy-values are negative and algebraically decreasing, with the result (in these particular cases) that the Pure Premiums *increase* as the rate of interest increases, while the Policy-values sometimes vary in one direction, and at other times in the contrary direction.

IV. VARIATIONS IN THE RATE OF *Mortality*, THE RATE OF INTEREST REMAINING UNCHANGED.

27. Putting $i'=i$ in the Equation of Equilibrium, there results

$$(P'-P)(1+i)=(q'-q)S+(p'-p)V_1 \quad . \quad . \quad (11)$$

which expresses that if the Normal Basis and the Special Basis produce identical Policy-values, the Excess Premium accumulated

to the end of the year must provide the Excess Claims then payable *plus* the Excess Reserves for the continuing Policies. For all cases in which $p+q=p'+q'=1$ (*see* Article 6) this relation may be put into the form

$$(P'-P)(1+i)=(q'-q)(S-V_1) \quad . \quad . \quad . \quad (12)$$

whence

$$q'-q=\frac{(P'-P)(1+i)}{S-V_1}\left(\text{varying as } \frac{1}{S-V_1} \text{ if } i \text{ constant}\right) \quad . \quad (13)$$

and this is the condition required to hold good each year in order to produce equivalent Policy-values. In the special case of an ordinary Whole-Life Policy assuring unity on a life aged x at entry, the result shows that the excess rate of mortality which will leave Policy-values unchanged must be equal for the $(n+1)$th year to

$$\frac{(1+i)(P'-P)}{1-V_{n+1}}, \text{ i.e., to } (1+i)(P'-P)\frac{1+a_x}{1+a_{x+n+1}} \quad . \quad (14)$$

i.e., proportional to $\dfrac{1}{1+a_{x+n+1}}$.

28. This may be shown to agree with the results obtained in the *Text-Book*, Part II, Chapter xviii, Articles 56–68. It is there shown that

$$p'_{x+n}=p_{x+n}\left(1-\frac{k}{a_{x+n}}\right)$$

$$=p_{x+n}-p_{x+n}\frac{k}{vp_{x+n}(1+a_{x+n+1})}.$$

Therefore $\qquad p'_{x+n}-p_{x+n}=-\dfrac{(1+i)k}{1+a_{x+n+1}},$

or $\qquad q'_{x+n}-q_{x+n}=\dfrac{(1+i)k}{1+a_{x+n+1}},$

which is proportional to $\dfrac{1}{1+a_{x+n+1}}$.

29. It is shown in the *Text-Book*, Chap. xviii, Article 51, that—

$$P'=(1+k)P+kd,$$

or $\qquad P'-P=kP+kd=k(P+d),$

whence $\quad (1+i)(P'-P)(1+a_x)=(1+i)k,$

so that $$q'_{x+n}-q_{x+n}=\frac{(1+i)k}{1+a_{x+n+1}}=\frac{(1+i)(P'-P)(1+a_x)}{1+a_{x+n+1}}. \quad (15)$$

Thus it is again seen that the analytical condition obtained for a particular kind of Policy is but a special case of a very general principle.

30. The essential point of the results obtained above is that the Excess Mortality is *proportional* to the Sum Assured less the Reserve at the end of the year, *i.e.*, to the Death Strain at risk. *Hence if any given difference in the rate of mortality leave Policy-values unchanged, any multiple of that difference, whether added to or subtracted from the normal rate of mortality, will also leave Policy-values unchanged.*

31. If the Excess Mortality take such a value that $(q'-q)S+(p'-p)V_1$ varies in comparison with $(P'-P)(1+i)$, instead of being always equal to that quantity, then the Special Reserves will differ from the Normal Reserves and the Critical Function will be

$$[\text{Excess Strain}]=[(q'-q)S+(p'-p)V_1].$$

The general investigation in Articles 6–15 shows that if this function is continually $\genfrac{}{}{0pt}{}{\text{increasing}}{\text{decreasing}}$, the Special Reserves will be throughout $\genfrac{}{}{0pt}{}{\text{greater}}{\text{less}}$ than the Normal Reserves.

32. Consider now the class of Policies (previously referred to) for which $p+q=p'+q'=1$, so that the Excess Strain $=(q'-q)(S-V_1)$, and suppose that $q'-q$ has a fixed value. If $(S-V_1)$ be continually diminishing, the Excess Strain, which $=(S-V_1)(q'-q)$, will be continually $\genfrac{}{}{0pt}{}{\text{decreasing}}{\text{increasing}}$ algebraically according as $q'-q$ is $\genfrac{}{}{0pt}{}{\text{positive}}{\text{negative}}$, and it has been shown that under these conditions the Special Reserves will be $\genfrac{}{}{0pt}{}{\text{less}}{\text{greater}}$ than the Normal Reserves. *Thus, whatever the form of benefit, if $p+q=1$ a constant* $\genfrac{}{}{0pt}{}{\textit{increase}}{\textit{decrease}}$ *in the rate of Mortality will* $\genfrac{}{}{0pt}{}{\textit{diminish}}{\textit{increase}}$ *the Policy-values provided that $(S-V_1)$ is constantly decreasing from year to year.*

33. Next, take the case in which (still considering the class of Policies for which $p+q=1$) the Excess Mortality is such that $p'=(1+k)p$, where k is any constant, positive or negative; then $q'-q=p-p'=-kp$. If p be always decreasing in numerical value $q'-q$ or $-kp$ will be so too: and if $S-V_1$ have a continually diminishing positive value, $(q'-q)(S-V_1)$, the Excess Strain, will always be diminishing in numerical value, and will always have the same sign as $q'-q$, *i.e.*, the opposite sign to k. Thus the Excess Strain, which in this case is the Critical Function, will have a continually $\begin{matrix}\text{increasing}\\\text{decreasing}\end{matrix}$ value according as k is $\begin{matrix}\text{positive}\\\text{negative}\end{matrix}$. It has been shown that under these conditions the Special Reserves will be $\begin{matrix}\text{greater}\\\text{less}\end{matrix}$ than the Normal Reserves. *Thus, for all benefits giving $p+q=1$ the effect of $\begin{matrix}\textit{increasing}\\\textit{diminishing}\end{matrix}$ p by a constant percentage will be to $\begin{matrix}\textit{increase}\\\textit{diminish}\end{matrix}$ the Policy-values, provided that* $(S-V_1)$ *be always positive and that p and* $(S-V_1)$ *be continually decreasing.* This result is stated for the particular case of a Whole-Life Policy in the *Text-Book*, Chapter xviii, Articles 59 and 60, and a numerical illustration will be found in the *J.I.A.*, xxiii, page 390.

34. Considering now Ordinary Whole-Life Policies, let $q'-q=kq$, *i.e.*, let the rate of mortality be increased or diminished in a constant ratio. Then if q, and therefore kq, or $q'-q$, be continually increasing and $(1-V_1)$ continually decreasing, the product $kq(1-V_1)$ may *primâ facie* be either increasing or decreasing, and it is necessary to investigate which is the case. Turning to the Fundamental Equation, Article 6, it will be seen that

$$q(1-V_1)=P(1+i)+V(1+i)-V_1$$

$$\therefore\quad \Delta[q(1-V_1)]=(1+i)\Delta V-\Delta V_1$$

$$=i\Delta V-\Delta^2 V$$

$$=(P+d)(\Delta^2 a-i\Delta a).$$

Therefore

$$\Delta[q(1-V_1)] \text{ is } +,\ 0, \text{ or } -, \text{ as } \Delta^2 a-i\Delta a \text{ is } +,\ 0, \text{ or } -,$$

i.e., as $\Delta^2 a >=< i\Delta a$ *.

* This result has been taken by the Author from Question 17 in the Examination Paper for Part III of the Institute Examinations, 1904.

Thus $kq\ (1-V_1)$ or $(q'-q)(1-V_1)$, will be increasing, constant or diminishing, according as $\Delta^2 a >=< i\Delta a$, and it will have the same sign as k because $(1-V_1)$ must necessarily be positive. Hence, if $\Delta^2 a$ be always $\genfrac{}{}{0pt}{}{\text{greater}}{\text{less}}$ than $i\Delta a$ the Excess Strain will be algebraically $\genfrac{}{}{0pt}{}{\text{increasing}}{\text{decreasing}}$ and the Special Reserves will be $\genfrac{}{}{0pt}{}{\text{greater}}{\text{less}}$ than the Normal Reserves.

35. *Thus, for ordinary Whole-Life Policies the effect of increasing or diminishing the rate of mortality by a constant percentage will be to change the Policy-values in the same direction or in the reverse direction, according as* $\Delta^2 a$ *is* $\genfrac{}{}{0pt}{}{\textit{greater}}{\textit{less}}$ *than* $i\Delta a$.* An example will be found in the *J.I.A.*, xxiii, page 389, showing that in the particular case there dealt with the effect of increasing the rate of mortality by 5 per-cent throughout is to increase Policy-values. In the case of the H^M Table it will be found that, for ages over 30, $\Delta^2 a$ is nearly always greater than $i\Delta a$, so that the effect of $\genfrac{}{}{0pt}{}{\text{increasing}}{\text{diminishing}}$ the rate of mortality in a constant ratio will be to $\genfrac{}{}{0pt}{}{\text{increase}}{\text{decrease}}$ the Policy-values for lives over that age at entry.

36. Dr. Sprague enunciated (*J.I.A.*, xxi, 109-110) the following conclusions, based upon an examination of the Whole-Life Policy-values produced by particular Mortality Tables:

"It therefore seems that we may fairly draw the following "conclusions; (1) if two tables show the same mortality at "young ages, and at higher ages an increasing difference in the "rate of mortality, then the one which shows the higher rate of "mortality will require larger Policy-values; (2) if two tables "show the same mortality at high ages, but an increasing "divergence as we proceed to younger ages, then the table which "shows the lower mortality at younger ages will require larger "Policy-values; and (3) if two tables, A and B, show the same "rate of mortality at the middle ages, say about 50, but at "younger ages the Table A shows the higher mortality and at "higher ages the lower mortality, then Table A will require the "lower Policy-values."

* When Tables of Policy-values are available, the criterion $(1+i)\Delta V >=< \Delta V_1$ will be found more convenient.

37. These conclusions, based merely on observation of particular cases of a limited class of benefit, may be shown by the method now under discussion to be very generally true; and at the same time their limitations may be indicated. The investigation will be restricted to Benefits for which $p+q=1$.

38. Let the Table stated to produce the lower Policy-values be treated as the Normal Basis. Then in case (1), the Excess Mortality of the other Table, namely, $q'-q$ will be first zero, then positive and numerically increasing. The value of $(S-V_1)$ will usually be positive and numerically decreasing: and if the rate of decrease be lower than the rate at which $q'-q$ is increasing, the Critical Function $(q'-q)(S-V_1)$ will be increasing algebraically.

39. In case (2), $q'-q$ will be negative and numerically decrease until it reaches zero. If $(S-V_1)$ be positive and numerically decreasing, $(q'-q)(S-V_1)$ must therefore be increasing algebraically.

40. In case (3), $q'-q$ will be first negative and numerically decreasing, then zero, afterwards positive and numerically increasing. Up to the zero point $(q'-q)(S-V_1)$ will be algebraically increasing, provided only that $(S-V_1)$ be positive; after that point it will be algebraically increasing provided $(S-V_1)$ be (*a*) positive (*b*) decreasing numerically at a lower rate than that at which $q'-q$ is increasing.

41. Thus in each case, subject to the specified limitations, the Critical Function is increasing, and the Special Basis, therefore, produces the higher Policy-values.

42. There is one special case of particular importance which should be mentioned here. From the nature of the construction of the O^M Table as compared with the $O^{M(5)}$ Table, these tables fall into Case (2). *Thus for all classes of benefits giving $p+q=1$, and $(S-V_1)$ continually decreasing, the O^M Policy-values must be higher than the $O^{M(5)}$ Policy-values for all durations, and for all ages at entry up to 74 inclusive. For older ages the two Tables, and therefore the Policy-Values, are practically identical.*

V. Modifications of the preceding Investigations when the Benefits and the Premiums are payable continuously.

43. The results hitherto obtained have been deduced on the assumption that the Premiums are payable at the beginning, and the Benefits at the end, of each year. For purposes connected with Industrial Assurances it is desirable to modify the formulæ

to meet the case of Premiums and Benefits payable continuously. This may be done without difficulty, and it will be found that the formulæ are in reality simplified through the absence of any distinction between the beginning and end of the momently intervals under consideration.

44. Turning to the Fundamental Equation in Article 6, it will be seen at once that if the interval be reduced from a year to $\frac{1}{m}$ year, the formula will become—

$$V\left(1+\frac{j}{m}\right)+\frac{P^{(m)}}{m}\left(1+\frac{j}{m}\right)=\frac{q}{m}S+{}_{\frac{1}{m}}p.V_{\frac{1}{m}} \quad . \quad . \quad (16)$$

in which j represents the nominal rate of Interest convertible $m$ly, $P^{(m)}$ the Annual Premium payable $m$ly, $\frac{q}{m}$ the chance of a claim arising at the end of the interval of $\frac{1}{m}$ year, and ${}_{\frac{1}{m}}p$ the chance of the status remaining in existence at the end of the interval. The Equation may be transformed as follows:

$$\frac{j}{m}V+\frac{P^{(m)}}{m}\left(1+\frac{j}{m}\right)=\frac{q}{m}S+{}_{\frac{1}{m}}p.V_{\frac{1}{m}}-V$$

$$=\frac{q}{m}S+(V_{\frac{1}{m}}-V)-(1-{}_{\frac{1}{m}}p)V_{\frac{1}{m}}$$

$$=\frac{q}{m}S+(V_{\frac{1}{m}}-V)-({}_0p-{}_{\frac{1}{m}}p)V_{\frac{1}{m}},$$

$$i.e., \quad jV+P^{(m)}\left(1+\frac{j}{m}\right)=qS+\frac{V_{\frac{1}{m}}-V}{\frac{1}{m}}-\frac{{}_0p-{}_{\frac{1}{m}}p}{\frac{1}{m}}V_{\frac{1}{m}} \quad . \quad . \quad (17)$$

45. If the interval $\frac{1}{m}$ be made infinitely small, this becomes—

$$\delta\bar{V}+P=\mu S+\left[\frac{d}{dt}\bar{V}_t\right]_{t=0}+\left[\frac{d}{dt}{}_tp\right]_{t=0}.V \quad . \quad . \quad (18)$$

which will be the new Fundamental Formula. If a different basis of valuation, denoted by accented symbols, give identical Policy-values, we shall have—

$$\delta'\bar{V}+P'=\mu'S+\left[\frac{d}{dt}\bar{V}_t\right]_{t=0}+\left[\frac{d}{dt}{}_tp'\right]_{t=0}V \quad . \quad . \quad (19)$$

and the new Equation of Equilibrium will be—

$$(\delta'-\delta)\overline{\mathrm{V}}+(\overline{\mathrm{P}}'-\overline{\mathrm{P}})=(\mu'-\mu)\mathrm{S}+\left[\frac{d}{dt}({}_tp'-{}_tp)\right]_{t=0}\mathrm{V} \quad . \quad . \quad (20)$$

that is, $\overline{\mathrm{P}}'-\overline{\mathrm{P}}=(\mu'-\mu)\mathrm{S}+\left[\frac{d}{dt}({}_tp'-{}_tp)\right]_{=0}\mathrm{V}-(\delta'-\delta)\overline{\mathrm{V}} \quad (21)$

or [Excess Premium] = [Excess Strain] − [Excess Interest].

By means of this Equation the results for Continuous Benefits may be obtained by reasoning precisely similar to that previously used for Annual Benefits. It is unnecessary to give the processes in full, but the values of the Critical Functions and the results obtained for the principal cases are collected together, and exhibited side by side with those already obtained, in the Summary appended to this Paper.

VI. Introduction of the Rate of *Discontinuance*, the Rate of Interest and the Rate of Mortality remaining unchanged.

46. So far as the writer is aware no general theoretical investigation has yet been given dealing with the effect of introducing the rate of Discontinuance in the calculation of Net Premiums and Reserve Values. Mr. Hunter, *J.I.A.*, xxxvi, pp. 51–2, quotes the following remarks by Mr. King :

> "The company which takes account of discontinuances in calculating its premiums requires to make—and I can prove it by mathematics—a considerably higher reserve than a company charging the ordinary level premiums. . . A premium calculated to allow for discontinuances is naturally lower than the premium calculated for mortality only. . . Now it is found that discontinuances take place in great proportion in the early days of the policies, and that when policies have been in existence, say, ten or fifteen years, discontinuances practically cease. . . Now the premium being lower than the ordinary level premium and the risk being the same [*i.e.*, after the discontinuances have ceased—G. J. L.], and the premium being part of the assets relied upon by the office in order to meet the claim, it follows that the other part of the assets, namely, the reserves, must be increased. So that if the premium is reduced by the application of lapses to lower the premium, then the reserves, on the other hand, must be increased. . ."

47. It does not appear that the mathematical proof in question has ever been published, and Mr. King's general reasoning—while it clearly shows that, if discontinuances entirely cease after a number of years, the Policy-values will thereafter be increased if the net Premiums and Reserves are modified to allow for Discontinuances—does not enable one to determine whether in the early years Reserves will be increased or diminished. On this point Messrs. Ackland and Bacon, in their paper, *J.I.A.*, xxxviii, 560, remark—" It does not appear to us to be " demonstrable that in the case of a Net Premium Valuation the " introduction of the element of lapse will always tend to increase " reserves."

48. The fact appears to be that just as special rates of Mortality and Interest may be found which will produce Policy-values either equal to, greater than, or less than Normal Policy-values, so there must be rates of Discontinuance which will produce similar results. This will now be shown to be the case, and it will be found that the new method of investigation enables the question to be treated with great facility, although a purely analytical method would be difficult, if not impracticable, owing to " the fact that the usual relation does not hold between the " annuity with allowance for lapse and mortality and the single " premium for the assurance payable on death (with allowance " for lapses)."

49. First suppose that all the withdrawals take place at the end of the year, immediately after the claims of the year have been paid: in this case the rate of withdrawal will be what Mr. Todhunter has called the " Rate of Non-Renewal." Let w be the rate, so that for each status in force at the beginning of the year wp will withdraw at the end, and let the surrender allowance be (SV); then the profit on withdrawals for the nth year will be $wp[\mathrm{V}_1-(\mathrm{SV})]$, and if this have a constant value it must be just sufficient to counterbalance the loss of Premium accumulated to the end of the year. Thus the following relation must hold each year, namely:

$$(\mathrm{P}-\mathrm{P}')(1+i)=wp[\mathrm{V}_1-(\mathrm{SV})]\ ;\ \text{or}\ w=\frac{(\mathrm{P}-\mathrm{P}')(1+i)}{p[\mathrm{V}_1-(\mathrm{SV})]}. \quad . \quad (22)$$

In these circumstances the Special Policy-values will be equal throughout to the Normal Policy-values, whatever the form of benefit may be. Take, for example, a case which admits of easy numerical verification, namely, a 15-year Endowment Assurance

on a life aged 35 at entry—Normal Basis H^M 3 per-cent, and suppose that there is no allowance on Withdrawal, so that $(SV)=0$. Then the following rates of non-renewal will reduce P by ·00971, [*i.e.* $(P-P')(1+i)=\cdot01$], but will leave Policy-values unaltered —a relation which has been verified by actual calculation.

Table D.

Rates of Non-Renewal which will leave Policy-values unaltered in the case of a 15-year Endowment Assurance on a life aged 35 at Entry. Normal Basis H^M *3 per-cent.*

Year	Rate of Non-Renewal
1	·1977
2	·0973
3	·0638
4	·0470
5	·0370
6	·0303
7	·0255
8	·0219
9	·0191
10	·0168
11	·0150
12	·0135
13	·0122
14	·0215*
15	·0000*

* The strict application of the formula would give the values ·0111 and ·0102 for the 14th and 15th years, but as it would be absurd to suppose withdrawals to take place at the moment when the Policy is maturing, a modification has been introduced, with the result that $\mathbf{V}'_{14}=\mathbf{V}_{14}$, instead of $V'_{14}=V_{14}$.

50. But if the rate of withdrawal take such a value that $-wp(V_1-SV)$ varies in value as compared with $(P'-P)(1+i)$, instead of being always equal to that quantity, it will be seen (as in the case of variations in mortality or interest) that $(P'-P)(1+i)$ must lie between the greatest and least values of $-wp(V_1-SV)$, which will therefore be the "Critical Function." And without repeating the formal demonstration at length, it will be clear that according as $-wp(V_1-SV)$ is $\begin{matrix}\text{increasing}\\ \text{decreasing}\end{matrix}$ (*i.e.*, according as the lowest values come in the early years or in the later years), there will be a *surplus* Premium in the early years to be carried forward and accumulated (at interest and with benefit of survivorship and continuance) in order to meet the deficiency of

Premium in subsequent years; or a *deficiency* in the early years which must be similarly accumulated and gradually repaid out of the Surplus Premium of the later years. In the one case the Special Policy-values (*i.e.*, those in which the element of discontinuance is introduced) will be greater, and in the other case less, than the Normal Policy-values.

51. Passing to the case of Benefits and Premiums payable continuously, the force of discontinuance must also be supposed to act continuously. Let $\overline{w}$ represent the continuous rate or "Force" of discontinuance: then it is evident that the Critical Function will be $-\overline{w}(\text{V}-\text{SV})$, and, according as this is increasing, constant or diminishing, the Special Policy-values will be greater than, equal to, or less than the Normal Policy-values.

52. Since it is only the progression of the Critical Function that is in question, and not its absolute value, it is evident that if it be increased or diminished in a constant ratio at all ages, the *direction* of the change in the Policy-values will not be altered. Again if the surrender-value be zero, the Critical Function $=-wp\text{V}_1$ for Annual Policies, or $-\overline{w}\text{V}$ for Continuous Policies; and if the surrender-value be a fixed proportion, say k, of the Normal Policy-value, the Critical Function $=-wp(\text{V}_1-k\text{V}_1)=-wp(1-k)\text{V}_1$ for Annual Policies, or $-\overline{w}(1-k)\text{V}$ for Continuous Policies; so that in the case both of the Annual and the Continuous Policies, if the surrender-value be k times the Normal Policy-value, the Critical Function is $(1-k)$ times (*i.e.*, a constant proportion of) the Critical Function applicable to the case in which no surrender-values are allowed.

53. *Hence, if any given rate of non-renewal for Annual Policies, or any given force of withdrawal for Continuous Policies, be found to produce* (1) *no change, or* (2) *an increase, or* (3) *a decrease in Policy-values, when any given fixed proportion of the Normal Value is allowed on surrender; the same rate or force* (*or any constant multiple thereof*) *will produce a similar effect* (*i.e.*, *a change in the same direction*) *if the surrender-value be any fixed proportion whatever of the Normal Value.**

* Table XXIII appended to Messrs. Bacon and Ackland's Paper (*J.I.A.*, xxxviii, 605) shows an apparent exception for duration 0, but the explanation appears to be (1) that a halved lapse *rate* is not consistent with a halved *force* of lapse, and the difference is very marked for duration 0; (2) that the Surrender-value there dealt with was *not* a fixed proportion of the Normal Reserve.

54. As an illustration of the application of the Critical Function to determine the relation between the Policy-values with and without the introduction of the lapse element, it will be convenient to take examples from the tables appended to the paper by Messrs. Ackland and Bacon already referred to. The Policy-values there given in Tables XIX to XXIV are the *Continuous* Policy-values (Form $\overline{V}$), and the Critical Function will therefore be $-\overline{w}(\overline{V}-\overline{SV})$, where $\overline{SV}$ equals zero for durations under 5 years, and equals 25 per-cent of the aggregate office premiums paid for greater durations. It is therefore necessary to calculate $\overline{w}$, since the figures given in Table I (*J.I.A.*, xxxviii, 570–1) are the Annual Rates of withdrawal amongst those who do not die in the year, which may be denoted by r. The values of $\overline{w}$ have been found from the approximate formula $\overline{w}_{t+\frac{1}{2}}=\dfrac{r_t}{1-\frac{1}{2}r_t}$, which will be sufficiently accurate for the purpose, although for the earliest durations no practical approximation will be very close to the exact value. It will be convenient to take specimen ages at entry, 1, 35, and 75 next birthday, corresponding to the assumed exact ages, $\frac{1}{2}$, $34\frac{1}{2}$, and $74\frac{1}{2}$ at entry. The Policy-values tabulated under curtate duration 0, 1, 2 . . . correspond to assumed completed durations $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$ . . . The values of $\overline{w}$ at the points $\frac{1}{2}$, $1\frac{1}{2}$ . . . will be found by the formula already given, namely, $\overline{w}_t=\dfrac{r_t}{1-\frac{1}{2}r_t}$, and for this purpose the values of r will be taken from the columns headed a in Messrs. Ackland and Bacon's Table I.

55. With these preliminary explanations the following Table showing the derivation of the Critical Function will be easily followed.

TABLE E.

Shewing the derivation of the Critical Function in the case of a Whole-Life Policy assuring a maximum of 100 (*with reduced benefit for the first* 10 *years in the case of Age* [1] *at Entry*—vide *J.I.A.*, xxxviii, 551). *Normal Basis, English Life No.* III, 3 *per-cent.*

	Curtate Duration	Average Completed Duration	$\overline{w}$	$\overline{V}$	$\overline{SV}$	$\overline{V}-\overline{SV}$	$-\overline{w}(\overline{V}-\overline{SV})$ =Critical Function	$(\overline{P}'-\overline{P})$ $-$ $[-\overline{w}(\overline{V}-\overline{SV})]$ $=\mathcal{R}$
Age at Entry 1 n.b.d. (0½)					+			
	0	½	·532	+ ·246	...	+ ·246	− ·131	+ ·208
	1	1½	·299	− ·453	...	− ·453	+ ·135	− ·058
	2	2½	·151	− ·514	...	− ·514	+ ·077	...
	3	3½	·111	− ·294	...	− ·294	+ ·033	+ ·044
	4	4½	·0833	+ ·083	...	+ ·083	− ·007	+ ·084
	5	5½	·0725	+ ·594	2·979	− 2·385	+ ·173	− ·096
	10	10½	·0492	+ 3·861	5·688	− 1·827	+ ·090	− ·013
	15	15½	·0377	+ 8·161	8·397	− ·236	+ ·009	+ ·068
	20	20½	·0284	+ 12·316	11·106	+ 1·210	− ·034	+ ·111
	25	25½	·0222	+ 16·375	13·815	+ 2·560	− ·057	+ ·134
	30	30½	·0187	+ 20·894	16·523	+ 4·371	− ·082	+ ·159
	35	35½	·0165	+ 25·943	19·232	+ 6·711	− ·111	+ ·188
Age at Entry 35 n.b.d. (34½)				+	+	+	−	
	0	½	·941	·669	...	·669	·630	+ ·264
	1	1½	·469	2·062	...	2·062	·967	+ ·601
	2	2½	·192	3·485	...	3·485	·668	+ ·302
	3	3½	·120	4·934	...	4·934	·591	+ ·225
	4	4½	·0888	6·410	...	6·410	·569	+ ·203
	5	5½	·0736	7·918	5·569	2·349	·173	− ·193
	10	10½	·0403	15·864	10·631	5·233	·211	− ·155
	15	15½	·0243	24·471	15·694	8·777	·213	− ·153
	20	20½	·0141	33·521	20·756	12·765	·180	− ·186
	25	25½	·0088	42·961	25·819	17·142	·151	− ·215
	30	30½	·0073	52·471	30·881	21·590	·158	− ·208
	35	35½	·0058	61·466	35·944	25·522	·148	− ·218
Age at Entry 75 n.b.d. (74½)				+	+	+		
	0	½	·424	2·324	...	2·324	− ·986	+ ·593
	1	1½	·208	6·936	...	6·936	− 1·439	+ 1·046
	2	2½	·0566	11·373	...	11·373	− ·644	+ ·251
	3	3½	·0450	15·638	...	15·638	− ·704	+ ·311
	4	4½	·0356	19·729	...	19·729	− ·702	+ ·309
	5	5½	·0305	23·647	35·050	− 11·403	+ ·348	− ·741
	10	10½	·0101	40·726	66·914	− 26·188	+ ·264	− ·657

56. Considering first age at entry 35, as representative of entry-ages neither very young nor very old, it is seen that the Critical Function is *negative* throughout, *i.e.*, the withdrawals produce a profit at each duration; and although the values progress somewhat irregularly, they are clearly on the whole numerically decreasing and therefore algebraically increasing. Hence it may be concluded, without calculating $\overline{P}'$, that the Special Policy-values (*i.e.*, those including the element of lapse) will be *greater* than the Normal; because the numerically decreasing profit is equated to a constant diminution of Premium and the *excess* in the early years is added to reserves and carried forward to meet the *deficiency* in later years. This is shown more conclusively by the last column, giving the values of $\mathbb{E}$, which involve the values of $\overline{P}'$. The Table shows that $\mathbb{E}$ is *positive* up to curtate duration 4 and thereafter negative; and hence (*see* Article 14) the Special Policy-values will be the greater throughout. The numerical examples given in Table XXI, *J.I.A.*, xxxviii, 603, show that this is actually the case.

57. Taking next age 75 at entry, the Critical Function is *negative* under duration 5 (*i.e.*, while no surrender-values are payable) and thereafter positive; in other words, the withdrawals produce a profit in the early years and a loss thereafter; and $\mathbb{E}$ is first positive and afterwards negative. Therefore in this case also the Special Policy-values must be the greater, as they are seen to be in Table XXIII, *J.I.A.*, xxxviii, 605.

58. These examples are sufficient to show that, when the Critical Function progresses with regularity, the theory will predict and explain the general relations between the Special and Normal Policy-values; and it becomes clear why the rapidly-diminishing force of withdrawal which is experienced in practice results in enhanced reserves. It is otherwise, however, with age at entry 1. There the Policy-values are sometimes positive and at other times negative, while even in the former case they are occasionally less than the guaranteed Surrender-value, and the result is seen in the great irregularity of the Critical Function, which changes sign no less than four times. In these circumstances, the Critical Function alone can hardly be expected to throw much light on the relations between the Policy-values, and it need be no matter for surprise that these relations are themselves somewhat anomalous, as pointed out in Messrs. Ackland and Bacon's Paper. To the present writer, however, it would appear that these anomalies are not in their entirety inherent in the problem,

but that they must be in some measure due to the inevitable inexactness of approximations to the values of functions which involve a rapidly changing lapse rate. This view of the matter is founded on the following considerations.

59. From the point of entry up to curtate duration 0 (*i.e.*, complete duration $\frac{1}{2}$) the Normal Policy-value V is positive throughout.* Hence, whatever the precise values of w, which are somewhat uncertain, $w(\mathrm{V}-\mathrm{SV})$, *i.e.*, $w\mathrm{V}$ since SV is zero, is positive, and according to Table XXIV of Messrs. Ackland and Bacon's Paper $\overline{\mathrm{P}}'-\overline{\mathrm{P}}$ has also a positive value, namely, ·077 per-cent. It follows that $(\overline{\mathrm{P}}'-\overline{\mathrm{P}})+w\overline{\mathrm{V}}$, *i.e.*, E, must be positive at all points from the date of entry up to complete duration $\frac{1}{2}$, and therefore the Special Policy-values must be greater than the Normal during that interval.

60.* The same argument may be expressed in more general language. For the period under consideration (namely, from entry to complete duration $\frac{1}{2}$) the Lapse-Basis values are credited with a higher net Premium than the Normal, and in addition a profit is made on each lapse, because the positive reserve in hand against the lapsed Policy is released. At the end of the period the accumulated excess Premium *plus* the accumulated profit on lapses will be in hand in addition to the Normal Policy-values for the continuing Policies, that is to say, the Lapse-Basis Policy-values must be greater than the Normal. The values are, however, given in Ackland and Bacon's Table XIX as ·246 per-cent excluding lapse rate, and ·019 per-cent including full lapse rate, so that the Special Policy-value is shown as less than the Normal—contrary to the general results obtained above.

61. It would appear, therefore, that the approximate methods used *faute de mieux*, in calculating the Policy-values, involve certain inaccuracies which disturb to some extent the natural relations between the Special and Normal Policy-values, and a glance at the column headed E in Table E would suggest that these disturbances are not improbably caused by an over-estimate of the value of $\overline{\mathrm{P}}'$, the Pure Premium on the Lapse Basis. Any such over-estimate would, of course, to some extent affect the Policy-values throughout the Table.

VII. Concluding Remarks.

62. The preceding description of the new method of investigation and the examples of its application already given are sufficient

* *See* Remarks on pp. 248, 249.

to show that it is one of considerable power and range of usefulness. The main idea—that of directly tracing out the effect upon pure premiums and reserves, caused by changes in the fundamental basis, instead of dealing analytically with mathematical expressions for such premiums and reserves—is one of very general applicability. It has, for example, been applied (*J.I.A.*, xxxiii, 354) by the present writer to obtain a very simple approximation to the amount of the pure premium for a Joint-Life Endowment Assurance—an approximation which is so close as to be amply sufficient for all practical purposes. The method will also find a useful application in dealing with questions of extra risk, and many of the general results given in the well-known papers on this subject by Mr. Ryan, and by Messrs. White and Whittall, may be deduced and illustrated by its means. Time, however, does not permit of the discussion of these further ramifications of the subject, and the author, therefore, now concludes this paper with the expression of the hope that the method which has often proved useful to himself in his daily practice, may also be found of service to others who have to deal with similar problems, as well as to students of the theory of the subject.

IN

IT

	...URE OF THE CRITICAL FUNCTION WHICH WILL ENSURE THAT THE SPECIAL POLICY-VALUES SHALL BE THROUGHOUT		
	L to Normal	LESS than Normal	GREATER than Nor...
. .	...onstant	Diminishing	Increasing
or $\mu = -\frac{d}{dt}p$.	,,	,,	,,
	. ...	C or $\bar{C}$ positive	C or $\bar{C}$ negative
...n Benefits and	...nstant	Diminishing	Increasing
	...	k negative	k positive
	...tant, *i.e.*, $\dots V_n = \Delta V_{n+1}$ $\dots \bar{V}_n = \frac{d^2}{dn}\bar{V}_n$	Diminishing, *i.e.*, $k(1+i)\Delta V_n < k\Delta V_{n+1}$ or $k\delta\frac{d}{dt}\bar{V}_n < k\frac{d^2}{dn}\bar{V}_n$	Increasing, *i.e.*, $k(1+i)\Delta V_n > k\Delta V_{n-\dots}$ or $k\delta\frac{d}{dn}\bar{V}_n > k\frac{d^2}{dn}\bar{V}\dots$
. .	...nstant	Diminishing	Increasing
. .	$\dots' = i$	$i' > i$	$i' < i$
Benefits and	,,	,,	,,

on
he
it
he
·d.
on
ne
ed
nt
·'s
ns
y.
Its
ch
w,
of
ng
er,
y-
rs
nd
al
he
he
of
he
nd
ne
er,
ct
at
ne
er
to
er,
ny
as
nd
n.
ct
re
as
ry
nd
e.
nt
ng
er
en
as

SUMMARY OF RESULTS

IN RESPECT OF VARIATIONS IN THE RATE OF MORTALITY OR IN THE RATE OF INTEREST

	Art. [illegible]	Nature of Benefit	Special Conditions: Annual Method	Special Conditions: Continuous Method	Critical Function: Annual Method	Critical Function: Continuous Method	Nature of [illegible]: [illegible]	[illegible]: Less	[illegible]: Greater
Variations in the Rate of Mortality, the Rate of Interest remaining unchanged	Art. 31, 45	All Classes			$(q'_n-q_n)S_n+(p'_n-p_n)V_{n+1}$	$(\mu'_n-\mu_n)S_n+\frac{d}{dt}(p'_n-p_n)\bar{V}_n$	Constant	Diminishing	Increasing
	32	Do. such that $p+q=1$, or $\mu=-$ [illegible]			$(q'_n-q_n)(S_n-V_{n+1})$	$(\mu'_n-\mu_n)(S_n-\bar{V}_n)$			
	[illegible]	Do. do.	q'_n-q_n constant and $=C$; S_n-V_{n+1} decreasing and positive	$\mu'_n-\mu_n$ constant and $=C$; $S_n-\bar{V}_n$ decreasing and positive	$C(S_n-V_{n+1})$	$C(S_n-\bar{V}_n)$		[illegible]	C [illegible]
	[illegible]	Ordinary Whole-Life, Uniform Benefits and Premiums			$(q'_n-q_n)(1-V_{n+1})$	$(\mu'_n-\mu_n)(1-\bar{V}_n)$	Constant	Diminishing	Increasing
	33	Do. do.	$p'_n=(1+k)p_n$, k constant, p decreasing		$-k(p_n)(S_n-V_{n+1})$			[illegible]	k positive
	34	Do. do.	$q'_n-q_n=kq_n$, k constant	$\mu'_n-\mu_n=k\mu_n$, k constant	$kq_n(1-V_{n+1})$	$k\mu_n(1-\bar{V}_n)$	Constant, [illegible] $(1+i)\Delta V_n$ [illegible] ΔV_n [illegible]	Diminishing, [illegible] ΔV_n [illegible] ΔV_{n+1} [illegible]	Increasing, [illegible] $(1+i)\Delta V_n$ [illegible] $k\Delta V_{n+1}$ [illegible]
Variations in the Rate of Interest, the Rate of Mortality remaining unchanged	35	All classes			$V_{n+1}(i'_n-i_n)$	$\bar{V}_n(\delta'_n-\delta_n)$	Constant	Diminishing	Increasing
	24	Do.	$i'-i$ constant, V increasing	$\delta'-\delta$ constant, V increasing	$V_{n+1}(i'-i)$	$\bar{V}_n(\delta'-\delta)$	[illegible]	[illegible]	[illegible]
	[illegible]	Ordinary Whole Life, Uniform Benefits and Premiums	$i'-i$ constant, i decreasing	$\delta'-\delta$ constant, δ decreasing					

Abstract of the Discussion.

Mr. J. E. Faulks said he had been asked to open the discussion on the paper, but if the word "discussion" was to be taken in the ordinary sense as implying that some of those who took part in it would bring forward points of adverse criticism, he thought all the members would agree there would be very little discussion indeed. Rather would they all join in an expression of very high appreciation of the paper, and very hearty thanks to the author. Mr. Lidstone had a habit of submitting papers, the central ideas of which seemed to be obvious after the papers had been read, not before. To put the matter rather differently, after reading some of Mr. Lidstone's papers everybody felt so thoroughly in agreement with his conclusions that their only wonder was they had not been arrived at previously. He need hardly remind the members of the very important results with regard to the practical valuation of endowment assurances, which Mr. Lidstone had shown to follow, as it seemed to everybody now, very simply and almost as a matter of course, from a consideration of the way in which the value of a temporary annuity varied according to the age, when the term remained constant. In the present paper, Mr. Lidstone had taken up the subject of the variation of policy-values, and shown how the results obtained by previous investigators were practically only illustrations of the application of broad and general principles. Bearing in mind the small domain of actuarial science as compared with the broad fields of science as a whole, he did not think he was going too far in saying that in considering the central ideas of those two papers, they were irresistibly reminded of the scientific revolutions that had been accomplished owing to the consideration by a master mind of some apparently simple and necessary occurrence—the falling apple of Sir Isaac Newton, and the steaming kettle of James Watt. The fundamental idea of the paper, the consideration of things in the concrete, instead of the abstract symbols representing those things, was, he thought, one of great interest and great power. The method suffered, perhaps, to some extent, from the fact that sooner or later in the argument, in order to avoid long and complicated explanations, it became necessary to have recourse to symbols, and there, he thought, lay a certain danger, or at any rate a certain difficulty. He had not been able to find any illustration of that in the paper itself, but in the note on premiums for joint-life endowment assurances, which was referred to at the end of the paper (see *J.I.A.*, xxxiii, 354), there was an illustration. Turning to that note, the argument could be followed with perfect clearness as long as words were employed, but when the words were translated into symbols, difficulties were apt to arise, unless it was carefully borne in mind that those symbols were not ordinary algebraic expressions, but merely a kind of description in shorthand of concrete things that had been previously explained in the note. For instance, the separation of the annual premium for an endowment assurance into two parts was followed with perfect clearness as long as words were employed, but when symbols were reached, the reader was apt, unless he bore carefully in mind the point that had been referred to, to question whether $P_{\overline{n|}}$ and $P_{x\overline{n|}}$ could be considered as

being payable for the same period.* He only referred to that difficulty, which was, probably, having regard to the limits of language, unavoidable, to show that there must be a certain amount of care in employing the method in other directions. In this paper Mr. Lidstone applied that method in the first instance to produce an Equation of Equilibrium, showing the relations that must exist between two bases to produce equal policy-values. His explanations of that equation, and of the variation fund, were as convincing as they were interesting. The variation fund had, to a certain extent, from one point of view, a counterpart in the actual working of a life assurance fund. Starting a valuation period with a policy-value calculated on a certain specified basis, and putting aside the question of loading, which could, of course, be dealt with easily in other ways, the usual method of considering the working of the fund was that the policy-value must remain on the same basis throughout. But alternatively one might perhaps consider that one operated on the policy-value by means of experienced rates throughout, instead of assumed rates—an experienced rate of interest and an experienced rate of mortality, and therefore also of premium. In that way was produced a policy-value on a kind of special basis, equivalent to the policy-value on the normal basis, plus an additional fund similar to the variation fund. At the end of the valuation period, the policy-value was twisted back, as it were, to its normal basis, and the variation fund left available for bonus or other purposes. On the application of the principle of the variation fund to the consideration of policy-values on different bases, and on the discovery—for he thought it was no less than a discovery—of the Critical Function, as it was called, by means of an inspection of which so much information could be derived, he thought there was very little to be said that was not already in the paper. The author had, for instance, shown how the question of changes in the rate of mortality as affecting policy-values could be most readily dealt with by means of that function, and how the results of previous investigators, valuable and important as they were in their day, were by the new method at one and the same time confirmed, and placed in their proper positions as simply illustrations of general theorems. Passing on to continuous functions, or functions involving the rate of discontinuance, the same principles were found to prevail. The members of the Institute had been accustomed very often to see the statement that every man was a debtor to his profession, and he thought it was a source of peculiar pride to the members that a great many of their own profession had repaid that debt, and accumulated a considerable balance on the other side of the account. Mr. Lidstone's credit balance, which was already a very large one, had been greatly added to by the interesting and valuable paper they had had the pleasure of listening to that evening.

Mr. T. E. Young said that detailed comments on the whole scope of the paper appeared almost to be needless, for every contribution which Mr. Lidstone presented to the Institute was not merely conceived in originality of motive and freshness of form, but had also evidently been so adequately tested that criticism naturally passed into praise. With regard to the

* *See* Remarks on pp. 247, 248.

method that had been so ably illustrated, it had been shown that in the first historical stage of the development of algebra, the reasoning was conducted solely on what historians had termed the Rhetorical plan, where the processes of that reasoning were verbally expressed with the conceptions of the subject directly present to consciousness, and without the mechanism of symbols or their relations. That course was absolutely necessary at that early period, for long-hand in mental investigation, equally as in handwriting, invariably preceded shorthand. Since the method which Mr. Lidstone had so well illustrated was the natural and spontaneous form in which the intellect evolved the relations of space and time, the system itself furnished the finest and surest mode—indeed, it might almost be expressed as the native mode—of mental discipline, and afforded the clearer vision of the road to truth, a steadier perception of the sequent steps in that road, and a firmer and more permanent grasp of the truth when obtained. It was an occasion to him accordingly of pleasure and congratulation that in this paper the Institute possessed so admirable a specimen, adapted to one of their professional problems, of that rigorous and illuminative method. The advantages of the plan which Mr. Lidstone had so fully mastered, and so ably exhibited, were so deep and prevailing that he thought a moment's attention was worthily devoted to their enumeration. The actual mental conceptions of Things and their relations, about which they reasoned, were retained directly and consciously before the intellect throughout the entire course of investigation; they thus reasoned concerning objects themselves, in virtue of the manner in which those objects were conceived in the mind. In the symbolic method, on the other hand, consciousness was altogether lost of the concrete things of which they treated, and the mind attended simply to the signs by which they were symbolized, so that they employed, not the real relations of things, but merely the accepted rules of combining and operating upon the symbols by which those things were represented. In fact, the difference between the two methods might be likened, with some slight exaggeration of statement, to the actual construction of a building, and a figured sketch of it. The larger generality of the method which Mr. Lidstone had so lucidly presented, was an attractive and decisive feature, and several instances occurred in the course of the paper in which results previously obtained by former enquirers were simply shown to be particular cases of the wider conclusions which Mr. Lidstone's plan indicated and reached. It was particularly interesting to observe the manner in which that extension of view was displayed in connection with Dr. Sprague's conclusions respecting the reserves for whole-life policies. While, however, pronouncing this judgment upon the efficiency of the method, he did not, of course, intend for a moment to imply any reflection upon the analytical method. He simply desired to assign its appropriate place in educational training, and in the most effective modes of enquiry. In the hands of a student who had mastered the conceptions and principles of a science, the analytical process was obviously a most potent instrument of verification and extension, accompanied by a considerably diminished amount of intellectual expenditure. But for the establishment of

those principles, for a secure and vivid grasp upon the conceptions forming the subject-matter of our reasonings, and for the most stable and intrinsic education of the intellect, the plan Mr. Lidstone had followed was conspicuous and supreme. He might add a practical remark suitable to the younger members of the profession. In their subsequent and more responsible life they would deal exclusively with realities—the realities of men's motives and actions, and of commercial affairs—and no more helpful auxiliary in that ultimate direction could be discovered than the employment of this method—in addition to, but in advance of, the analytical process—during their years of study, and the consequent habituation of the mind to its general and ready use. He should like to add also that he was particularly pleased to observe, in these days of loose and unscientific language, that Mr. Lidstone had introduced the term "critical" in his Critical Function, in the precise and accepted manner in which it was employed in mathematical physics. A practice of that nature tended not merely to clearness of thought and expression, but also aided the more effectively, in the strict sense in which Mr. Lidstone had rightly employed the term, to bring their professional studies definitely within the range of science. The members always confidently expected, and as certainly found, so much mental aid and enlightenment from the contributions of Mr. Lidstone, that Mr. Lidstone might not deem it impertinent, either in the ancient or the modern sense of that term, if he expressed the hope that he would apply his distinctive capacity of research and exposition to other problems within the range of actuarial work. Looking to the importance of the method which this paper exemplifies, the speaker had intended—had not time suggested curtailment—to add a few remarks upon the ancient controversy upon the comparative merits of the geometrical and analytical methods which culminated in Dr. Whewell's book, in 1845, upon the Elements of a Liberal Education. The author there luminously proved—and, he thought, finally—the vastly superior utility of the former method as a mental discipline and as the most distinctive course of investigation—though the more laborious and severe strain upon the intellect—involving, as it necessarily did throughout the entire process, the retention and employment of mental concrete conceptions in place of their symbols. He had himself been accustomed to express the difference between the two as that of Presentation and Representation. The term "geometrical," it was needless to mention, corresponds to the old term, "rhetorical," and the method is that which Mr. Lidstone has so excellently applied.

Mr. George King could not let the opportunity pass without joining with other speakers, in congratulating Mr. Lidstone on his paper, and in thanking him for it. In saying that, he wished to add that it would require a great deal more study than he had so far been able to give to it, as it was a paper that involved a great deal of thought to grasp it. It was simple, he believed, when once grasped, but the ideas required thinking out and applying in one's mind to practical instances, and he had not yet had time or opportunity to undertake that investigation. Yet there were one or two points he should like to refer to. Table B was, at first sight, certainly rather startling:—"Rates

of Interest yielding policy-values equal to those based on the uniform rate of 3 per-cent. (H^M mortality ordinary whole-life assurance.)" It had hitherto always been thought that it was not possible to have different rates of interest yielding the same policy-values with the same mortality table, but he gathered from the table that it was possible, only the special rate was variable from year to year, a fixed rate not being taken throughout the whole time. He must confess he did not see how these rates were arrived at, and he asked whether P′ varied from year to year.

Mr. LIDSTONE said that having fixed P′ the different rates of interest could be calculated. There would obviously be a separate rate of interest for every year.

Mr. KING said he wished to mention one point about the O^M and $O^{M(5)}$ Tables. The tables came under Dr. Sprague's Class 2, and he thought it would be found that from age 74 onwards the O^M Table and the $O^{M(5)}$ Table gave identical annuity-values, so that it would be necessary to add (in Article 42) the words " For all ages at entry not later than 74," as, for later ages, the two tables would give equal values. On p. 228, a quotation was given from one of his speeches. On that occasion he spoke for an hour with only a few notes before him, and therefore, perhaps, did not weigh his words so carefully as he ought to have done, and the speech was published without his having had an opportunity of revising it after it had been taken down in shorthand. If he had now to write it, he should leave out the words, "I can prove it by mathematics"; not that he thought under the circumstances of the case that that could not be done. The quotation was rather taken away from its context. He was speaking about the duration of the policies in an ordinary Company, which was much longer than the assessment people maintained, and said that, for ordinary whole-life business, and for all rates of withdrawal practically possible, the company, making allowance for lapse in its premiums and valuations, must after a number of years set aside the larger reserves. That was incontrovertible, and Mr. Lidstone practically proved it in his paper, although he also showed that by assuming a certain hypothetical lapse-rate it was possible to get the two methods of valuation to produce the same reserves. But that would not be a practical lapse-rate under the conditions which he was discussing.* That also applied to the remarks of Messrs. Ackland and Bacon in their paper, because in speaking of very young ages the rule did not apply. Policies on lives aged 1 or 2 at entry could not be regarded practically as whole-life assurances, and it was not possible to have a net premium valuation for the infantile ages. The policies had to be treated as term policies for one year, increasing year by year up to age 10. He noticed, however, that Mr. Lidstone, in Table D, had given the hypothetical lapse rates for a 15-year endowment assurance. That was a table that interested him very much, but he did not quite see how it applied. Mr. Lidstone said that whatever the form of benefit might be the principle held, but if a whole-life benefit were involved, it would be necessary to prepare a table up to the end of life, and no doubt a very extraordinary lapse-rate would be obtained. He

* *See* Remarks on page 251.

did not think it was possible to have 15 years' endowment assurances lapsing at the rates given in that table, but it was an interesting table, and showed that one must be very careful in making a general statement too emphatically. The statement might be perfectly true for the purposes for which it was introduced, but it might not be exactly true under every conceivable theoretical condition. He had made those few remarks, not in any way in criticism of the paper, because he felt it was a paper that on further study would be most interesting and most useful.

Mr. T. G. Ackland also expressed to the author his congratulations on the paper, which appeared to him to be a classical one, worthy of comparison, if he might say so, with that which Mr. Woolhouse had published, in vol. xv of the *Journal*, on the Continuous Method, as applicable to Insurance Benefits, or to the important paper of Mr. G. F. Hardy, in vol. xxiv, upon methods of Approximate Summation. The paper was classical also as regarded its style, which was lucid and concise; and though brief, it was yet so full of matter, that, whilst not difficult to follow, the steps of the argument being abundantly clear, it required close attention in perusal, because the arguments and demonstration were condensed. Mr. Lidstone had taken a well-known formula, one that had been used by previous writers, and had been familiar to actuaries as long as actuarial science had been in existence, and had shown relations in that formula which were hitherto unknown to most if not all of them. Previous investigators, like Mr. Meikle and Dr. Sprague, had gone into the matter of relative policy reserves with great fulness and great skill, but Mr. Lidstone had shown the law which underlay the fundamental formulas, and had enabled them, by the use of his most interesting and valuable "Equation of Equilibrium" and "Critical Function", to see under what circumstances policy reserves were greater than, equal to, or less than, those on different bases. As an illustration, he might point out that the relation of decreased policy-values to an increased rate of interest was demonstrated in the paper with a clearness that, so far as he knew, had never hitherto been equalled. As students they had understood that, although it was a recognized fact that a net policy-reserve at 4 per-cent, other circumstances being equal, was less than a net policy-reserve at 3 per-cent, it was a somewhat difficult matter to prove mathematically, and one had to proceed by analytical methods and the like, and after all the proof was not entirely satisfactory. Mr. Lidstone, by the use of his elegant method, and the investigation of the laws underlying his formulas, showed exactly under what circumstances—for it did not appear to be an invariable law—the relation in question obtained. He had been a little puzzled with a point in Art. 32, and in other places in the paper, where Mr. Lidstone brought out in a formal statement the conditions under which policy-values varied, and put it that "$(S-V_1)$ is constantly decreasing from year to year." He should have thought that as S (the sum assured) was presumably constant, the conditions would have been more simply stated by saying that V_1 constantly increased from year to year.

Mr. Lidstone said that the sum assured S was not intended to be constant.

Mr. Ackland said that was, no doubt, an answer. In the matter of withdrawal, he probably would be expected to say a word, as his own name and that of Mr. Bacon were referred to more than once in connection with their recently-published paper in the *Journal* (vol. xxxviii, p. 539, *et seq.*). Mr. King had explained that the paragraph, which was quoted in Art. 46 of the present paper from some remarks Mr. King made years ago, was not intended to be taken *au pied de la lettre.* The statement that a relation, which seemed to have been shown by other writers not to be a true relation, could be mathematically proved, had puzzled a good many people, and it cleared up the point to have the statement now qualified by Mr. King. As he understood it, however, Mr. Lidstone's Table D was only a particular illustration, and showed the very curious effect, arising from his previous demonstration, that, on a given rate of lapse, and by no means an unreasonable or exceptional one, the policy-values would be throughout identical, other things being equal, whether the rate of lapse was introduced or not. But it was proved throughout the demonstration, apart altogether from that particular case, that policy-values, including the element of lapse, might be greater or less than, or equal to, those excluding such element, according to the value of the "Critical Function" appropriate to the case. This was also demonstrated by figures, although by no means so elegantly and fully, in the paper by Mr. Bacon and himself, that had been referred to. Nor did that condition only apply to the figures given in that paper at age at entry 1, but also to other ages at entry. With reference to Mr. Lidstone's treatment of the element of withdrawal, in connection with the figures given in the paper written by Mr Bacon and himself, everyone must have been struck with the elegant way in which a law had been developed, showing the conditions under which certain relative variations in policy-values obtained, and thus their own limited investigation had been confirmed. With regard to the policy reserves at age at entry 1, he had nothing to say on the remarks of the author, except that he thought that Mr. Lidstone's demonstrations and arguments were quite conclusive. It was Mr. Warner, he believed, who said in a humorous way, and before he had the responsibilities of office upon him, that the symbol Z, associated with the name of Mr. Lidstone, was expressive of finality; and upon the present point he thought that Mr. Lidstone's demonstrations left very little to be said by way of comment or criticism. It was no doubt the case, and he believed Mr. Bacon agreed with him, that the net premium for age 1 was probably not quite correctly deduced in their paper. It was pointed out in the paper that this value was only deduced approximately, the chief difficulty being that the force of withdrawal at the moment of entry was almost impossible to ascertain by mathematical processes, and they felt, when they submitted it, some doubt as to its accuracy. This applied to all entry ages, but especially to office entry age 1, where the mortality was rapidly changing, and the sum assured was variable. He would be glad to see a demonstration of a point stated in Art. 59, where Mr. Lidstone said that during the first six months from entry the normal reserve, exclusive of lapses, was positive throughout. He thought that

would depend upon the sum assured that was payable in the early months, and perhaps also upon the incidence of mortality and withdrawal. In the particular case that was before them there was, however, no doubt, he thought, that Mr. Lidstone was correct. He had had the advantage of seeing a very neat demonstration, which Mr. Lidstone had kindly sent to him, proving the statement in question, and he hoped that Mr. Lidstone would see no objection to including that demonstration in his paper. The application of Mr. Lidstone's method to extra risks would be extremely interesting, and it might be hoped that he would write a further paper developing his present method in some of its further applications. He did not know whether he was right in supposing that the method would also be applicable to valuations by Select Tables. That question had been under notice for some time past, and a most interesting letter, which most of the members had probably seen, appeared in the *Insurance Record* of 24 February 1905, from Mr. O. F. Diver, showing an approximate method of dealing with the matter, which, so far as he had been able to look into it, appeared to be extremely valuable and interesting. There was one little point, in conclusion, about which he was in some doubt, and that was as to the definition of the word "status." Mr. Lidstone gave on p. 212 a summary of the special terms and defined their meaning, but he supposed he did not consider that the word status would need any special definition. He was, however, a little puzzled as to the particular class of benefits to which the formulas given would respectively apply. Mr. Lidstone gave certain formulas applicable to the case where $p+q=1$, and it occurred to him whether that was quite the same thing as saying that they would apply where the annuity and the assurance benefit had the well-known relation formally stated in Mr. Peter Gray's "Lemma" in "Tables and Formulas for the Computation of Life Contingencies" and elsewhere; as, for instance, in the case of last-survivor benefits.*

Mr. R. P. Hardy thought the full drift and ultimate bearing of this paper, as well as the many present and subservient uses to which the principles unfolded in it might be applied, could be hardly adequately dealt with in the short space of time at the disposal of the members for their examination. He hoped, therefore, that Mr. Lidstone would supplement his monograph with some illustration of the various uses to which it could be put. While in his own mind those were manifold, he rejoiced to recognize much of what he had for many years past been vainly seeking, namely, a general relation as distinguished from those special processes daily employed to meet the actual cases that came before them. He had long been of opinion that all the formulas (which exhibited in a highly compact form the mere balance of the forces brought to measurement) were merely particular cases of problems of high generality, the expression of which in all comprehensive terms needed an intellectual uplifting into a wider sphere. In part confirmation of that, he referred as a single instance to the several special investigations in the Text Book which students found so helpful. Mr. Lidstone showed that those were merely particular cases falling within the sweep of his general method. An outline of the general

* *See* Remarks on pp. 250, 251.

method, but intentionally very limited in design, would be found in the "Companion to the Almanack for 1842." But in the present paper, entirely new ground had been broken up, and a considerable advance had been made towards that end, and the path indicated along which like investigations must in future proceed. The effect of the conjoint operation of the various elements contributing to the formation of what was called a value, had hitherto been perceived only in its final result, whether such was expressed algebraically or arithmetically. But now the curtain was withdrawn, and the silent activities of the forces at work, and their curious interplay by advance and retreat, by consolidation and diffusion, could be observed in the Variation Fund with which he had been for some years familiar, and the same could be studied during their actual progress. When that idea had been fully mastered, they would be no longer slaves of mere x and y, nor be memory-burdened with detached formulas, but could expatiate in the freedom that recognized that "all are but parts of one stupendous whole." He had in view a class of critics, who were content to abide in the existing state of things, as a matter of finality; but he thought that scarcely represented the prevailing temper of the times, nor would it be characteristic in the future, which, as it grew in power, would expand in aim. If to-day, with the scarcely perfect means at hand, so much intellectual pleasure was derived in tracking results to their influencing sources, and in exhibiting the deflection on either side of the anticipated experience, how much more would there be with the new and powerful instrument in their hands, by means of which they could exercise almost a fore-knowledge, and study both the locus and the manifestation of the complex effects that followed from any new condition that might be purposely introduced.

The PRESIDENT said that the Institute had once more to thank Mr. Lidstone for a most valuable and interesting paper, in which freshness of thought and clearness of expression were, as was always the case with him, very noticeable. He was sure it was the wish of the members to offer to Mr. Lidstone their very best thanks for the valuable paper that he had presented that evening.

Mr. LIDSTONE, in reply, said that when he had prepared the paper he felt very doubtful whether it was one really suitable for submission to a sessional meeting, because he feared it was so technical that it would hardly lend itself to oral discussion. He was afraid it was due to that more than to the merits of the paper, that he had now rather to thank the members for their very generous reception than to reply to any very numerous criticisms. Mr. Young had compared the method of argument to what he called the Rhetorical plan as contrasted with the analytical. He himself had considered it to be, in a sense, analogous to geometrical work as compared with analytical. He thought it was generally recognised that the study of Euclid formed a very good corrective to too much purely analytical work, and in the same way, if one could get down to ideas of a general character in actuarial matters, instead of dealing with purely analytical expressions, there was a certain gain of simplicity and clearness. He took it that the general idea of the paper had found acceptance, and that would certainly encourage him in the future, if

he found time and opportunity, to follow up some of the other lines of investigations to which Mr. Ralph Hardy and others had referred. In the first instance he thought it better merely to put forward the general idea. He hoped Mr. King might find the time to give the paper further study, and that when he had done so the method would prove acceptable to him as it had to others. Mr. King had made reference to Table B, and asked how the difference in premiums $P'-P$ was arrived at. The fact was simply that there were an infinite number of differences of premium, each of which was associated with its own set of differences in the rate of interest, so as to give policy-values absolutely equivalent to those based on the ordinary 3 per cent. Therefore to form a table he had to assume some particular difference of premium, and he took 2*s.* per-cent, and then worked out what rates of interest that would involve. He was indebted to Mr. King for pointing out what was certainly too great a generality in Art. 42, in speaking of the O^{M} and $O^{M(5)}$ tables. The result there given should have been limited to ages at entry up to 74, inclusive, and he would modify the statement when the paper appeared in its final form. Fortunately that practically amounted to being universal, and for ordinary purposes the proposition might be almost taken as it stood. He was glad the paper had given Mr. King an opportunity for stating that he did not wish his language with regard to discontinuances to be taken absolutely *au pied de la lettre*, because some had certainly felt that as it stood the argument was neither sufficiently general nor sufficently precise to be considered demonstrative. The word "practically," of course, was a difficult one to fit in with a mathematically rigid demonstration, and when Mr. King in his lecture spoke of discontinuances "practically" ceasing after 10 or 15 years, it led one to ask at what point "practically" in his mind became sufficiently close to the absolute zero which was required for the purposes of his argument. For instance, he found in Mr. Ackland's recent tables that after 10 years the rate of discontinuance for age at entry 35 was 4 per-cent, after 15 years it was nearly $2\frac{1}{2}$ per-cent, after 26 years it was still $1\frac{1}{2}$ per-cent.

Mr. King said that that was industrial business. The other was ordinary business.

Mr. Lidstone said he thought the proposition was put forward generally, and while the rates might possibly be considered to disappear as compared with the very much higher initial rates, it left one's mind in a little doubt as to the logical effects of the argument. Even if the arguments were taken to prove that after a certain point had been reached the reserve values were greater, there was nothing to show what had happened to policy-values up to that point, and how far they might have counterbalanced the greater reserves after 10 or 15 years. It was satisfactory to know now that Mr. King's remarks were to be considered merely as an illustration, and certainly a very interesting illustration, of the results brought out by actual figures, and by other methods of argument. Table D related only to the particular case of a 15-year endowment assurance, which he took because he wished to verify the policy-values, and a 15-year endowment assurance led to considerably less calculation than a whole-life policy. Every particular kind of policy, if one wished to produce

exact equivalence of policy-values, would involve its own particular scale of discontinuance—excepting, of course, that having found any one scale, then double those rates, or half those rates, or any constant multiple thereof, would produce precisely the same results. In the case of a whole-life policy the rates of discontinuance would not—and he was glad Mr. Ackland agreed with him—differ so very much in type from the rapidly-descending rate actually found in practice, at any rate in industrial business.

It was gratifying to him to find that Mr. Ackland agreed with the remarks in Art. 58 with reference to the age at entry 1. Mr. Ackland had to make the best of his materials, and in calculating an annuity-value allowing for discontinuance, unless one assumed merely theoretical rates, it was no doubt absolutely impossible to get at the exact truth. A very slight deviation, quite within the limits one could consider reasonable in the actual result brought out, might—and, he thought he had shown, did—throw out such a very delicate matter as the relative policy-values after duration six months. Mr. Ackland had asked him to demonstrate the statement that the normal policy-value in Table E for age at entry 1, from duration 0 to $\frac{1}{2}$, was positive at every point—and if that was worth adding to the paper, he should be happy to do so. He did not at present flatter himself that anything in the paper would assist in the very burning question of select reserves. With regard to the particular statuses to which the method would apply, he would say, in answer to Mr. Ackland's remarks, that he did not think the condition of $p+q=1$, and the condition that the annuity-value and the assurance-value should bear the ordinary relationship, need, necessarily, embrace the same class of cases. It might possibly, so far as mere statuses were concerned; but in his paper he had contemplated a variable sum assured—a thing which one might very well get, for instance, if one were allowing for a sum assured with a guaranteed bonus. Then, of course, there could be no clear relationship between the assurance-value and the annuity-value.

[Mr. Lidstone has sent us the following additional remarks for publication.—Ed. *J.I.A.*]

Mr. Faulks has remarked that when verbal arguments are clothed in mathematical dress, certain difficulties of interpretation, not observable in the verbal arguments themselves, may arise; and he instanced the expression $P_{x\overline{n}|}-P_{\overline{n}|}$ which occurs in the author's note "On a Method of Approximately Calculating Net Premiums for Endowment Assurances on Two Joint Lives" (*J.I.A.*, xxxiii, 354). Mr. Faulks observed that *primâ facie* it was not easy to see how P_{xn}, a Premium depending on the life of x, could be associated with $P_{\overline{n}}$ which is payable for n years certain, the two quantities apparently being essentially different in their nature. This difficulty has doubtless occurred to others, and it appears desirable to indicate briefly the solution, which is

both simple and instructive. The symbol $P_{\overline{n|}}$ represents primarily the Annual Premium payable for n years certain to provide a payment of unity at the end of that period. But since the Pure Premium Policy-value at any point is mathematically and financially equivalent to the difference between the value of the benefit and the value of the remaining net Premiums, $P_{\overline{n|}}$ may also be regarded as the Premium payable for one year to secure ${}_1V_{\overline{n|}}$ at the end of one year; or as the Annual Premium payable for two years to secure ${}_2V_{\overline{n|}}$ at the end of that time; or generally as the Premium payable for t years (where t is not $>n$) to secure ${}_tV_{\overline{n|}}$ at the end of that period. If then $P_{\overline{n|}}$ per annum be payable by l_x persons aged x, the Premiums ceasing at death or after n payments, this Premium will secure ${}_1V_{\overline{n|}}$ at the end of one year to the d_x persons who die in the first year, ${}_2V_{\overline{n|}}$ at the end of the second year to the d_{x+1} who die in that year; and so on, up to ${}_nV_{\overline{n|}}=1$ for the d_{x+n-1} who die in the nth year and the l_{x+n} who survive the full term of n years. In this way it will be seen that $P_{x\overline{n|}}$, which secures a uniform benefit of unity, can be legitimately compared with $P_{\overline{n|}}$, which secures the varying benefit ${}_tV_{\overline{n|}}$, and that both these Premiums can be regarded as ceasing at the death of x.

The arguments of Art. 60 can be put into the following mathematical form, which to some minds may be more conclusive. By the retrospective method the Normal Policy-value is the difference between the Pure Premiums and the Claims, accumulated at interest and with benefit of survivorship, *i.e.*:

$$ {}_n\overline{V}=\int_0^n(\overline{P}-\mu_t S_t)\frac{D_{x+t}}{D_{x+n}}\,dt. $$

When the element of Discontinuance is introduced, $\overline{P}$ will be changed to $\overline{P}'$ or say $\overline{P}+\phi$, and the further accumulative factor $\frac{{}^wl_{x+t}}{{}^wl_{x+n}}$ will be introduced.* This factor may be written $e^{\int_t^n \overline{w}_k\,.\,dk}$, where $\overline{w}_k$ is the force of lapse, equal to $-\frac{1}{{}^wl_{x+k}}\cdot\frac{d\,{}^wl_{x+k}}{dk}$. Thus—

$$ \begin{aligned} {}_n\overline{V}' &= \int_0^n(\overline{P}+\phi-\mu_t S_t)\frac{D_{x+t}}{D_{x+n}}\cdot e^{\int_t^n \overline{w}_k\,.\,dk}\,.\,dt \\ &= \phi\int_0^n\frac{D_{x+t}}{D_{x+n}}\cdot e^{\int_t^n \overline{w}_k\,.\,dk}\,.\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A)} \\ &\quad + \int_0^n(\overline{P}-\mu_t S_t)\frac{D_{x+t}}{D_{x+n}}\cdot e^{\int_t^n \overline{w}_k\,.\,dk}\,.\,dt \quad . \quad . \quad \text{(B)} \end{aligned} $$

Expression (A) represents the accumulated amount of the Excess

* It is here assumed that no Surrender-values are payable.

Premium, ϕ. Expression (B) may be transformed as follows by Integration by Parts, the formula for which (slightly modified) is

$$\int_0^n u_t v_t dt = u_n \int_0^n v_t dt - \int_0^n \frac{du_t}{dt}\left(\int_0^t v_k . dk\right)dt.$$

Putting $e^{\int_t^n \overline{w}_k . dk}$ for u_t, and $(\mathrm{P}-\mu_t \mathrm{S}_t)\frac{\mathrm{D}_{x+t}}{\mathrm{D}_{x+n}}$ for v_t, this gives,

(since $u_n = e^{\int_n^n \overline{w}_k . dk} = e^0 = 1$),

$$\int_0^n (\overline{\mathrm{P}}-\mu_t \mathrm{S}_t)\frac{\mathrm{D}_{x+t}}{\mathrm{D}_{x+n}} \cdot dt, \text{ or } {}_n\mathrm{V}$$

$$+\int_0^n w_t . e^{\int_t^n \overline{w}_k . dk}\left[\int_0^t (\overline{\mathrm{P}}-\mu_k \mathrm{S}_k)\frac{\mathrm{D}_{x+k}}{\mathrm{D}_{x+n}} dk\right]dt$$

$$= {}_n\overline{\mathrm{V}} + \int_0^n \overline{w}_t . e^{\int_t^n \overline{w}_k . dk} . \frac{\mathrm{D}_{x+t}}{\mathrm{D}_{x+n}}\left[\int_0^t (\overline{\mathrm{P}}-\mu_k \mathrm{S}_k)\frac{\mathrm{D}_{x+k}}{\mathrm{D}_{x+t}} . dk\right]dt$$

$$= {}_n\overline{\mathrm{V}} + \int_0^n \overline{w}_t . e^{\int_t^n \overline{w}_k . dk}\frac{\mathrm{D}_{x+t}}{\mathrm{D}_{x+n}} . {}_t\overline{\mathrm{V}} . dt$$

= Normal Policy-value *plus* accumulated Profit from lapses. Adding expression (A) which has already been shewn to be equal to the Accumulated Excess Premium, there results—

Special Policy-value = Accumulated Excess Premium . . (A)
plus Normal Policy-value . . . . } (B)
plus Accumulated Profit from lapses . }

The following is the proof that under the given conditions and with the given Scale of Benefits (*vide J.I.A.*, xxxviii, 551), the Normal Policy-value for age at entry 1 is positive up to duration $\frac{1}{2}$. From $n=0$ to $n=\frac{1}{4}$, the sum assured is zero, and therefore $\overline{\mathrm{V}}$ is the accumulated amount of the Pure Premiums; it is thus essentially positive between those limits. It is also found to be positive when $n=\frac{1}{2}$ (*vide* Table XIX, *J.I.A.*, xxxviii, 601). Now, if μ_x be calculated for monthly intervals in the first year of age (by means of the monthly table of l_x given by Farr in the introduction to the English Life Table No. III) it will be found that from $n=\frac{1}{4}$ to $n=\frac{1}{2}$, μ is always greater than ·08; and as the sum assured, S, during that period is ·15 per unit Policy, μS is always greater than ·0120, *i.e.*, always greater than $\overline{\mathrm{P}}$ which is equal to ·01147. Therefore from $n=\frac{1}{4}$ to $n=\frac{1}{2}$ the Reserves must be continually drawn upon to assist the current premium income to meet the current claims. But it has been seen that the Reserve is positive at duration $\frac{1}{4}$ and remains positive at duration $\frac{1}{2}$,

notwithstanding that it is being so drawn upon. It must therefore be positive at every point throughout this interval.

It has been suggested that it would be desirable to amplify the remarks at the end of Art. 4, with reference to last survivor statuses, and for this purpose it will be sufficient to take as an example the simplest form, namely, an ordinary assurance for the uniform amount of 1 on the status $\overline{xy}$. In such a case after n years both x and y may be living, or x may be living and y dead, or y may be living and x dead; and the Policy-value will vary according to which one of these events shall happen. If $l_{\overline{xy}}$ Policies be effected on the status $\overline{xy}$ the following Table shews the number of cases of each kind, and the relative Policy-values—

Event	Number of Cases	Individual Policy-value	Aggregate Policy-values
nd y living	$l_{x+n:y+n}$	$1-(\mathrm{P}+d)\mathrm{a}_{\overline{x+n:y+n}}$	$l_{x+n:y+n}-(\mathrm{P}+d)[l_{x+n:y+n}\mathrm{a}_{x+n} + l_{x+n:y+n}\mathrm{a}_{y+n}-l_{x+n:y+n}:\mathrm{a}_{x+n:y+n}]$
iving . . lead . .	$l_{x+n}(l_y-l_{y+n})$	$1-(\mathrm{P}+d)\mathrm{a}_{x+n}$	$l_{x+n}(l_y-l_{y+n}) -(\mathrm{P}+d)[(l_y-l_{y+n})l_{x+n}\,.\,\mathrm{a}_{x+n}]$
lead . . iving . .	$l_{y+n}(l_x-l_{x+n})$	$1-(\mathrm{P}+d)\mathrm{a}_{y+n}$	$l_{y+n}(l_x-l_{x+n}) -(\mathrm{P}+d)[(l_x-l_{x+n})l_{y+n}\,.\,\mathrm{a}_{y+n}]$
Total	$l_y\,.\,l_{x+n}+l_x\,.\,l_{y+n} -l_{x+n\ y+n}$	Mean $={}_n\mathrm{V}_{xy}$	$[l_y\,.\,l_{x+n}+l_x\,.\,l_{y+n}-l_{x+n:y+n}] -(\mathrm{P}+d)[l_y\,.\,l_{x+n}\,.\,\mathrm{a}_{x+n} +l_x\,.\,l_{y+n}\,.\,\mathrm{a}_{y+n}-l_{x+n:y+n}\mathrm{a}_{x+n:y+n}]$

Here it is seen that while individual Policies may have any one of three different values, yet *in the aggregate* the continuing Policies may be said to have the *mean* value

$$1-(\mathrm{P}+d)\frac{l_y\,.\,l_{x+n}\mathrm{a}_{x+n}+l_x\,.\,l_{y+n}\,.\,\mathrm{a}_{y+n}-l_{x+n:y+n}\mathrm{a}_{x+n:y+n}}{l_y\,.\,l_{x+n}+l_x\,.\,l_{y+n}-l_{x+n:y+n}}.$$

To this mean value the symbol ${}_n\mathrm{V}_{\overline{xy}}$ may be assigned, for this symbol is stated in the Institute Notation to represent the "value" after n years of a Policy effected on the status $\overline{xy}$, and it is only as representing such a *mean value* that the symbol can be said to have any definite signification.

If $l_{x+n}l_y+l_xl_{y+n}-l_{x+n:y+n}$ be denoted by ${}^nl_{\overline{xy}}$, then in the application of the Equation of Equilibrium to the last survivor status, p_n will take the mean value $\dfrac{{}^{n+1}l_{\overline{xy}}}{{}^nl_{\overline{xy}}}$, and q_n the mean value $\dfrac{{}^nl_{\overline{xy}}-{}^{n+1}l_{\overline{xy}}}{{}^nl_{\overline{xy}}}$, and V_n will represent the mean Policy-value which has been denoted by ${}_n\mathrm{V}_{xy}$.

Similar formulæ might be deduced for the survivor of 3, 4 . . . lives, and it would not be difficult to shew, by means of the well-known expressions for $p_{\overline{xyz\ldots(t)}}^{\;r}$ and $a_{xyz\ldots(t)}^{\;r}$, that the general expression for the mean Policy-value may be put into the form—

$$ {}_nV_{xyz\ldots wv\ldots(t)}^{\;r} = 1-(P+d)\,\frac{X^r - rX^{r+1} + \frac{r.(r-1)}{2}X^{r+2} - \ldots}{\Omega^r - r\Omega^{r+1} + \frac{r.(r-1)}{\lfloor 2}\Omega^{r+2} - \ldots} $$

where

X^r = the sum of all terms of the type $l_{xy\ldots(t-r)} . {}^nl_{wv\ldots(r)} . {}^n\text{a}_{wv\ldots(r)}$

and

Ω^r = ,, ,, ,, ,, $l_{xy\ldots(t-r)} . {}^nl_{wv\ldots(r)}$.

It must be admitted, however, that this formula is of purely theoretical interest.

In illustration of the statement made in the course of the discussion, that, in the case of a Whole-Life Policy, the rates of lapse which would leave Policy-values unaltered were not essentially different from those obtaining in practice, the following Table has been prepared, showing a set of values of the force of Discontinuance, $\overline{w}_t$, which will cause no change in the continuous Policy-values, V, for an Ordinary Whole-Life Assurance on a life aged [$34\frac{1}{2}$] at entry; the Normal Basis being English Life No. III, 3 per-cent and no surrender-values being payable. It is thought that these values cannot be fairly described as "very extraordinary," as suggested by Mr. King, and in general progression they are not markedly dissimilar from those of practice in industrial business. It is not, however, intended (nor is it required for the purposes of this Paper) to suggest that the values should be regarded as practical ones. Messrs. Ackland and Bacon have shown (*J.I.A.*, xxxviii, pp. 559–562) that, in the case of Industrial Assurance business, the observed rates of Discontinuance tend to increase Reserves on a Pure Premium Basis with allowance for lapses, thus confirming Mr. King's general conclusions; and the present writer is in entire agreement with Mr. King in thinking that the increase of Reserves would doubtless be more pronounced in the case of ordinary (non-industrial) business.

Age attained	w_t	Age attained	w_t
35	·748	60	·0116
36	·243	65	·0095
37	·143	70	·0081
38	·101	75	·0072
39	·0780	80	·0066
40	·0631	85	·0061
45	·0316	90	·0058
50	·0204	95	·0056
55	·0149	100	·0055

G. J. L.

On Mortgages of Expectancies under Wills.

By James Robert Hart, F.I.A.

Insurance Companies are sometimes asked to lend on a prospective interest under the will of a living person. In most cases, such an expectancy would of course be quite out of the question as a security; for besides other circumstances that may arise to defeat it, there is the possibility that the person who has executed the will may change his mind, or get rid of all his property in his lifetime. But in certain circumstances the chance of realizing such an expectancy may be materially increased. For example, the person who has made the will may become lunatic and incapable of dealing with his property or making a testamentary disposition of it. The property would then be placed in the custody of a trustee or the Court, and, subject to a proper allowance for his maintenance, would be kept intact till his death or recovery. And, if there is strong evidence that there is practically no chance of the lunatic recovering his reason, the prospective interest can almost be regarded as an ordinary contingent reversion to fall in on the death of the lunatic, if the beneficiary survive him. In such circumstances, the prospective beneficiary may desire to raise a loan on his interest; and life offices are sometimes approached in this connection. Though, as the writer shows in the following remarks, such an expectancy cannot form a mortgage security, even when the chances of its realization are as favourable as those mentioned above, it may be useful to examine the chief points that come up for consideration in a proposal of this sort, and the reasons for which it should be declined.

We have to keep in view the following risks, namely, of (*a*) the legatee predeceasing the lunatic, (*b*) there being another

will in existence dated subsequent to that put forward as the last will, (*c*) the will in question being disputed on the ground of the lunatic being of unsound mind when it was made, (*d*) the lunatic recovering his mental powers, and altering his will to the detriment of the borrower's interest.

The first risk can, of course, be covered by an ordinary survivorship policy on the life of the applicant against that of the lunatic. And there would be but little difficulty in arranging to cover (*b*), (*c*), and (*d*) if, as sometimes happens, strong evidence from trustworthy sources can be produced as to the circumstances affecting these risks. A declaration from the solicitor who always acted for the lunatic and who made the will, may go to show that it is practically certain that the one on which the proposed security is based is the last will, and that it was made under conditions that leave very little doubt as to the lunatic's sanity at the time of its execution. On the latter point, of course, medical reports would also be required, as well as sound professional opinion as to the lunatic's chance of recovery.

But, assuming the necessary covering insurances can be satisfactorily arranged, there still remains a very important question affecting the lender's position from its legal aspect. How is he to secure that on the death of the lunatic the legacy would not be paid over to the legatee himself, or to some other incumbrancer, without his loan being satisfactorily discharged ?

A full consideration of these points in the light of a case *in re* Dallas, 1904, 2 Ch. 385, that recently appeared in the *Law Reports,* shows that a lender could not protect himself in respect of these risks. To make this clear, the writer proposes to refer briefly to the principal points in the case, though, as it was a very lengthy one, and was, moreover, carried to the Court of Appeal, it is impossible to give more than an outline.

One Frederick Dallas was the legatee and sole executor under the will of his father. During the latter's lifetime he charged his legacy of £10,000 for sums far in excess of that amount; and, on the father's death, disputes arose among the incumbrancers as to their rights of priority in sharing in the legacy. Some of the incumbrancers had given notice to Dallas's solicitors during the lifetime of the father; others to the beneficiary immediately on the death of his father. F. Dallas renounced probate on that event happening, and administration was granted to his sister; and some incumbrancers gave notice to her. In the course of the actions, it was argued (1) that no notice was necessary, (2) that, because he was

guilty of no negligence, an incumbrancer who gave notice to the sister as soon as he heard that she was administrator, should rank before an incumbrancer whose charge was of later date, but who had given earlier notice, and (3) that notice to F. Dallas was effectual. Without going into the arguments, however, it may be sufficient to state that it was held that the incumbrancer who first gave notice to the sister obtained priority over those who gave notice to her subsequently. The judgment was based on the ground that (*a*) the first to give notice to the first person having legal dominion of the fund must be preferred, (*b*) the fact that on the death of the testator, when the fund came into existence, there was no trustee or person having legal dominion of the fund to whom effective notice could be given, was immaterial to alter the priority obtained by notice given to the person who subsequently had such legal dominion, and (*c*) notice given by an assignee of a fund to the person who is himself the assignor is not an effective notice so as to alter priorities, and (*d*) notice given by an incumbrancer to an executor, who afterwards renounces without having in any way acted in the office, is invalid.

It will be seen, therefore, that, unless a lender is in a position to serve notice immediately the executor takes up his duties, or, in the event of the executor renouncing, on the administrator, he runs the risk of his charge being ousted or postponed. And, even when he fulfils these conditions, he may not be safe. Several, or all, of the incumbrancers may give notice to the proper person on the same day. In such event, the priorities will rank according to the date of the deeds of charge to which the notices relate; and a lender who believed that his charge was the first, may find that it was really subsequent to others. For it is to be specially noted that there are no trustees, when the charge is created, from whom the lender could enquire as to prior dealings. In fact, no deed or fund exists while the lunatic is alive. The result is that a lender has to rely for his protection only on the declaration of the borrower. It is hardly necessary to state that no prudent lender would consider such a declaration sufficient to secure his advance. In the case of Dallas, an incumbrancer was misled in this way. For this reason, apart from the risks attaching to the question of serving notice on the executor or administrator, it is clear that an investor in the position of a life office could not lend upon such an expectancy as that dealt with.

Note on the Methods followed in the Registration of Deeds affecting Land in Scotland. By A. GRAHAM DONALD, M.A., F.F.A., A.I.A., *of the Scottish Provident Institution.*

[A Paper on the English Land Registration system was recently read by Mr. J. R. HART, F.I.A., before the Faculty of Actuaries in Scotland (*see* T.F.A., vol. ii, p. 269 *et seq.*). It is thought that the following note, dealing with the methods adopted in Scotland as to the registration of Deeds, may be of interest to readers of the *Journal.*—ED. *J.I.A.*].

UNDER the Act of 1617, cap. 16 (of the Scots Parliament) and subsequent Acts, all deeds relating to land require to be registered, such deeds ranking according to the date on which they are registered and not according to the date of signature. There are other kinds of Registration in Scotland, notably for Preservation and Execution. Under this the principal writs are retained in the permanent custody of the Registrar, an extract (official copy) being furnished which has the same validity as the original. In the case of *Registration for Publication,* with which this note deals, the deed, bearing a *Certificate of Registration* by the Keeper of the Register, is in due course returned to the persons registering.

Except when the heritable property (real property) is within the ancient boundaries of burghs, all deeds must be registered at Edinburgh in the *General Register of Sasines* (deeds giving title to feudal property). Each deed presented there must bear a *Warrant of Registration* by the person (or his agent) at whose instance it is to be registered, and this warrant must state the county or counties in which the lands are situated. It may be noted that by introducing a suitable *Clause of Direction* into the deed and referring to it in the Warrant of Registration, a specific part only of the deed will be registered.

For convenience, the Register is kept in *Divisions* corresponding to the 33 counties, so that each county possesses a complete Register of its own. Should land in different counties be disponed (conveyed) by one deed, the deed is recorded at length in the books of the Division applicable to one county, and by memorandum in the books of the Divisions applicable to the other counties.

Keeping in view this last mentioned provision, the following remarks as to the various books apply to each county. When the deed is presented for registration, an entry of a single line

is made in the appropriate *Presentment Book* (or Books); this shows between what hours it was presented on the given day, the names of the granter or grantee, of the person presenting, and of the Agent (Solicitor).

A short précis of the deed is made, and a printed copy of this forms the *Minute,* the minutes being printed in annual volumes for each County (minute books). The order in which the writs appear in the minute book, fixes priority and preference of title. The deed is then copied at length in MS. in one of the *Register Volumes,* the engrossment being twice collated, but reference is not often required to these full copies.

Complete *Indexes* of all the names of persons, and of all the names of places are prepared annually, and are printed and bound up with the respective volumes of minutes. The Presentment Books, Minutes, Indexes and the Register Volumes themselves, constitute the statutory system of registration in Scotland. In the year 1871, however, the department commenced the *Search Sheet* (bound volumes also relating to each county) with relative indexes of persons and places. As occasion requires, an account is opened in the Search Sheet for each separate property in Scotland, and all the entries relating to that property are posted in the order in which the writs have been registered. Thus, by simply turning up the proper heading, a note will be found of all changes in ownership or burdens put on the property since the commencement of the Search Sheet. *Searches* disclosing the state of the title to properties are obtainable either from certain firms of professional (unofficial) searchers, or from the Keeper of the Register.

The fees of registration are commendably moderate. Writing fees at the rate of 2*s.* per 200 words are charged for the MS. copy of the writ engrossed in the Register Volumes, and in addition a uniform fee of 5*s.* is charged to cover the whole process of registration including the Search Sheet. Searches are charged at the rate of 1*s.* 6*d.* for each year embraced in the period of search, plus writing fees at 1*s.* 6*d.* per sheet of 250 words.

The whole books and indexes of the Register including the Search Sheet are open to the use of the public at small fees of from 2*s.* 6*d.* to 10*s.* according to the number of volumes examined or the length of the period covered by the examination.

From 43,000 to 45,000 deeds are registered annually, of which about 13,000 are received at the term of Whit-Sunday (15th May), and 6,500 at Martinmas (11th November).

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Bonuses in Model Office Valuations and their relations to Reserves. By James Buchanan, D.Sc., F.I.A., F.F.A., *of the Scottish Widows Fund Life Assurance Society.*

[Read before the Institute, 27 March 1905.]

Following on the reading of Mr. King's paper "On the Comparative Reserves of Life Assurance Companies" (*J.I.A.*, xxxvii, 453), there appeared in the *Insurance Record* an interesting correspondence, in which certain developments of that paper were asked for. As these promised to lead to results of considerable interest and to throw much light on the working of bonus systems, I communicated with Mr. King, and it was with his concurrence that the present investigation was undertaken. I desire now to express my thanks to him for his kindness in placing at my disposal his manuscript tables, which greatly shortened the work of certain parts of the investigation.

To quote from Mr. Lidstone's suggestive letter—"When an "office, for one reason or another, is considering the advisability "of changing its valuation basis, it requires, first and foremost, "to form an estimate of the difference which such a change will "produce in its reserves; this the Model Office enables it to do "with great accuracy and facility. But it is necessary to go "much further than this, and to consider the effect on the future

"margin of loading; on the margin between the actual and "expected claims, and between the actual and expected "death strain; on the amount of the accruing surplus "(arising from the three main sources, namely: surplus loading, "surplus interest and mortality profit) and its incidence as "between policies of different ages at entry and durations; and "on the cash surplus necessary to allow of a distribution of "profits on a given scale." It is the object of this paper to answer some of these questions.

The most popular form of bonus distribution in this country is that which divides surplus as a percentage per annum on the sums assured, or on the sums assured and existing bonus additions. In a paper read before the Institute in 1896 (*J.I.A.*, xxxii, 320), Mr. Andras gave some interesting statistics regarding offices which follow this mode of division, and stated that, out of 66 ordinary life offices, 33 had at that time adopted one or other of these systems—18 dividing surplus on the "simple" and 15 on the "compound" reversionary bonus plan. More recently (*Transactions of the Third International Congress of Actuaries*, p. 268) Mr. Ryan drew attention to the fact that, of 52 selected offices—as against 20 which followed this method in the first distribution after 1870—30 had adopted it in the last valuation prior to 1900. It is of interest to quote his reasons for this growth of popularity. "Great simplicity "in its application is, perhaps, its chief claim to favour; a "smoothness or regularity in its results, the ease with which "its basis and principles can be explained to the non-expert, "and comprehended by them; these advantages also count "for something. But they are in themselves not enough; "and the methods would never have become so generally "adopted had it not been that the investigations of "trained actuaries had proved that they could be defended "on grounds of equity, having regard to the proper "relation that should always exist between benefit and "contribution." . . . . "The practical requirements of "business and a due regard to scientific considerations combined "to make the uniform and compound reversionary bonus "methods the staple systems for the division of surplus among "participating policies."

The present-day tendency appears to be rather towards an increase in the number of offices following the compound bonus plan, probably because, in addition to possessing the above

requirements, it meets the popular desire for bonuses, increasing with duration of the policy in reversionary amount as well as in cash value. The tables given in the present paper relate to one or other of these bonus systems; but they will probably be found applicable to the case of offices which follow some other method of distribution. As in Mr. King's recent paper, they are based on the data of the Model Office constructed from the Experience of the Twenty Companies, called by him Model Office No. 1.

Amounts of Simple and Compound Bonus.

Denoting by s_{x+n} the sums assured in the Model Office on lives aged x at entry and subsisting after n years, and by b the annual rate of bonus, assumed uniform throughout the history of the office, the amounts of existing bonus and new bonus, at the end of each quinquennium, are shown briefly in the following tabular forms:

(i). *Simple Bonus Office.*

Policies of durations	Existing Bonus	New Bonus
1–5 years	...	$\Sigma_1^5 nbs_{x+n}$
6–10 ,,	$\Sigma_1^5 nbs_{x+5+n}$	$5b\Sigma_1^5 s_{x+5+n}$
11–15 ,,	$\Sigma_1^5(5+n)bs_{x+10+n}$	$5b\Sigma_1^5 s_{x+10+n}$
16–20 ,,	$\Sigma_1^5(10+n)bs_{x+15+n}$	$5b\Sigma_1^5 s_{x+15+n}$
⋮	⋮	⋮

(ii). *Compound Bonus Office.*

Policies of durations	Existing Bonus	New Bonus
1–5 years	...	$\Sigma_1^5 nbs_{x+n}$
6–10 ,,	$\Sigma_1^5 nbs_{x+5+n}$	$5b\Sigma_1^5(1+nb)s_{x+5+n}$
11–15 ,,	$\Sigma_1^5[(1+5b)(1+nb)-1]s_{x+10+n}$	$5b(1+5b)\Sigma_1^5(1+nb)s_{x+10+n}$
16–20 ,,	$\Sigma_1^5[(1+5b)^2(1+nb)-1]s_{x+15+n}$	$5b(1+5b)^2\Sigma_1^5(1+nb)s_{x+15+n}$
⋮	⋮	⋮

where $\Sigma_1^5 nbs_{x+n}$ denotes the sum of the values of nbs_{x+n} obtained by giving to n the values 1 to 5.

To construct the table of amounts for the simple bonus office, we require the values of

$$\Sigma_1^5 s_{x+5+n},\ \Sigma_1^5 s_{x+10+n},\ \ldots\ldots$$

and $$\Sigma_1^5 nbs_{x+n},\ \Sigma_1^5 nbs_{x+5+n},\ \Sigma_1^5 nbs_{x+10+n},\ \ldots\ldots$$

and, for the compound bonus office, we require in addition the values of

$$\Sigma_1^5(1+nb)s_{x+5+n},\ (1+5b)\Sigma_1^5(1+nb)s_{x+10+n},$$
$$(1+5b)^2\Sigma_1^5(1+nb)s_{x+15+n},\ \ldots\ .$$

The expressions in the above schemes can then be readily written down; and, assuming a given rate of bonus to have been maintained in the past, and all bonus to have been added by way of reversion, these give the amounts of bonus subsisting at the end of each quinquennium, and the amount of new bonus required to maintain that rate.

The assumptions made here differ considerably from what occurs in practice; for the rate of bonus may vary from quinquennium to quinquennium, while policyholders have the option of taking their bonus in cash, or in the form of reduction of premium. Further, the Model Office is one which has been transacting a uniform new business during its whole past history. But there are so many factors contributing to disturb the age distribution of existing bonus additions, that the assumption of a uniform rate of bonus will probably not have much effect on the result. The object is to construct a Model Office for bonus, by the valuation of which it may be possible to estimate—

(i) What it will cost to make a reserve for existing bonus on a given basis,

(ii) What it will cost to declare a new bonus at a given rate.

In the case of an office allotting bonus on the uniform reversionary plan, the value which we assign to b is immaterial—for the amounts for one value can be got from any other by simple proportion; but with the compound reversionary plan the case is different, for the effect of compounding at a higher rate is to give a relatively larger share to policies of long duration, where the reserve value of the bonus is greatest. A rate of 30*s.* per-cent per annum was assumed throughout, as this is a rate approximating to that declared by many first-class offices. In answer to question 2 of the 6th schedule of the Act of 1870, Companies are required to make a return, according to present age, of the amounts assured under ordinary whole of life policies, and are required to state the amounts of reversionary bonuses

TABLE I.

SIMPLE BONUS OFFICE.

Amount of Existing Bonus calculated at the rate of 30s. per-cent per annum of the Sums Assured in Model Office.

Central Age at Entry	Age of Office								
	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	Existing Bonus grouped according to Central Age at Entry								
20	1,052	3,417	6,741	10,804	15,436	20,550	26,001	31,534	36,615
25	3,246	10,844	21,776	35,214	50,466	66,403	82,129	96,769	109,036
30	4,104	13,740	27,555	44,274	62,548	80,846	97,727	112,031	122,015
35	3,651	12,194	24,233	38,552	53,745	68,159	80,248	88,389	92,488
40	2,724	8,973	17,531	27,303	37,048	45,629	51,660	54,724	55,786
45	1,831	5,925	11,289	17,010	22,062	25,581	27,374	28,004	28,142
50	1,158	3,679	6,828	9,892	12,164	13,395	13,794	13,875	13,876
55	618	1,877	3,317	4,196	5,193	5,484	5,558	5,559	5,559
60	309	864	1,392	1,746	1,892	1,922	1,922	1,922	1,922
65	120	314	470	557	579	579	579	579	579
Total	18,813	61,827	121,132	189,848	261,133	328,548	386,992	433,386	466,018
Ages attained	Existing Bonus grouped according to Ages attained								
26–30	1,052	1,052	1,052	1,052	1,052	1,052	1,052	1,052	1,052
31–35	3,246	5,611	5,611	5,611	5,611	5,611	5,611	5,611	5,611
36–40	4,104	11,702	15,026	15,026	15,026	15,026	15,026	15,026	15,026
41–45	3,651	13,287	24,219	28,282	28,282	28,282	28,282	28,282	28,282
46–50	2,724	11,267	25,082	38,520	43,152	43,152	43,152	43,152	43,152
51–55	1,831	8,080	20,119	36,838	52,090	57,204	57,204	57,204	57,204
56–60	1,158	5,252	13,810	28,129	46,403	62,340	67,791	67,791	67,791
61–65	618	3,139	8,503	18,275	33,468	51,766	67,492	73,025	73,025
66–70	309	1,568	4,717	10,438	20,183	34,597	51,478	66,118	71,199
71–75	120	675	2,115	5,179	10,231	18,812	30,901	45,205	57,472
76–80	...	194	722	1,901	4,173	7,692	13,723	21,864	31,848
81–85	...	...	156	510	1,207	2,438	4,231	7,295	11,394
86–90	...	...	...	87	233	524	923	1,553	2,615
91 & over	...	...	...	...	22	52	126	208	347
Total	18,813	61,827	121,132	189,848	261,133	328,548	386,992	433,386	466,018

Table II.

Compound Bonus Office.

Amount of Existing Bonus calculated at the rate of 30s. *per-cent per annum of the Sums Assured in Model Office and previously declared Bonus.*

Central Age at Entry	Age of Office								
	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	Existing Bonus grouped according to Central Age at Entry								
20	1,052	3,484	7,022	11,508	16,819	22,909	29,655	36,774	43,572
25	3,246	11,055	22,691	37,528	55,009	73,982	93,439	112,270	128,679
30	4,104	14,008	28,712	47,171	68,114	89,894	110,777	129,168	142,512
35	3,651	12,432	25,242	41,051	58,460	75,615	90,563	101,022	106,494
40	2,724	9,147	18,256	29,013	40,208	50,417	57,869	61,804	63,219
45	1,831	6,038	11,745	18,059	23,844	28,027	30,241	31,051	31,235
50	1,158	3,748	7,101	10,481	13,079	14,542	15,035	15,140	15,142
55	618	1,911	3,444	4,745	5,542	5,887	5,978	5,979	5,979
60	309	879	1,440	1,830	1,996	2,033	2,034	2,034	2,034
65	120	319	484	579	603	603	603	603	603
Total	18,813	63,021	126,137	201,995	283,674	363,909	436,194	495,845	539,469
Ages attained	Existing Bonus grouped according to Ages attained								
26–30	1,052	1,052	1,052	1,052	1,052	1,052	1,052	1,052	1,052
31–35	3,246	5,678	5,678	5,678	5,678	5,678	5,678	5,678	5,678
36–40	4,104	11,913	15,451	15,451	15,451	15,451	15,451	15,451	15,451
41–45	3,651	13,555	25,191	29,677	29,677	29,677	29,677	29,677	29,677
46–50	2,724	11,505	26,209	41,016	46,357	46,357	46,357	46,357	46,357
51–55	1,831	8,254	21,064	39,523	57,004	63,094	63,094	63,094	63,094
56–60	1,158	5,365	14,474	30,283	51,226	70,199	76,945	76,945	76 945
61–65	618	3,208	8,915	19,702	37,111	58,891	78,348	85,467	85,467
66–70	309	1,602	4,955	11,269	22,434	39,589	60,472	79,303	86,101
71–75	120	690	2,223	5,603	11,388	21,597	36,545	54,936	71,345
76–80	...	199	760	2,061	4,659	8,842	16,294	26,753	40,097
81–85	...	...	165	555	1,352	2,815	5,029	8,964	14,436
86–90	...	...	...	95	261	606	1,099	1,909	3,324
91 & over	...	...	.	...	24	61	153	259	445
Total	18,813	63,021	126,137	201,995	283,674	363,909	436,194	495,845	539,469

Table III.

Amount of New Bonus of 30s. per-cent per annum.

Central Age at Entry	Age of Office									
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	Simple Bonus Office									
20	1,431	3,244	4,735	6,019	7,151	8,160	9,074	9,901	10,630	11,222
25	4,066	9,613	14,393	18,613	22,355	25,678	28,531	30,919	32,851	34,282
30	5,005	12,010	18,071	23,405	28,061	32,018	35,326	37,892	39,782	40,950
35	4,421	10,644	16,022	20,673	24,665	27,980	30,565	32,406	33,485	33,966
40	3,326	7,981	11,919	15,229	17,956	20,085	21,626	22,547	22,955	23,080
45	2,280	5,421	8,006	10,085	11,681	12,791	13,426	13,701	13,785	13,801
50	1,487	3,482	5,076	6,299	7,158	7,657	7,880	7,941	7,952	7,952
55	825	1,898	2,697	3,258	3,591	3,745	3,798	3,810	3,810	3,810
60	448	994	1,349	1,556	1,656	1,689	1,695	1,695	1,695	1,695
65	192	409	534	595	620	625	625	625	625	625
Total	23,481	55,696	82,802	105,732	124,900	140,458	152,546	161,437	167,570	171,383
	Compound Bonus Office									
20	1,431	3,324	4,996	6,515	8,011	9,421	10,792	12,125	13,388	14,490
25	4,066	9,856	15,222	20,314	25,169	29,802	34,078	37,925	41,268	43,930
30	5,005	12,317	19,121	25,559	31,602	37,158	42,069	46,201	49,170	51,639
35	4,421	10,919	16,955	22,566	27,744	32,365	36,236	39,198	41,061	41,952
40	3,326	8,186	12,605	16,598	20,135	23,101	25,408	26,888	27,590	27,821
45	2,280	5,557	8,457	10,963	13,036	14,576	15,524	15,965	16,110	16,140
50	1,487	3,569	5,357	6,831	7,944	8,638	8,971	9,070	9,088	9,088
55	825	1,941	2,841	3,517	3,947	4,161	4,240	4,258	4,258	4,258
60	448	1,018	1,415	1,664	1,793	1,838	1,847	1,847	1,847	1,847
65	192	418	557	630	661	668	668	668	668	668
Total	23,481	57,108	87,526	115,187	140,045	161,728	179,833	194,145	204,748	211,833

separately. The amounts of existing bonus in the above tables have accordingly been arranged in quinquennial groups according to age attained, as well as according to age at entry; and by comparing the amounts in any actual case with the figures of these tables—taken, in the case of the compound bonus office, in connection with Table 5 of Mr. King's paper (*J.I.A.*, xxxvii, 461)—it will be possible to select a Model Office of suitable age. The case of an office which has been doing a large new business, or in which bonuses have been applied to a large extent in some other way than by addition to the sum assured, will probably be met by selecting a Model Office younger than its calendar years would show.

Values of Sums Assured. In the above schemes, giving the amounts of existing bonus and new bonus at the end of each quinquennium, the expressions will still hold good, if for amounts we substitute values, so that, to make the valuations of the bonus additions, it is easiest first to value the original sums assured under policies in the Model Office. Mr. King has shown how these valuations may be made with great facility for each quinquennium; but for the present purpose we want the values for each age, and each duration, separately. The valuations were accordingly made in the ordinary detailed way, and the results are set out in Table XVIII.

Values of Simple and Compound Bonus. The values of the sums assured were then substituted in the expressions given above for the amounts of existing and new bonus, and the complete valuations of the bonus additions—simple and compound—are given in Tables XX–XXIX.

In estimating reserves the age distribution of the bonus additions is important, and Tables I and II, with the corresponding tables for reserve-values, may be useful for this purpose. Thus, taking an O^M 3 per-cent reserve, the percentage of the reserve-value on amount of existing bonus additions is, according to age of the office, as follows—

	Age of Office				
	10 years	20 years	30 years	40 years	50 years
Simple Bonus Office . .	50·5	56·1	61·1	65·0	67·6
Compound Bonus Office .	50·5	56·2	61·3	65·5	68·3

These percentages appear to show that the selection of a Model

Office of suitable age is of some importance, but the way in which the bonus is allotted has little effect on the reserves. Mr. King has constructed a Model Office for bonus from the actual results of five representative companies, and it is interesting to compare his figures with those given above. According to his table the O^M 3 per-cent reserve is 67·3 per-cent of the amount of the bonus additions, and this practically agrees with the percentage for the Model Office of 50 years standing. In practice the reserve for bonus is generally small as compared with the total liability; but, if a suitable model be selected, it seems that these tables should enable an estimate of reserve for existing bonus to be made with great accuracy; while Tables XXII–XXIX, giving the value of the new bonus, show how much surplus the office ought to have in hand in order to declare a new bonus at a given rate.

Sources of Surplus. This leads us to consider the principal sources of surplus: namely, surplus loading, surplus interest, and mortality profit. The late Mr. Sunderland (*J.I.A.*, xxvi, 357) constructed an elaborate series of tables showing, in the case of an office making a valuation by H^M 3 per-cent, experiencing H^M mortality, and giving interim bonuses on the same scale, the bonuses accruing from surplus loading and surplus interest for experience rates of interest varying from 3 to 5 per-cent. More recently (*J.I.A.*, xxxii, 73) Mr. Lidstone has discussed the same question, and has given, for certain ages at entry, the bonuses which would accrue from each source to policies of different durations. The principle, on which these are based, is to convert the cash surplus annually into bonus additions according to the valuation table of mortality and rate of interest—thus providing for interim bonuses on policies which become claims during the quinquennium.

Assume that surplus interest is earned at the rate ρ per annum, and let κ be the annual cash allotment, per unit sum assured, made at the end of each year; let V' be the reserve in respect of the sum assured immediately after payment of the premium, B the bonus existing at the beginning of the quinquennium, and A the assurance value on the valuation basis; and let subscripts 0, 1, 2 . . . denote that the factors are calculated at the beginning of the quinquennium, one year later, and so on. Then the amounts of bonus arising from this annual cash allotment, from surplus interest on policy reserve, and from surplus interest on bonus reserve—increased in each case by bonus

arising from surplus interest on profit from the same source in previous years of the quinquennium—are given approximately by

$$\frac{5(1+\rho)^2\kappa}{A_3}, \quad \frac{5\rho(1+\rho)^2}{A_3}V'_2, \quad \frac{5\rho(1+\rho)^2}{A_3}BA_2.$$

These are applicable to any policy in the Model Office which has been in force for a complete five years; so that the cash values of the bonus additions, which have accrued from each source, in respect of s_{x+n} policies in force at the end of the quinquennium, are

$$\frac{5(1+\rho)^2\kappa_x A_{x+n}}{A_{x+n-2}}s_{x+n}$$

$$\frac{5\rho(1+\rho)^2 A_{x+n}}{A_{x+n-2}}\,{}_{n-3}V'_x s_{x+n}$$

$$\frac{5\rho(1+\rho)^2 A_{x+n}}{A_{x+n-2}}BA_{x+n-3}s_{x+n}.$$

The advantages of these expressions are twofold—(1) they are simple,—a very important matter in applying them to the Model Office, where some thousands of expressions have to be evaluated; and (2) the difference between the experience and valuation rates of interest appears in the form of a constant factor in each term. The other factors depend on the scale of premiums charged, the valuation table of mortality and rate of interest; so that, if we tabulate, as is done in Tables IV–X, for certain valuation bases the values of

$$\frac{\cdot 05A_{x+n}}{A_{x+n-2}}s_{x+n}, \quad \frac{\cdot 05A_{x+n}}{A_{x+n-2}}\,{}_{n-3}V'_x s_{x+n}, \quad \frac{\cdot 05A_{x+n}}{A_{x+n-2}}BA_{x+n-3}s_{x+n},$$

we can write down for any experience rate of interest the amount of surplus accruing from each source by a few easy multiplications.

The values of the first expression are tabulated only for the O^M table with 3 per-cent interest, as the corresponding figures for the other tables of mortality did not differ as a rule in more than the last figure; and these figures may be used for any valuation basis, if we remember that ρ is, in this case, the excess of the experience rate of interest above that used in the construction of this table, *i.e.*, 3 per-cent.

For the combined O^M and $O^{M(5)}$ tables, the values of the expression giving the surplus interest on policy reserves, were

obtained from the corresponding figures for the $O^{M(5)}$ table by a method similar to that used by Mr. King in constructing the O^{M} and $O^{M(5)}$ reserves for the Model Office.

If V′ be the policy value after payment of the premium,

$$V' = 1 - \frac{\mathbf{a}_{x+n}}{\mathbf{a}_x} + \frac{1}{\mathbf{a}_x} - d$$

$$= v - \frac{a_{x+n}}{\mathbf{a}_x}$$

or

$$a_{x+n} = (v - V')\mathbf{a}_x.$$

Hence, if ω_1 and ω_2 be the pure premiums according to the O^{M} and $O^{M(5)}$ Tables,

$$V'(O^{M}O^{M(5)}) = V'(O^{M(5)}) + (\omega_2 - \omega_1)a_{x+n}(O^{M(5)})$$

$$= [V' + \psi(v - V')](O^{M(5)})$$

where $\psi = (\omega_2 - \omega_1)\mathbf{a}_x(O^{M(5)})$. Thus we have

$$\left\{\begin{array}{c}\text{Surplus Interest on Policy}\\ \text{Reserve according to the}\\ \text{Combined Tables}\end{array}\right\} = \left\{\begin{array}{c}\text{Surplus Interest on Policy}\\ \text{Reserve according to the}\\ O^{M(5)}\text{ Table}\end{array}\right\}$$

$$+ 5\rho(1+\rho)^2\psi\Sigma(v - {}_{n-3}V'_x)\frac{A_{x+n}}{A_{x+n-2}} \cdot s_{x+n}.$$

In the last term the factor $\dfrac{A_{x+n}}{A_{x+n-2}}$ does not differ much from unity and, being multiplied by the small factor $\rho\psi$, may be omitted.

In valuations by the combined tables, where an O^{M} reserve is made for policies in the first quinquennium, the surplus interest, earned in the second quinquennium, will be earned on these reserves, and the figures for the 10-year office are given on this basis. In the same way, for the combined H^{M} and $H^{M(5)}$ tables, the figures of the first two columns are the same as the corresponding figures for the H^{M} Table.

In the expressions for surplus interest on bonus reserve, the factor $\dfrac{A_{x+n-3}}{A_{x+n-2}}$ differs very little from unity, and the effect of replacing it for each quinquennium by its mean value seldom altered the figures by more than unity. As these figures are multiplied by ρ, any error introduced by making this substitution is quite negligible.

The above expressions require modification in the case of policies in the first quinquennium, which have been less than five years in force. Those depending on surplus loading are the only

ones of much importance; and, taking the expressions given in Mr. Lidstone's paper *(J.I.A.,* xxxii, 94), the following approximations were used—

Duration of Policy	Cash Value of Bonus Addition arising from annual cash allotment of κ
1 year . .	$\kappa_x(1+\rho)s_{x+1}$
2 years . .	$2\kappa_x(1+\rho)\dfrac{A_{x+2}}{A_{x+1\cdot 5}}s_{x+2}$
3 ,, . .	$3\kappa_x(1+\rho)\dfrac{A_{x+3}}{A_{x+2}}s_{x+3}$
4 ,, . .	$4\kappa_x(1+\rho)\dfrac{A_{x+4}}{A_{x+2\cdot 5}}s_{x+4}$
5 ,, . .	$5\kappa_x(1+\rho)\dfrac{A_{x+5}}{A_{x+3}}s_{x+5}$

the principle followed being to substitute twice, three times, . . . the central value of the detailed expression, and then to write $1+\rho$ as a common factor in each term. Practically the same result will be got, if we assume a common factor $(1+\rho)^2$ throughout, and deduct ρ times the figures of the first column.

To get the loading surplus, the figures of Table IV should be multiplied by $100\kappa(1+\rho)^2$, where κ is the loading allotted annually by way of bonus and ρ has the meaning explained above, and a deduction made for policies in the first quinquennium; while surplus interest on policy and bonus reserves will be found by multiplying the figures of Tables V–X by $100\rho(1+\rho)^2$, where ρ is the excess of the experience rate above the valuation rate of interest.

Table IV.

Loading Surplus.

Central Age at Entry	Age of Office									
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
20	979	2,240	3,275	4,166	4,950	5,648	6,280	6,850	7,352	7,758
25	2,783	6,633	9,947	12,871	15,462	17,759	19,727	21,370	22,693	23,671
30	3,425	8,284	12,485	16,179	19,399	22,148	24,403	26,162	27,451	28,245
35	3,026	7,338	11,061	14,275	17,029	19,309	21,081	22,336	23,070	23,396
40	2,274	5,498	8,220	10,503	12,379	13,838	14,890	15,517	15,793	15,878
45	1,558	3,728	5,510	6,940	8,036	8,792	9,224	9,410	9,467	9,478
50	1,015	2,391	3,487	4,325	4,911	5,251	5,403	5,444	5,452	5,452
55	563	1,301	1,850	2,233	2,459	2,563	2,599	2,607	2,607	2,607
60	304	679	922	1,063	1,131	1,153	1,157	1,157	1,157	1,157
65	129	277	362	404	421	425	425	425	425	425

TABLE V.

Surplus Interest on Policy-Reserves.

Central Age at Entry	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	O^M Mortality Table—Interest $2\frac{1}{2}$ per-cent									
20	27	112	243	411	611	838	1,091	1,363	1,641	1,896
25	89	387	862	1,486	2,233	3,075	3,956	4,826	5,633	6,302
30	127	557	1,243	2,140	3,191	4,326	5,454	6,483	7,341	7,926
35	130	569	1,267	2,159	3,180	4,239	5,223	6,029	6,556	6,812
40	114	495	1,084	1,810	2,598	3,356	4,000	4,436	4,648	4,718
45	93	393	839	1,357	1,875	2,310	2,598	2,737	2,783	2,793
50	72	295	611	956	1,265	1,479	1,588	1,621	1,627	1,627
55	48	188	369	546	677	748	775	781	781	781
60	31	113	204	276	319	335	338	338	338	338
65	16	54	90	114	125	128	128	128	128	128
Total	747	3,163	6,812	11,255	16,074	20,834	25,151	28,742	31,476	33,321
	O^M Mortality Table—Interest $2\frac{3}{4}$ per-cent									
20	26	107	232	393	586	806	1,052	1,317	1,589	1,840
25	85	371	828	1,430	2,153	2,972	3,832	4,685	5,478	6,138
30	122	535	1,198	2,067	3,090	4,199	5,304	6,317	7,164	7,743
35	125	550	1,227	2,096	3,094	4,133	5,102	5,898	6,420	6,674
40	111	482	1,057	1,767	2,540	3,286	3,922	4,354	4,564	4,633
45	90	383	820	1,329	1,839	2,268	2,553	2,691	2,737	2,747
50	70	288	599	939	1,244	1,456	1,564	1,597	1,603	1,603
55	47	184	362	537	667	738	765	771	771	771
60	31	112	202	274	316	332	335	335	335	335
65	16	53	89	112	123	126	126	126	126	126
Total	723	3,065	6,614	10,944	15,652	20,316	24,555	28,091	30,787	32,610
	O^M Mortality Table—Interest 3 per-cent									
20	24	101	220	375	561	774	1,013	1,272	1,539	1,786
25	82	356	795	1,376	2,076	2,872	3,712	4,548	5,329	5,980
30	118	516	1,156	1,999	2,995	4,078	5,162	6,159	6,995	7,568
35	122	533	1,190	2,036	3,012	4,031	4,985	5,771	6,287	6,539
40	108	469	1,029	1,723	2,482	3,217	3,845	4,272	4,481	4,550
45	88	374	802	1,302	1,805	2,229	2,511	2,648	2,694	2,703
50	69	283	589	924	1,226	1,436	1,513	1,576	1,582	1,582
55	46	182	358	531	660	730	757	763	763	763
60	30	110	199	270	312	328	331	331	331	331
65	16	53	88	111	122	125	125	125	125	125
Total	703	2,977	6,426	10,647	15,251	19,820	23,984	27,465	30,126	31,927
	O^M Mortality Table—Interest $3\frac{1}{2}$ per-cent									
20	22	92	201	343	515	714	939	1,186	1,442	1,681
25	78	330	736	1,277	1,933	2,685	3,486	4,289	5,044	5,678
30	109	479	1,076	1,867	2,809	3,843	4,885	5,850	6,664	7,225
35	114	499	1,118	1,920	2,852	3,833	4,757	5,523	6,029	6,276
40	102	443	975	1,639	2,370	3,082	3,695	4,114	4,319	4,387
45	84	357	767	1,250	1,738	2,152	2,426	2,561	2,606	2,615
50	66	272	567	892	1,187	1,393	1,498	1,530	1,536	1,536
55	45	176	347	516	642	711	738	744	744	744
60	30	108	195	265	307	323	326	326	326	326
65	15	51	86	109	120	123	123	123	123	123
Total	665	2,807	6,068	10,078	14,473	18,859	22,873	26,246	28,833	30,591

TABLE VI.

Surplus Interest on Policy-Reserves.

Central Age at Entry	Age of Office									
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	$O^{M(5)}$ Mortality Table—Interest 3 per-cent									
20	23	93	202	315	520	723	954	1,206	1,468	1,711
25	79	335	748	1,302	1,978	2,754	3,577	4,401	5,173	5,819
30	115	498	1,117	1,939	2,917	3,986	5,059	6,048	6,879	7,449
35	120	521	1,171	2,008	2,975	3,987	4,936	5,719	6,234	6,485
40	108	465	1,021	1,711	2,466	3,197	3,823	4,219	4,457	4,526
45	88	373	799	1,297	1,798	2,221	2,503	2,640	2,685	2,691
50	69	282	586	920	1,221	1,431	1,538	1,571	1,577	1,577
55	46	181	356	529	658	728	755	761	761	761
60	30	110	199	270	312	328	331	331	331	331
65	16	53	88	111	122	125	125	125	125	125
Total	694	2,914	6,287	10,432	14,967	19,480	23,601	27,051	29,690	31,478
	Combined O^M, $O^{M(5)}$ Mortality Tables—Interest 3 per-cent									
20	24	101	232	392	581	795	1,035	1,294	1,562	1,809
25	82	356	808	1,402	2,110	2,911	3,753	4,591	5,372	6,023
30	118	516	1,176	2,031	3,034	4,122	5,208	6,206	7,042	7,614
35	122	533	1,204	2,059	3,040	4,062	5,017	5,804	6,321	6,573
40	108	469	1,037	1,736	2,497	3,232	3,860	4,287	4,495	4,564
45	88	374	806	1,308	1,812	2,236	2,519	2,656	2,701	2,710
50	69	283	590	926	1,228	1,438	1,545	1,578	1,584	1,584
55	46	182	358	532	661	731	758	764	764	764
60	30	110	200	271	313	329	332	332	332	332
65	16	53	88	111	122	125	125	125	125	125
Total	703	2,977	6,499	10,768	15,398	19,981	24,152	27,637	30,298	32,098
	H^M Mortality Table—Interest 3 per-cent									
20	24	96	207	353	529	734	966	1,220	1,484	1,729
25	82	346	766	1,324	2,004	2,784	3,612	4,441	5,218	5,869
30	117	502	1,120	1,943	2,922	3,994	5,073	6,068	6,906	7,481
35	121	523	1,170	2,007	2,977	3,995	4,949	5,738	6,257	6,509
40	108	468	1,027	1,721	2,482	3,219	3,852	4,282	4,491	4,560
45	89	375	803	1,305	1,810	2,237	2,522	2,659	2,704	2,714
50	70	286	594	931	1,236	1,448	1,555	1,588	1,594	1,594
55	47	184	362	537	667	738	765	771	771	771
60	31	112	202	274	317	333	336	336	336	336
65	16	54	90	113	124	127	127	127	127	127
Total	705	2,946	6,341	10,508	15,068	19,609	23,757	27,230	29,888	31,690
	Combined H^M and $H^{M(5)}$ Mortality Tables—Interest 3 per-cent									
20	24	96	223	379	563	774	1,010	1,267	1,532	1,778
25	82	346	807	1,394	2,096	2,893	3,732	4,568	5,349	6,002
30	117	502	1,165	2,022	3,026	4,114	5,202	6,202	7,042	7,618
35	121	523	1,206	2,070	3,057	4,085	5,045	5,837	6,357	6,609
40	108	468	1,052	1,761	2,531	3,273	3,908	4,339	4,549	4,618
45	89	375	816	1,326	1,835	2,264	2,550	2,688	2,733	2,743
50	70	286	601	942	1,248	1,461	1,569	1,602	1,608	1,608
55	47	184	364	541	672	743	770	776	776	776
60	31	112	203	275	318	334	337	337	337	337
65	16	54	90	113	124	127	127	127	127	127
Total	705	2,946	6,527	10,823	15,470	20,068	24,250	27,743	30,410	32,216

Table VII.

Surplus Interest on Bonus-Reserves.

Simple Bonus Office.

Central Age at Entry	Age of Office: 10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	O[M] Mortality Table—Interest 2½ per-cent								
20	22	75	158	268	403	565	751	953	1,151
25	75	263	559	952	1,435	1,978	2,553	3,121	3,624
30	103	363	768	1,298	1,921	2,589	3,245	3,831	4,260
35	99	350	731	1,220	1,775	2,335	2,830	3,180	3,362
40	80	279	571	928	1,306	1,659	1,917	2,053	2,101
45	58	199	395	617	824	974	1,054	1,083	1,090
50	40	132	254	380	477	532	550	554	554
55	23	72	131	182	213	226	230	230	230
60	12	35	58	73	80	82	82	82	82
65	5	14	21	25	26	26	26	26	26
Total	517	1,782	3,646	5,943	8,460	10,966	13,238	15,113	16,180
	O[M] Mortality Table—Interest 2¾ per-cent								
20	21	71	148	252	381	536	716	912	1,105
25	70	247	526	901	1,364	1,889	2,446	3,002	3,496
30	97	343	729	1,236	1,837	2,486	3,127	3,703	4,124
35	94	333	699	1,171	1,709	2,257	2,743	3,087	3,268
40	77	267	549	896	1,266	1,611	1,866	2,001	2,050
45	56	191	382	598	802	951	1,030	1,058	1,065
50	38	128	247	371	467	521	539	543	543
55	22	70	128	178	209	222	225	225	225
60	12	34	57	72	79	81	81	81	81
65	5	13	20	24	25	25	25	25	25
Total	492	1,697	3,485	5,699	8,139	10,579	12,798	14,637	15,982
	O[M] Mortality Table—Interest 3 per-cent								
20	19	66	139	237	360	509	682	873	1,062
25	65	232	497	854	1,299	1,805	2,347	2,890	3,374
30	91	324	692	1,179	1,761	2,390	3,016	3,581	3,997
35	89	316	668	1,123	1,647	2,182	2,659	2,998	3,175
40	73	256	528	865	1,226	1,565	1,817	1,950	1,998
45	54	184	369	582	781	928	1,005	1,034	1,041
50	37	124	241	362	457	510	528	532	532
55	21	68	125	175	205	218	221	221	221
60	12	34	56	71	78	79	79	79	79
65	5	13	20	24	25	25	25	25	25
Total	466	1,617	3,335	5,472	7,839	10,211	12,379	14,183	15,504
	O[M] Mortality Table—Interest 3½ per-cent								
20	16	58	122	211	323	461	622	803	982
25	57	206	444	769	1,179	1,652	2,165	2,684	3,151
30	81	291	626	1,076	1,618	2,215	2,812	3,358	3,762
35	80	287	612	1,037	1,532	2,042	2,503	2,832	3,007
40	67	236	490	808	1,154	1,481	1,725	1,856	1,902
45	50	172	347	550	743	885	961	989	995
50	35	117	229	345	437	490	508	512	512
55	20	65	120	168	197	210	213	213	213
60	11	33	54	69	76	76	76	76	76
65	5	12	19	23	24	24	24	24	24
Total	422	1,477	3,063	5,056	7,283	9,536	11,609	13,347	14,624

TABLE VIII.

Surplus Interest on Bonus-Reserves.

COMPOUND BONUS OFFICE.

Central Age at Entry	Age of Office: 10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	O^{M} Mortality Table—Interest $2\frac{1}{2}$ per-cent								
20	22	78	166	287	442	635	865	1,125	1,389
25	75	269	583	1,018	1,571	2,218	2,929	3,661	4,334
30	103	371	802	1,386	2,101	2,897	3,708	4,162	5,034
35	99	357	763	1,303	1,939	2,605	3,217	3,666	3,909
40	80	284	595	989	1,423	1,842	2,162	2,336	2,401
45	58	202	411	656	893	1,073	1,171	1,208	1,217
50	40	135	265	403	515	580	602	607	607
55	23	73	136	192	227	243	247	247	247
60	12	36	60	77	85	86	86	86	86
65	5	14	21	25	27	27	27	27	27
Total	517	1,819	3,802	6,336	9,223	12,206	15,014	17,425	19,251
	O^{M} Mortality Table—Interest $2\frac{3}{4}$ per-cent								
20	21	72	155	269	418	602	825	1,077	1,335
25	70	252	550	963	1,494	2,118	2,808	3,523	4,183
30	97	350	760	1,321	2,010	2,783	3,575	4,315	4,879
35	94	339	729	1,249	1,868	2,519	3,120	3,562	3,803
40	77	272	572	955	1,379	1,790	2,105	2,278	2,342
45	56	195	398	637	870	1,047	1,144	1,180	1,189
50	38	131	258	394	504	568	591	595	595
55	22	71	133	188	223	239	243	243	243
60	12	35	59	76	83	85	85	85	85
65	5	13	21	25	26	26	26	26	26
Total	492	1,730	3,635	6,077	8,875	11,777	14,522	16,884	18,680
	O^{M} Mortality Table—Interest 3 per-cent								
20	19	67	145	253	395	572	787	1,032	1,284
25	65	237	518	913	1,422	2,026	2,697	3,395	4,043
30	91	331	722	1,260	1,926	2,677	3,452	4,178	4,734
35	89	323	697	1,200	1,800	2,436	3,027	3,463	3,701
40	73	261	551	923	1,337	1,740	2,051	2,222	2,286
45	54	188	385	619	848	1,022	1,118	1,155	1,163
50	37	127	251	385	493	557	579	584	584
55	21	69	130	184	219	234	238	238	238
60	12	34	57	74	82	84	84	84	84
65	5	13	20	25	26	26	26	26	26
Total	466	1,650	3,476	5,836	8,548	11,374	14,059	16,377	18,143
	O^{M} Mortality Table—Interest $3\frac{1}{2}$ per-cent								
20	16	59	128	225	354	517	718	950	1,190
25	57	209	462	822	1,292	1,855	2,490	3,158	3,783
30	81	297	653	1,150	1,772	2,482	3,222	3,923	4,463
35	80	293	639	1,109	1,676	2,283	2,853	3,276	3,509
40	67	240	511	863	1,258	1,647	1,949	2,116	2,179
45	50	175	361	585	806	975	1,069	1,105	1,113
50	35	119	238	367	472	535	556	561	561
55	20	66	125	177	211	227	231	231	231
60	11	33	56	72	80	81	81	81	81
65	5	13	20	24	25	25	25	25	25
Total	422	1,504	3,193	5,394	7,946	10,627	13,194	15,426	17,135

Table IX.

Surplus Interest on Bonus-Reserves.

Simple Bonus Office.

Central Age at Entry	Age of Office: 10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	$O^{M(5)}$ Mortality Table—Interest 3 per-cent								
20	20	67	141	239	363	512	686	877	1,066
25	66	235	500	859	1,303	1,811	2,353	2,896	3,380
30	92	327	696	1,182	1,766	2,397	3,023	3,588	4,004
35	90	318	670	1,126	1,650	2,184	2,662	3,001	3,179
40	73	256	530	867	1.229	1,568	1,819	1,952	2,000
45	54	185	369	582	781	928	1,006	1,035	1,041
50	37	124	241	362	457	510	528	532	532
55	21	68	125	175	205	218	221	221	221
60	12	34	56	71	78	79	79	79	79
65	5	13	20	24	25	25	25	25	25
Total	470	1,627	3,348	5,487	7,857	10,232	12,402	14,206	15,527
	H^{M} Mortality Table—Interest 3 per-cent								
20	20	68	143	243	367	516	691	883	1,073
25	67	237	505	865	1,312	1,823	2,368	2,913	3,401
30	93	329	700	1,191	1,776	2,410	3,039	3,608	4,026
35	90	320	674	1,133	1,660	2,197	2,677	3,018	3,196
40	74	258	532	871	1,234	1,575	1.828	1,961	2,009
45	54	186	372	585	787	933	1.012	1,040	1,047
50	37	125	242	364	459	513	530	534	534
55	22	69	126	176	206	219	223	223	223
60	12	34	56	71	78	79	79	79	79
65	5	13	20	24	25	25	25	25	25
Total	474	1,639	3,370	5,523	7,904	10,290	12,472	14,284	15,613
	$H^{M(5)}$ Mortality Table—Interest 3 per-cent								
20	20	70	146	246	372	523	698	891	1,081
25	68	241	512	877	1,327	1,840	2,388	2,934	3,422
30	94	334	708	1,203	1,791	2,428	3,058	3,627	4.045
35	91	323	680	1,141	1,670	2,207	2,688	3,029	3,207
40	75	260	536	875	1,239	1,581	1,833	1,967	2,015
45	55	187	373	588	789	936	1,014	1,043	1,050
50	38	126	243	366	461	514	532	536	536
55	22	69	126	176	206	219	223	223	223
60	12	34	56	71	78	79	79	79	79
65	5	13	20	24	25	25	25	25	25
Total	480	1,657	3,400	5,567	7,958	10,352	12,538	14,354	15,683

Table X.

Surplus Interest on Bonus Reserves.

Compound Bonus Office.

Central Age at Entry	Age of Office: 10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	$O^{M(5)}$ Mortality Table—Interest 3 per-cent								
20	20	69	147	256	398	576	790	1,036	1,288
25	66	239	522	919	1,429	2,033	2,704	3,403	4,051
30	92	333	726	1,265	1,932	2,683	3,458	4,184	4,740
35	90	324	699	1,203	1,803	2,440	3,030	3,466	3,704
40	73	261	552	924	1,338	1,742	2,052	2,223	2,287
45	54	188	385	620	849	1,023	1,119	1,156	1,164
50	37	127	251	385	493	557	579	584	584
55	21	69	130	184	219	234	238	238	238
60	12	34	57	74	82	84	84	84	84
65	5	13	20	25	26	26	26	26	26
Total	470	1,657	3,489	5,855	8,569	11,398	14,080	16,400	18,166
	H^{M} Mortality Table—Interest 3 per-cent								
20	20	69	148	258	401	579	795	1,042	1,296
25	67	241	526	924	1,438	2,045	2,719	3,421	4,074
30	93	336	730	1,273	1,943	2,698	3,477	4,209	4,767
35	90	326	703	1,210	1,814	2,453	3,047	3,485	3,723
40	74	263	555	929	1,346	1,752	2,063	2,235	2,299
45	54	189	387	623	853	1,028	1,125	1,161	1,170
50	37	128	253	387	496	559	582	586	586
55	22	70	131	185	220	236	240	240	240
60	12	34	58	75	82	84	84	84	84
65	5	13	20	25	26	26	26	26	26
Total	474	1,669	3,511	5,889	8,619	11,460	14,158	16,489	18,265
	$H^{M(5)}$ Mortality Table—Interest 3 per-cent								
20	20	71	152	263	407	587	804	1,052	1,306
25	68	246	531	937	1,454	2,064	2,741	3,444	4,097
30	94	340	739	1,285	1,959	2,717	3,497	4,229	4,788
35	91	330	709	1,218	1,824	2,464	3,059	3,497	3,736
40	75	265	558	934	1,351	1,757	2,070	2,241	2,305
45	55	190	389	625	856	1,031	1,127	1,164	1,172
50	38	128	254	388	497	560	583	588	588
55	22	70	131	186	220	236	240	240	240
60	12	34	58	75	82	84	84	84	84
65	5	13	20	25	26	26	26	26	26
Total	480	1,687	3,544	5,936	8,676	11,526	14,231	16,565	18,342

Expected Claims. In finding the expected claims it is usual to make adjustments for new entrants and withdrawals. In the Model Office, however, that for new entrants is not required, as all policies are supposed to be issued at the beginning of the year; and, if we make the further assumption that withdrawals take place at the end of the year, the correction for these may be omitted. Whatever error is thereby introduced is not of much importance; for the object of the table of expected claims is a comparative one, and the effect is to alter all the figures in the same direction and in nearly the same proportion. In the case of an office valuing by combined tables—such as the H^M and $H^{M(5)}$—the H^M reserves, made for policies in the first quinquennium, will together with the H^M pure premiums provide for claims according to that table during each year of the ensuing period, and for increased reserves for policies in force at the end of it. The figures giving the expected claims for the second quinquennium are therefore the same as the figures for the H^M Table.

TABLE XI.

Expected Claims in Model Office according to various Tables of Mortality.

Age of Office	$O^{[M]}$	$O^{[NM]}$	O^M	$O^{M(5)}$	H^M	Combined H^M and $H^{M(5)}$
5 years	4,210	5,615	5,522	6,141	6,296	6,296
10 ,,	9,735	12,033	11,119	12,077	12,402	12,402
15 ,,	16,002	18,767	17,183	18,347	18,839	19,216
20 ,,	22,811	26,063	23,876	25,174	25,831	26,523
25 ,,	30,211	33,967	31,214	32,605	33,435	34,398
30 ,,	37,999	42,269	38,983	40,410	41,455	42,639
35 ,,	45,889	50,666	46,881	48,384	49,603	50,953
40 ,,	53,441	58,694	54,474	55,996	57,440	58,920
45 ,,	60,221	65,889	61,302	62,834	64,502	66,073
50 ,,	65,793	71,796	66,920	68,456	70,326	71,957

Expected Death Strain. But it is well known that "the actual cost to the "office of the death of an assured life is not the "sum assured, but the difference between that amount and the "reserve-value retained to meet such a payment, and it is this "balance which should be the measure in any attempt to "ascertain the strain of the mortality on the funds" (*J.I.A.*, xx, 154). A mere statement of the percentage by which the actual claims fall short of the expected is not sufficient to

indicate the profit from mortality; for that depends on the incidence of the claims, as well as on their number and amount; and, as was pointed out by Mr. Higham in the paper above quoted, it is possible for the death claims largely to exceed or fall short of the estimate without causing loss or gain to the office. In order to get this measure of the mortality profit, we require the expected claims and their values, and these should be calculated according to the valuation rate of mortality; for, even though it "may not be that which will most probably be "experienced, yet it nevertheless governs the amount of "mortality profit, which is from time to time released and thrown "into surplus."

Table XII gives the expected death strain for various valuation tables and rates of interest. Where reserves have been made at the preceding valuation according to the combined O^{M} and $O^{M(5)}$ tables, the figures for the expected strain were derived from the corresponding figures for the $O^{M(5)}$ table by the following method. Let V and V' be the policy-reserves, and ω and ω' the pure premiums, according to the $O^{M(5)}$ and the combined Tables. Then we know that

$$V' = V + \psi(1-V)$$

where

$$\psi = (\omega - \omega')\mathrm{a}_x.$$

Hence

$$1 - V' = (1-\psi)(1-V)$$

or,

$$\left\{\begin{matrix}\text{Expected Strain according}\\ \text{to the Combined Tables}\end{matrix}\right\} = (1-\psi)\left\{\begin{matrix}\text{Expected Strain according}\\ \text{to the } O^{M(5)} \text{ Table.}\end{matrix}\right\}$$

As in the case of the expected claims, the figures given for the expected strain in the second quinquennium according to the combined O^{M} and $O^{M(5)}$, and H^{M} and $H^{M(5)}$ Tables, are the same as those for the O^{M} and H^{M} Tables respectively.

From the figures of this table it should be possible, if a suitable Model Office be selected, to estimate the expected strain in respect of original sums assured, and the mortality profit is the amount by which this exceeds the actual strain on the funds. There would of course be profit in respect of bonus additions; but these are small in the early policy years, where the difference between amount and reserve-value is greatest; and the construction of similar tables for bonus additions would scarcely repay the great amount of labour which would be involved.

TABLE XII.

Expected Death Strain in Model Office according to various Tables of Mortality.

Age of Office	O^M 2½ per-cent	O^M 2¾ per-cent	O^M 3 per-cent	O^M 3½ per-cent
5 years	5,181	5,192	5,201	5,219
10 "	9,840	9,875	9,907	9,968
15 "	14,254	14,328	14,395	14,525
20 "	18,168	18,590	18,706	18,929
25 "	22,132	22,612	22,788	23,124
30 "	26,032	26,278	26,518	26,981
35 "	29,162	29,474	29,780	30,373
40 "	31,723	32,097	32,468	33,183
45 "	33,678	34,105	34,530	35,353
50 "	35,031	35,500	35,965	36,871
	$O^{M(5)}$ 3 per-cent	Combined O^M & $O^{M(5)}$ 3 per-cent	H^M 3 per-cent	Combined H^M & $H^{M(5)}$ 3 per-cent
5 years	5,809	5,201	5,951	5,951
10 "	10,833	9,907	11,111	11,111
15 "	15,507	14,545	15,901	16,148
20 "	19,941	18,944	20,422	20,845
25 "	24,111	23,081	24,657	25,215
30 "	27,909	26,845	28,510	29,155
35 "	31,222	30,127	31,870	32,565
40 "	33,917	32,823	34,638	35,366
45 "	36,038	34,889	36,766	37,512
50 "	37,495	36,326	38,254	39,007

Effect of Selection. It is common to see statements in annual reports that the mortality, or claim experience, has been better than the expected. Much of this favourable mortality experience is without doubt due to selection, and it might readily happen that it would largely disappear if a more accurate standard of comparison were employed. The publication of the select tables according to the new experience enables us to measure the effect of selection on estimates of expected claims; and in Table XI are given the expected claims according to the new select tables as well as according to the aggregate tables.

In the case of participating policies the $O^{[M]}$ claims may fairly be taken as the true expected claims; and these figures show that offices, using the valuation tables of the old experience and experiencing select mortality, ought to show a very favourable claim experience. It must be remembered that the Model Office is one which has been doing a uniform new business during its whole past history; and in any progressive office the proportion

of new business is likely to be much larger, and the claim experience should be correspondingly better.

The mortality may be better than the expected for two reasons, (1) because the lives assured are selected by the office, and (2) because they are select lives of a selected class. It is shown elsewhere (*T.F.A.*, ii, 199) that an office working at a moderate initial expenditure, experiencing select mortality and a normal lapse rate, should be able from the profits accruing from selection to write off its initial expenses in respect of participating assurances within a limited number of years. The figures of Table XIII, giving the actual strain in an office experiencing $O^{[M]}$ mortality and basing reserves on an aggregate table, show that, where valuations are made by the tables of the old experience, this benefit of selection lasts for a longer period than ten years. But where the O^M Table is used in valuations the mortality profit due to selection soon begins to decrease, and it is not unlikely that an office doing a large non-profit business might show an unfavourable mortality experience.

Table XIII.

Actual Death Strain in Model Office experiencing $O^{[M]}$ Mortality, and basing Reserves on various Valuation Tables.

Age of Office	O^M $2\frac{1}{2}$ per-cent	O^M $2\frac{3}{4}$ per-cent	O^M 3 per-cent	O^M $3\frac{1}{2}$ per-cent
5 years	3,939	3,947	3,955	3,967
10 „	8,545	8,575	8,608	8,664
15 „	13,124	13,192	13,263	13,392
20 „	17,425	17,543	17,663	17,888
25 „	21,434	21,612	21,792	22,132
30 „	25,050	25,295	25,539	26,008
35 „	28,179	28,491	28,802	29,400
40 „	30,729	31,104	31,480	32,198
45 „	32,672	33,099	33,530	34,352
50 „	34,015	34,481	34,955	35,856
	$O^{M(5)}$ 3 per-cent	Combined O^M & $O^{M(5)}$ 3 per-cent	H^M 3 per-cent	Combined H^M & $H^{M(5)}$ 3 per-cent
5 years	3,960	3,955	3,960	3,960
10 „	8,630	8,608	8,621	8,621
15 „	13,308	13,249	13,286	13,238
20 „	17,734	17,640	17,692	17,602
25 „	21,889	21,762	21,824	21,700
30 „	25,665	25,504	25,572	25,422
35 „	28,958	28,766	28,833	28,660
40 „	31,664	31,444	31,506	31,316
45 „	33,738	33,493	33,553	33,352
50 „	35,182	34,914	34,976	34,769

Mr. Lidstone has suggested that "trading profits"—including profits from surrenders and profit from non-participating business—should be set off against expenses; and has stated that, if mortality profit be thrown in, the balance will generally be small. In support of this he shows (*J.I.A.*, xxxii, 84), that average office premiums for a whole of life participating assurance may be reproduced by loading the $H^{[M]}$ pure premium taken at an experience rate of interest ($4\frac{1}{8}$ per-cent) with a compound reversionary bonus of 30*s.* per-cent; so that practically the effect is to return the whole of the loading as well as all the surplus interest in the shape of bonus.

Taking an office of 50 years' standing, valuing by H^M and $H^{M(5)}$ 3 per-cent, earning $4\frac{1}{8}$ per-cent interest on its funds, and declaring a compound reversionary bonus, the following example shows the effect of assuming that all the H^M loading of his premiums (*J.I.A.*, xxxii, 83, Table A), and all the surplus interest is returned in the form of bonus.

Age at Entry	Office Premium	100κ	Loading Surplus	Surplus Interest on Policy-Reserve	Surplus Interest on Bonus-Reserve	Total	Amount required to provide 30*s.* bonus *v.* Table xxvii
20	1·950	·523	4,143	2,044	1,502	7,689	7,626
25	2·175	·550	13,295	6,904	4,713	24,912	24,738
30	2·450	·570	16,440	8,763	5,508	30,711	30,813
35	2·800	·607	14,500	7,603	4,299	26,402	26,355
40	3·225	·636	10,309	5,312	2,652	18,273	18,321
45	3·800	·686	6,636	3,156	1,348	11,140	11,064
50	4·550	·749	4,167	1,849	676	6,692	6,526
55	5·525	·800	2,128	891	277	3,296	3,197
60	6·900	·913	1,077	387	96	1,560	1,443
...	...	...	...	...	...	130,675	130,083

With another scale of office premiums, calculated on the basis explained in Mr. Andras's paper (*J.I.A.*, xxxii, 332), it will be found that the same result is attained with an interest margin of 1 per-cent. This confirms, from the Model Office point of view, the result of Mr. Lidstone's paper, that the effect of returning the whole of the H^M loading and all the surplus interest with an interest margin of about 1 per-cent is to produce bonuses closely agreeing with those of the compound reversionary system (*J.I.A.*, xxxii, 91).

Where the O^M table is used in valuations, mortality profit is very much reduced, but the loading is increased; and the combined effect of loading and mortality profit on the total

accruing surplus will probably not differ much from that resulting from the use of the tables of the old experience.

In the discussion which followed the reading of Mr. Lidstone's paper, Mr. King considered the effect of the adoption of a lower rate of interest on surplus accruing in respect of loading and surplus interest (*J.I.A.*, xxxii, 110-1), and the tables of the present paper enable us to estimate the effect of such a change in Model Offices of different ages. Let us suppose that a compound bonus office, valuing by O^M 3 per-cent and earning $3\frac{3}{4}$ per-cent on its funds, is considering the advisability of a reduction of its valuation rate of interest to $2\frac{1}{2}$ per-cent. The effect of such a change will be to withdraw a certain amount from loading surplus, and to increase the profit arising from surplus interest; and, in the early years, the loss will be greater than the gain, while later on the position will be reversed. From the point of view of accruing surplus, an office, which has been doing a large new business, is in the position of a Model Office much younger than its calendar years; and the following example shows the effect of such a change in the case of an office of 25 years' and in one of 50 years' standing—

Age at Entry	$100(\kappa-\kappa')$	25-Year Office: Decrease in Loading Surplus	25-Year Office: Increase in Surplus Interest	50-Year Office: Decrease in Loading Surplus	50-Year Office: Increase in Surplus Interest
20	·116	582	...	913	...
25	·121	1,977	...	2,905	...
30	·126	2,478	...	3,609	...
35	·131	2,261	...	3,107	...
40	·135	1,694	...	2,173	...
45	·139	1,132	...	1,336	...
50	·143	712	...	790	...
55	·145	361	...	383	...
60	·146	169	...	171	...
65	·146	62	...	63	...
		11,428	...	15,450	...
Policy-Reserve ...		...	8,986	...	18,389
Bonus-Reserve ...		...	3,676	...	10,855
Net Increase ...		...	1,234	...	13,794

If we turn to Table XXVIII, we see that, in the case of the office of 50 years' standing, the increase in the amount of surplus required to maintain the same rate of bonus at the lower valuation rate of interest is 9783; and this is more than met by the net increase shown above. But, in the case of the younger office, the net increase in surplus arising from these two sources is not sufficient to provide for the increase in the reserve-value of the bonus; so that, unless the deficiency is made good from some other source, the change will apparently result in a reduction of the bonus rate although the office is financially stronger and is building up better bonus prospects for the future.

Net Premiums and their Values.

In Tables XIV–XVII are given the net premiums corresponding to the business in force in the Model Office, and these are arranged according to age attained; as in that form they may be more useful for certain purposes, to which they may be applied. Table XVIII gives the values of the sums assured, and, by deducting from the figures of this table the reserve-values given in Mr. King's paper, we get the values of the net premiums. By comparing the valuation premiums, arranged according to present age, with the figures of Tables XIV–XVII, it should be possible from the figures of Table XIX to estimate their values on any given basis in cases, where it is thought desirable to find them separately.

Methods of Verification.

In conclusion, I would indicate briefly the methods which have been adopted at different stages of the work to verify results. Following Mr. King, I used, except in a few cases, four-figure logarithms, and checks similar to those employed by him were applied. For example, in the valuation of the sums assured the second differences of the values at four rates of interest were taken out and placed alongside one another. This proved a most efficient check, an error of even a unit being sometimes readily detected. These values, however, were made use of again and again in subsequent work, and as a further test of their accuracy, the values for the 50-year office were found by Mr. King's method (*J.I.A.*, xx, 259), using the formula

$$\Sigma A_{x+n} = \Sigma_n V_x + A_x\{R - \Sigma_n V_x\}$$

where R is the number of policies in force. The aggregate values brought out by the two methods seldom differed by more than one in 100,000. Similar checks were applied to the other tables; but in a few cases—such as the tables of amounts of

bonus and of expected claims—the calculations were made in duplicate. A final check was in most cases applied in the following way. Taking for example, the values of the bonus additions by the O^{M} and $O^{M(5)}$ Tables, the differences of corresponding figures were taken. The sum of these added to the total of the one table ought to give the total of the other; and, by placing these differences at four rates of interest alongside of one another, a check was applied to the whole of the preceding work. It is hoped that with the steps that have been taken to ensure accuracy, the results are free from any error of importance; and that the tables which accompany this paper will be found useful in enabling an actuary to estimate the effect of a change of valuation basis on future as well as on present surplus.

Table XIV.

Net Premiums corresponding to business in force in Model Office.

$2\frac{1}{2}$ PER-CENT.

Ages attained	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	O^M Mortality Table									
21–25	484	484	484	484	484	484	484	484	484	484
26–30	1,557	1,901	1,901	1,901	1,901	1,901	1,901	1,901	1,901	1,901
31–35	2,213	3,430	3,713	3,713	3,713	3,713	3,713	3,713	3,713	3,713
36–40	2,287	4,077	5,125	5,369	5,369	5,369	5,369	5,369	5,369	5,369
41–45	2,036	3,901	5,449	6,375	6,589	6,589	6,589	6,589	6,589	6,589
46–50	1,675	3,325	4,936	6,299	7,120	7,311	7,311	7,311	7,311	7,311
51–55	1,331	2,665	4,061	5,451	6,644	7,373	7,546	7,546	7,546	7,546
56–60	914	1,944	3,042	4,216	5,412	6,430	7,056	7,213	7,213	7,213
61–65	631	1,316	2,139	3,022	3,989	4,982	5,819	6,343	6,481	6,481
66–70	346	784	1,294	1,925	2,605	3,360	4,134	4,790	5,214	5,326
71–75	...	223	508	866	1,310	1,780	2,326	2,878	3,361	3,675
76–80	...	...	128	294	506	764	1,034	1,361	1,684	1,982
81–85	...	...	...	63	143	241	356	473	617	761
86–90	...	...	...	...	25	51	85	117	153	197
91 & over	...	...	...	...	...	5	10	17	23	30
Total	13,474	24,050	32,780	39,981	45,810	50,353	53,733	56,105	57,659	58,578
	$O^{M(5)}$ Mortality Table									
21–25	518	518	518	518	518	518	518	518	518	518
26–30	1,628	1,995	1,995	1,995	1,995	1,995	1,995	1,995	1,995	1,995
31–35	2,275	3,547	3,849	3,849	3,849	3,849	3,849	3,849	3,849	3,849
36–40	2,326	4,166	5,262	5,522	5,522	5,522	5,522	5,522	5,522	5,522
41–45	2,059	3,955	5,547	6,515	6,744	6,744	6,744	6,744	6,744	6,744
46–50	1,688	3,357	4,996	6,397	7,255	7,460	7,460	7,460	7,460	7,460
51–55	1,338	2,682	4,094	5,510	6,734	7,496	7,681	7,681	7,681	7,681
56–60	918	1,954	3,061	4,248	5,464	6,510	7,164	7,332	7,332	7,332
61–65	632	1,319	2,147	3,037	4,015	5,025	5,886	6,434	6,582	6,582
66–70	347	786	1,298	1,933	2,618	3,381	4,169	4,843	5,286	5,406
71–75	...	223	509	869	1,315	1,789	2,341	2,902	3,398	3,726
76–80	...	...	129	295	508	767	1,039	1,369	1,698	2,005
81–85	...	...	...	63	144	243	359	477	623	769
86–90	...	...	...	...	26	52	86	118	154	198
91 & over	...	...	...	...	...	5	10	17	23	30
Total	13,729	24,502	33,405	40,751	46,707	51,356	54,823	57,261	58,865	59,817
	H^M Mortality Table									
21–25	523	523	523	523	523	523	523	523	523	523
26–30	1,649	2,021	2,021	2,021	2,021	2,021	2,021	2,021	2,021	2,021
31–35	2,313	3,601	3,907	3,907	3,907	3,907	3,907	3,907	3,907	3,907
36–40	2,363	4,234	5,344	5,607	5,607	5,607	5,607	5,607	5,607	5,607
41–45	2,085	4,012	5,630	6,610	6,842	6,842	6,842	6,842	6,842	6,842
46–50	1,710	3,400	5,065	6,489	7,358	7,565	7,565	7,565	7,565	7,565
51–55	1,355	2,717	4,147	5,587	6,831	7,603	7,790	7,790	7,790	7,790
56–60	930	1,979	3,100	4,302	5,538	6,602	7,265	7,434	7,434	7,434
61–65	643	1,340	2,178	3,079	4,069	5,095	5,970	6,525	6,675	6,675
66–70	353	800	1,319	1,960	2,654	3,427	4,227	4,912	5,361	5,483
71–75	...	227	518	882	1,334	1,814	2,373	2,943	3,448	3,780
76–80	...	...	131	300	516	779	1,054	1,388	1,722	2,034
81–85	...	...	...	64	146	246	363	482	630	779
86–90	...	...	...	...	26	53	87	119	156	201
91 & over	...	...	...	...	...	5	10	18	24	31
Total	13,924	24,854	33,883	41,331	47,372	52,089	55,604	58,076	59,705	60,672

TABLE XV.

Net Premiums corresponding to business in force in Model Office.

$2\frac{3}{4}$ PER-CENT.

Ages attained	Age of Office									
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	O^M Mortality Table									
21–25	464	464	464	464	464	464	464	464	464	464
26–30	1,498	1,827	1,827	1,827	1,827	1,827	1,827	1,827	1,827	1,827
31–35	2,139	3,310	3,580	3,580	3,580	3,580	3,580	3,580	3,580	3,580
36–40	2,219	3,948	4,957	5,190	5,190	5,190	5,190	5,190	5,190	5,190
41–45	1,983	3,792	5,288	6,179	6,385	6,385	6,385	6,385	6,385	6,385
46–50	1,638	3,246	4,809	6,125	6,915	7,098	7,098	7,098	7,098	7,098
51–55	1,306	2,610	3,970	5,322	6,472	7,173	7,339	7,339	7,339	7,339
56–60	901	1,912	2,985	4,128	5,288	6,272	6,874	7,024	7,024	7,024
61–65	623	1,297	2,105	2,968	3,910	4,874	5,683	6,187	6,319	6,319
66–70	343	776	1,278	1,897	2,561	3,296	4,017	4,681	5,089	5,197
71–75	...	221	503	856	1,291	1,751	2,283	2,818	3,284	3,586
76–80	...	...	127	291	500	753	1,017	1,335	1,649	1,937
81–85	...	...	...	62	142	239	352	466	607	746
86–90	...	...	...	...	25	51	84	115	150	193
91 & over	...	...	...	...	...	5	10	17	23	30
Total	13,114	23,403	31,893	38,889	44,550	48,958	52,233	54,526	56,028	56,915
	$O^{M(5)}$ Mortality Table									
21–25	498	498	498	498	498	498	498	498	498	498
26–30	1,571	1,924	1,924	1,924	1,924	1,924	1,924	1,924	1,924	1,924
31–35	2,202	3,430	3,721	3,721	3,721	3,721	3,721	3,721	3,721	3,721
36–40	2,259	4,040	5,098	5,348	5,348	5,348	5,348	5,348	5,348	5,348
41–45	2,006	3,847	5,388	6,322	6,543	6,543	6,543	6,543	6,543	6,543
46–50	1,651	3,277	4,868	6,224	7,052	7,249	7,249	7,249	7,249	7,249
51–55	1,314	2,629	4,005	5,381	6,565	7,300	7,478	7,478	7,478	7,478
56–60	904	1,921	3,004	4,160	5,341	6,354	6,985	7,147	7,147	7,147
61–65	625	1,302	2,115	2,985	3,938	4,919	5,752	6,280	6,422	6,422
66–70	344	778	1,282	1,905	2,575	3,319	4,084	4,736	5,163	5,279
71–75	...	221	503	857	1,295	1,759	2,297	2,842	3,322	3,639
76–80	...	...	127	292	502	756	1,022	1,344	1,663	1,960
81–85	...	...	...	62	142	239	353	468	610	752
86–90	...	...	...	...	25	51	84	116	151	195
91 & over	...	...	...	...	...	5	10	17	23	30
Total	13,374	23,867	32,533	39,679	45,469	49,985	53,348	55,711	57,262	58,185
	H^M Mortality Table									
21–25	504	504	504	504	504	504	504	504	504	504
26–30	1,592	1,950	1,950	1,950	1,950	1,950	1,950	1,950	1,950	1,950
31–35	2,240	3,484	3,778	3,778	3,778	3,778	3,778	3,778	3,778	3,778
36–40	2,296	4,108	5,180	5,433	5,433	5,433	5,433	5,433	5,433	5,433
41–45	2,033	3,905	5,472	6,419	6,642	6,642	6,642	6,642	6,642	6,642
46–50	1,673	3,321	4,939	6,318	7,157	7,356	7,356	7,356	7,356	7,356
51–55	1,330	2,662	4,056	5,455	6,660	7,405	7,585	7,585	7,585	7,585
56–60	916	1,946	3,043	4,215	5,416	6,446	7,086	7,249	7,249	7,249
61–65	635	1,321	2,144	3,026	3,991	4,988	5,836	6,372	6,516	6,516
66–70	350	791	1,302	1,933	2,612	3,366	4,143	4,807	5,240	5,357
71–75	...	225	512	871	1,314	1,784	2,329	2,883	3,372	3,693
76–80	...	...	131	298	511	769	1,038	1,364	1,689	1,991
81–85	...	...	...	64	145	243	358	475	619	764
86–90	...	...	...	...	26	52	86	118	154	198
91 & over	...	...	...	...	...	5	10	17	23	30
Total	13,569	24,217	33,011	40,264	46,139	50,721	54,134	56,533	58,110	59,046

TABLE XVI.

Net Premiums corresponding to business in force in Model Office.

3 PER-CENT.

Ages attained	Age of Office									
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	O^M Mortality Table									
21–25	445	445	445	445	445	445	445	445	445	445
26–30	1,412	1,758	1,758	1,758	1,758	1,758	1,758	1,758	1,758	1,758
31–35	2,068	3,195	3,455	3,455	3,455	3,455	3,455	3,455	3,455	3,455
36–40	2,154	3,826	4,797	5,021	5,021	5,021	5,021	5,021	5,021	5,021
41–45	1,932	3,688	5,135	5,993	6,190	6,190	6,190	6,190	6,190	6,190
46–50	1,602	3,168	4,686	5,959	6,719	6,895	6,895	6,895	6,895	6,895
51–55	1,282	2,558	3,883	5,195	6,307	6,982	7,141	7,141	7,141	7,141
56–60	887	1,879	2,929	4,043	5,169	6,120	6,700	6,844	6,844	6,844
61–65	615	1,279	2,072	2,916	3,834	4,770	5,552	6,037	6,164	6,164
66–70	340	768	1,262	1,870	2,520	3,236	3,965	4,578	4,971	5,074
71–75	...	219	497	844	1,272	1,722	2,240	2,759	3,210	3,501
76–80	...	...	126	288	494	742	1,000	1,310	1,614	1,893
81–85	...	...	...	62	140	235	346	458	595	731
86–90	...	...	...	...	25	50	83	114	148	190
91 & over	...	...	...	...	...	5	10	17	22	29
Total	12,767	22,783	31,015	37,849	43,349	47,626	50,801	53,022	54,473	55,331
	$O^{M(5)}$ Mortality Table									
21–25	480	480	480	480	480	480	480	480	480	480
26–30	1,517	1,858	1,858	1,858	1,858	1,858	1,858	1,858	1,858	1,858
31–35	2,133	3,319	3,599	3,599	3,599	3,599	3,599	3,599	3,599	3,599
36–40	2,195	3,920	4,912	5,183	5,183	5,183	5,183	5,183	5,183	5,183
41–45	1,956	3,745	5,237	6,139	6,352	6,352	6,352	6,352	6,352	6,352
46–50	1,615	3,201	4,747	6,060	6,860	7,050	7,050	7,050	7,050	7,050
51–55	1,290	2,576	3,917	5,254	6,401	7,111	7,283	7,283	7,283	7,283
56–60	891	1,890	2,949	4,076	5,224	6,205	6,815	6,970	6,970	6,970
61–65	617	1,284	2,082	2,933	3,862	4,815	5,622	6,133	6,270	6,270
66–70	340	769	1,266	1,878	2,533	3,258	4,001	4,633	5,046	5,157
71–75	...	219	498	847	1,277	1,730	2,255	2,784	3,249	3,555
76–80	...	...	126	289	495	745	1,005	1,319	1,629	1,917
81–85	...	...	...	62	141	236	348	460	599	737
86–90	...	...	...	...	25	50	83	114	148	191
91 & over	...	...	...	...	...	5	10	17	22	29
Total	13,034	23,261	31,701	38,658	44,290	48,677	51,944	54,235	55,738	56,631
	H^M Mortality Table									
21–25	486	486	486	486	486	486	486	486	486	486
26–30	1,538	1,883	1,883	1,883	1,883	1,883	1,883	1,883	1,883	1,883
31–35	2,171	3,373	3,656	3,656	3,656	3,656	3,656	3,656	3,656	3,656
36–40	2,232	3,988	5,024	5,268	5,268	5,268	5,268	5,268	5,268	5,268
41–45	1,982	3,802	5,321	6,235	6,451	6,451	6,451	6,451	6,451	6,451
46–50	1,637	3,244	4,816	6,153	6,964	7,156	7,156	7,156	7,156	7,156
51–55	1,306	2,610	3,969	5,329	6,497	7,217	7,391	7,391	7,391	7,391
56–60	903	1,914	2,987	4,130	5,297	6,296	6,914	7,071	7,071	7,071
61–65	627	1,303	2,111	2,974	3,916	4,885	5,706	6,223	6,362	6,362
66–70	346	782	1,286	1,905	2,569	3,304	4,060	4,703	5,122	5,235
71–75	...	223	507	861	1,296	1,756	2,288	2,826	3,299	3,609
76–80	...	...	128	293	502	755	1,019	1,337	1,652	1,945
81–85	...	...	...	63	143	240	353	467	608	749
86–90	...	...	...	...	25	51	84	115	150	193
91 & over	...	...	...	...	...	5	10	17	23	30
Total	13,228	23,608	32,174	39,236	44,953	49,409	52,725	55,050	56,578	57,485

Table XVII.

Net Premiums corresponding to business in force in Model Office.

$3\frac{1}{2}$ per-cent.

Ages attained	Age of Office									
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
					O^M Mortality Table					
21–25	410	410	410	410	410	410	410	410	410	410
26–30	1,340	1,631	1,631	1,631	1,631	1,631	1,631	1,631	1,631	1,631
31–35	1,937	2,985	3,224	3,224	3,224	3,224	3,224	3,224	3,224	3,224
36–40	2,033	3,599	4,502	4,708	4,708	4,708	4,708	4,708	4,708	4,708
41–45	1,837	3,494	4,849	5,646	5,828	5,828	5,828	5,828	5,828	5,828
46–50	1,534	3,023	4,455	5,648	6,355	6,517	6,517	6,517	6,517	6,517
51–55	1,236	2,457	3,717	4,956	5,998	6,625	6,772	6,772	6,772	6,772
56–60	861	1,818	2,823	3,882	4,945	5,836	6,375	6,507	6,507	6,507
61–65	601	1,245	2,009	2,817	3,690	4,573	5,306	5,757	5,874	5,874
66–70	333	751	1,231	1,817	2,439	3,120	3,808	4,382	4,747	4,842
71–75	...	215	486	823	1,235	1,666	2,159	2,649	3,071	3,341
76–80	...	...	124	282	482	721	968	1,263	1,550	1,811
81–85	...	...	...	61	138	230	337	444	574	702
86–90	...	...	...	...	24	49	81	111	144	184
91 & over	...	...	...	...	...	5	9	16	22	28
Total	12,122	21,628	29,461	35,905	41,107	45,143	48,133	50,219	51,579	52,379
					$O^{M(5)}$ Mortality Table					
21–25	447	447	447	447	447	447	447	447	447	447
26–30	1,419	1,736	1,736	1,736	1,736	1,736	1,736	1,736	1,736	1,736
31–35	2,005	3,114	3,375	3,375	3,375	3,375	3,375	3,375	3,375	3,375
36–40	2,075	3,696	4,652	4,877	4,877	4,877	4,877	4,877	4,877	4,877
41–45	1,861	3,553	4,956	5,799	5,997	5,997	5,997	5,997	5,997	5,997
46–50	1,548	3,057	4,519	5,753	6,501	6,677	6,677	6,677	6,677	6,677
51–55	1,244	2,477	3,753	5,017	6,095	6,759	6,919	6,919	6,919	6,919
56–60	865	1,828	2,842	3,915	5,000	5,922	6,492	6,637	6,637	6,637
61–65	602	1,249	2,018	2,834	3,718	4,619	5,378	5,855	5,983	5,983
66–70	334	753	1,235	1,825	2,453	3,143	3,846	4,440	4,826	4,929
71–75	..	215	487	826	1,241	1,675	2,175	2,675	3,112	3,398
76–80	...	..	124	283	483	724	973	1,272	1,565	1,835
81–85	...	...	...	61	138	231	338	416	578	709
86–90	...	...	...	...	25	50	82	111	144	185
91 & over	...	...	...	...	...	5	10	17	22	28
Total	12,400	22,125	30,144	36,748	42,086	46,237	49,322	51,481	52,895	53,732
					H^M Mortality Table					
21–25	453	453	453	453	453	453	453	453	453	453
26–30	1,439	1,761	1,761	1,761	1,761	1,761	1,761	1,761	1,761	1,761
31–35	2,043	3,168	3,432	3,432	3,432	3,432	3,432	3,432	3,432	3,432
36–40	2,113	3,765	4,734	4,962	4,962	4,962	4,962	4,962	4,962	4,962
41–45	1,888	3,610	5,039	5,895	6,096	6,096	6,096	6,096	6,096	6,096
46–50	1,569	3,099	4,587	5,845	6,604	6,783	6,783	6,783	6,783	6,783
51–55	1,260	2,510	3,804	5,091	6,190	6,864	7,026	7,026	7,026	7,026
56–60	877	1,853	2,879	3,967	5,072	6,012	6,590	6,736	6,736	6,736
61–65	613	1,269	2,048	2,875	3,772	4,689	5,462	5,946	6,075	6,075
66–70	340	766	1,255	1,853	2,490	3,190	3,905	4,510	4,901	5,006
71–75	...	219	496	839	1,259	1,700	2,206	2,715	3,161	3,451
76–80	...	...	126	287	490	734	987	1,290	1,589	1,864
81–85	...	...	...	62	140	234	343	452	586	719
86–90	...	...	...	...	25	50	82	112	146	187
91 & over	...	...	...	...	...	5	10	17	23	29
Total	12,595	22,473	30,614	37,322	42,746	46,965	50,098	52,291	53,730	54,580

TABLE XVIII.

Values of Sums Assured under Policies in force in Model Office.

Age of Office	$2\frac{1}{2}$ per-cent	$2\frac{3}{4}$ per-cent	3 per-cent	$3\frac{1}{2}$ per-cent
	H^M Mortality Table			
5 years	282,020	266,244	251,696	225,839
10 ,,	522,546	494,756	469,031	423,099
15 ,,	739,593	702,172	667,428	605,119
20 ,,	935,612	890,536	848,576	773,028
25 ,,	1,109,735	1,058,732	1,011,139	925,167
30 ,,	1,258,986	1,203,580	1,151,778	1,057,948
35 ,,	1,380,835	1,322,343	1,267,582	1,168,159
40 ,,	1,474,468	1,413,960	1,357,242	1,254,119
45 ,,	1,541,526	1,479,791	1,421,883	1,316,484
50 ,,	1,584,658	1,522,262	1,463,708	1,357,065
	Combined H^M and $H^{M(5)}$ Mortality Tables			
5 years	282,020	266,244	251,696	225,839
10 ,,	525,103	497,407	471,781	425,988
15 ,,	743,889	706,644	672,065	610,010
20 ,,	941,121	896,276	854,537	779,335
25 ,,	1,116,058	1,065,327	1,017,999	932,452
30 ,,	1,265,819	1,210,720	1,159,209	1,065,857
35 ,,	1,387,976	1,329,809	1,275,351	1,176,449
40 ,,	1,481,773	1,421,603	1,365,201	1,262,620
45 ,,	1,548,916	1,487,532	1,429,943	1,325,101
50 ,,	1,592,087	1,530,047	1,471,813	1,365,736
	O^M Mortality Table			
5 years	277,642	261,704	247,058	221,020
10 ,,	515,461	487,400	461,510	415,263
15 ,,	730,636	692,866	657,902	595,173
20 ,,	925,291	879,800	837,572	761,510
25 ,,	1,098,374	1,046,901	999,004	912,436
30 ,,	1,246,828	1,190,910	1,138,764	1,044,272
35 ,,	1,368,076	1,309,043	1,253,893	1,153,772
40 ,,	1,461,274	1,400,192	1,343,068	1,239,200
45 ,,	1,528,038	1,465,714	1,407,379	1,301,205
50 ,,	1,571,000	1,508,001	1,449,006	1,341,575
	$O^{M(5)}$ Mortality Table			
5 years	280,115	264,322	249,717	223,822
10 ,,	519,183	491,330	465,517	419,490
15 ,,	735,059	697,538	662,663	600,211
20 ,,	930,119	884,899	842,776	767,023
25 ,,	1,103,435	1,052,245	1,004,461	918,226
30 ,,	1,252,014	1,196,382	1,144,357	1,050,212
35 ,,	1,373,318	1,314,580	1,259,556	1,159,778
40 ,,	1,466,537	1,405,755	1,348,754	1,245,234
45 ,,	1,533,309	1,471,283	1,413,072	1,307,246
50 ,,	1,576,272	1,513,572	1,454,700	1,347,618

TABLE XIX.

Value of Net Premiums corresponding to business in force in Model Office.

Age of Office	2½ per-cent	2¾ per-cent	3 per-cent	3½ per-cent
	H^M Mortality Table			
5 years	256,042	241,193	227,530	203,336
10 ,,	438,844	413,849	390,808	349,941
15 ,,	575,537	543,239	513,440	460,489
20 ,,	676,847	639,339	604,706	543,068
25 ,,	750,010	708,872	670,857	603,132
30 ,,	800,622	757,054	716,771	644,953
35 ,,	833,691	788,595	746,874	672,437
40 ,,	853,915	807,903	765,321	689,341
45 ,,	865,393	818,874	775,822	698,976
50 ,,	871,214	824,442	781,173	703,880
	Combined H^M and $H^{M(5)}$ Mortality Table			
5 years	256,042	241,193	227,530	203,336
10 ,,	436,529	411,706	388,830	348,236
15 ,,	571,666	539,660	510,116	457,622
20 ,,	671,925	634,783	600,475	539,410
25 ,,	744,387	703,664	666,010	598,945
30 ,,	794,563	751,449	711,550	640,436
35 ,,	827,396	782,762	741,431	667,738
40 ,,	847,483	801,951	759,768	684,530
45 ,,	858,893	812,868	770,200	694,115
50 ,,	864,678	818,406	775,500	698,990
	O^M Mortality Table			
5 years	250,888	235,893	222,140	197,809
10 ,,	429,651	404,126	381,232	340,142
15 ,,	563,091	530,508	500,505	447,271
20 ,,	661,872	624,022	589,137	527,166
25 ,,	733,128	691,612	653,311	585,189
30 ,,	782,379	738,406	697,791	625,533
35 ,,	814,551	769,010	726,924	652,029
40 ,,	834,200	787,720	744,762	668,282
45 ,,	845,335	798,337	754,883	677,524
50 ,,	850,965	803,712	760,007	682,213
	$O^{M(5)}$ Mortality Table			
5 years	254,338	239,591	225,763	201,543
10 ,,	436,045	411,010	387,869	346,917
15 ,,	571,987	539,633	509,686	456,600
20 ,,	672,826	635,230	600,426	538,598
25 ,,	745,732	704,481	666,273	598,302
30 ,,	796,244	752,546	712,042	639,941
35 ,,	829,323	784,085	742,114	667,373
40 ,,	849,600	803,437	760,594	684,280
45 ,,	861,134	814,457	771,125	693,941
50 ,,	866,993	820,057	776,478	698,868
	Combined O^M and $O^{M(5)}$ (throughout) Mortality Tables			
5 years	248,886	234,127	220,469	196,395
10 ,,	426,642	401,748	378,716	337,995
15 ,,	559,516	527,334	497,516	444,710
20 ,,	657,965	620,558	585,890	524,367
25 ,,	729,040	687,985	649,916	582,256
30 ,,	778,192	734,689	694,323	622,530
35 ,,	810,310	765,263	723,425	648,987
40 ,,	829,942	783,964	741,250	665,229
45 ,,	841,070	794,573	751,368	674,169
50 ,,	846,700	799,941	756,486	679,156

Table XX.

Reserves for Existing Bonus in Model Office.

Simple Bonus Office.

Age of Office	2½ per-cent	2¾ per-cent	3 per-cent	3½ per-cent
	H^M Mortality Table			
10 years	10,637	10,114	9,629	8,753
15 ,,	36,557	34,889	33,337	30,512
20 ,,	74,652	71,504	68,557	63,165
25 ,,	121,527	116,791	112,328	104,139
30 ,,	172,861	166,613	160,708	149,823
35 ,,	223,865	216,327	209,186	195,964
40 ,,	270,046	261,514	253,410	238,368
45 ,,	308,113	298,887	290,111	273,777
50 ,,	335,798	326,151	316,958	299,830
	$H^{M(5)}$ Mortality Table			
10 years	10,746	10,228	9,748	8,872
15 ,,	36,872	35,221	33,676	30,869
20 ,,	75,201	72,079	69,150	63,796
25 ,,	122,296	117,592	113,163	105,030
30 ,,	173,804	167,602	161,738	150,928
35 ,,	224,936	217,453	210,359	197,229
40 ,,	271,195	262,726	254,678	239,737
45 ,,	309,312	300,157	291,438	275,214
50 ,,	337,021	327,448	318,315	301,301
	O^M Mortality Table			
10 years	10,523	9,996	9,503	8,625
15 ,,	36,222	34,541	32,972	30,136
20 ,,	74,054	70,879	67,906	62,489
25 ,,	120,649	115,870	111,373	103,138
30 ,,	171,707	165,405	159,448	148,497
35 ,,	222,460	214,857	207,645	194,346
40 ,,	268,427	259,816	251,631	236,488
45 ,,	306,328	297,013	288,143	271,693
50 ,,	333,906	324,158	314,866	297,608
	$O^{M(5)}$ Mortality Table			
10 years	10,576	10,050	9,560	8,683
15 ,,	36,356	34,681	33,117	30,289
20 ,,	74,264	71,102	68,136	62,731
25 ,,	120,925	116,155	111,672	103,452
30 ,,	172,027	165,735	159,795	148,863
35 ,,	222,804	215,211	208,020	194,736
40 ,,	268,781	260,183	252,017	236,891
45 ,,	306,688	297,384	288,533	272,101
50 ,,	334,266	324,530	315,258	298,018

Table XXI.

Reserves for Existing Bonus in Model Office.

Compound Bonus Office.

Age of Office	2½ per-cent	2¾ per-cent	3 per-cent	3½ per-cent
	H^M Mortality Table			
10 years	10,637	10,114	9,629	8,753
15 ,,	37,281	35,584	34,000	31,124
20 ,,	77,827	74,562	71,486	65,885
25 ,,	129,578	124,558	119,812	111,127
30 ,,	188,407	181,660	175,258	163,473
35 ,,	249,111	240,834	232,961	218,396
40 ,,	306,233	296,729	287,662	270,850
45 ,,	355,181	344,784	334,853	316,380
50 ,,	392,198	381,238	370,754	351,216
	$H^{M(5)}$ Mortality Table			
10 years	10,746	10,228	9,748	8,872
15 ,,	37,599	35,915	34,343	31,484
20 ,,	78,401	75,150	72,104	66,537
25 ,,	130,395	125,399	120,699	112,063
30 ,,	189,425	182,715	176,366	164,664
35 ,,	250,291	242,047	234,241	219,774
40 ,,	307,519	298,048	289,062	272,351
45 ,,	356,538	346,178	336,325	317,966
50 ,,	393,593	382,669	372,263	352,844
	O^M Mortality Table			
10 years	10,523	9,996	9,503	8,625
15 ,,	36,946	35,228	33,625	30,739
20 ,,	77,221	73,914	70,808	65,180
25 ,,	128,662	123,589	118,803	110,059
30 ,,	187,177	180,362	173,908	162,046
35 ,,	247,582	239,215	231,279	216,619
40 ,,	304,441	294,822	285,683	268,745
45 ,,	353,178	342,657	332,639	314,018
50 ,,	390,050	378,956	368,371	348,669
	$O^{M(5)}$ Mortality Table			
10 years	10,576	10,050	9,560	8,683
15 ,,	37,077	35,370	33,776	30,895
20 ,,	77,432	74,144	71,051	65,433
25 ,,	128,941	123,891	119,119	110,393
30 ,,	187,506	180,711	174,276	162,440
35 ,,	247,941	239,595	231,681	217,043
40 ,,	304,812	295,215	286,100	269,187
45 ,,	353,554	343,054	333,058	314,463
50 ,,	390,426	379,354	368,791	349,117

Table XXII.

Reserves for New Bonus in Model Office.

H^M Simple Bonus Office. H^M

Central Age at Entry	Age of Office									
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	H^M Mortality Table—Interest $2\frac{1}{2}$ per-cent									
20	583	1,375	2,079	2,735	3,360	3,964	4,555	5,130	5,673	6,141
25	1,793	4,412	6,854	9,188	11,429	13,577	15,560	17,336	18,860	20 052
30	2,388	5,967	9,319	12,513	15,523	18,293	20,729	22,754	24,327	25,341
35	2,282	5,723	8,942	11,948	14,723	17,188	19,228	20,761	21,697	22,128
40	1,859	4,646	7,190	9,491	11,519	13,199	14,481	15,280	15,646	15,762
45	1,378	3,406	5,203	6,748	8,011	8,932	9,483	9,729	9,807	9,822
50	970	2,356	3,541	4,506	5,221	5,655	5,855	5,912	5,923	5,923
55	579	1,377	2,009	2,475	2,763	2,901	2,949	2,960	2,960	2,960
60	336	767	1,063	1,213	1,333	1,363	1,369	1,369	1,369	1,369
65	153	333	441	496	519	524	524	524	524	524
Total	12,321	30,362	46,641	61,343	74,401	85,596	94,733	101,755	106,786	110,022
	H^M Mortality Table—Interest $2\frac{3}{4}$ per-cent									
20	539	1,274	1,932	2,550	3,144	3,720	4,287	4,843	5,371	5,828
25	1,668	4,118	6,417	8,629	10,768	12,831	14,749	16,477	17,968	19,139
30	2,235	5,604	8,781	11,829	14,721	17,400	19,770	21,752	23,298	24,298
35	2,150	5,411	8,483	11,370	14,054	16,452	18,447	19,953	20,877	21,305
40	1,763	4,422	6,867	9,092	11,061	12,708	13,970	14,759	15,120	15,234
45	1,317	3,265	5,002	6,505	7,740	8,646	9,189	9,433	9,510	9,525
50	933	2,274	3,427	4,371	5,074	5,502	5,700	5,755	5,766	5,766
55	560	1,335	1,952	2,411	2,696	2,833	2,881	2,891	2,891	2,891
60	327	748	1,039	1,216	1,305	1,335	1,341	1,341	1,341	1,341
65	149	326	433	487	510	514	514	514	514	514
Total	11,641	28,777	44,333	58,460	71,076	81,941	90,848	97,718	102,656	105,841
	H^M Mortality Table—Interest 3 per-cent									
20	500	1,185	1,802	2,384	2,947	3,497	4,043	4,581	5,094	5,541
25	1,554	3,849	6,016	8,115	10,158	12,141	13,996	15,677	17,136	18,288
30	2,095	5,271	8,286	11,197	13,978	16,569	18,876	20,814	22,333	23,320
35	2,028	5,121	8,055	10,830	13,426	15,760	17,712	19,192	20,104	20,527
40	1,674	4,213	6,563	8,716	10,636	12,244	13,483	14,261	14,618	14,731
45	1,258	3,131	4,811	6,274	7,483	8,373	8,908	9,149	9,226	9,241
50	897	2,194	3,316	4,240	4,931	5,353	5,549	5,605	5,616	5,616
55	543	1,298	1,902	2,353	2,634	2,769	2,817	2,827	2,827	2,827
60	318	730	1,015	1,190	1,278	1,308	1,314	1,314	1,314	1,314
65	146	320	425	479	501	505	505	505	505	505
Total	11,013	27,312	42,191	55,778	67,972	78,519	87,203	93,925	98,773	101,910
	H^M Mortality Table—Interest $3\frac{1}{2}$ per-cent									
20	432	1,029	1,572	2,091	2,599	3,103	3,608	4,112	4,599	5,028
25	1,357	3,380	5,312	7,207	9,074	10,910	12,649	14,243	15,641	16,754
30	1,850	4,682	7,403	10,065	12,639	15,068	17,255	19,112	20,582	21,543
35	1,812	4,605	7,288	9,858	12,291	14,503	16,372	17,803	18,691	19,105
40	1,514	3,836	6,011	8,028	9,848	11,388	12,586	13,344	13,695	13,806
45	1,152	2,886	4,461	5,847	7,004	7,865	8,387	8,624	8,699	8,714
50	832	2,047	3,111	3,996	4,661	5,075	5,267	5,322	5,332	5,332
55	509	1,225	1,804	2,240	2,513	2,645	2,691	2,701	2,701	2,701
60	302	697	973	1,144	1,231	1,259	1,264	1,264	1,264	1,264
65	138	306	409	462	484	489	489	489	489	489
Total	9,898	24,693	38,344	50,938	62,347	72,305	80,568	87,014	91,693	94,736

Table XXIII.

Reserves for New Bonus in Model Office.

$H^{M(5}$ Simple Bonus Office. $H^{M(5)}$

Central Age at Entry	Age of Office									
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	$H^{M(5)}$ Mortality Table—Interest $2\frac{1}{2}$ per-cent									
20	609	1,421	2,137	2,802	3,435	4,044	4,638	5,215	5,759	6,227
25	1,837	4,502	6,976	9,333	11,592	13,752	15,743	17,523	19,030	20,243
30	2,427	6,053	9,439	12,659	15,687	18,468	20,911	22,910	24,515	25,529
35	2 310	5,785	9,031	12,054	14,810	17,311	19,354	20,888	21,825	22,257
40	1,876	4,685	7,244	9,554	11,587	13,270	14,554	15,354	15,720	15,835
45	1,389	3,429	5,232	6,781	8,046	8,968	9,519	9,765	9,843	9,858
50	975	2,367	3,555	4,521	5,237	5,671	5,871	5,928	5,938	5,938
55	581	1,381	2,013	2,481	2,769	2,907	2,956	2,967	2,967	2,967
60	336	768	1,061	1,244	1,334	1,364	1,370	1,370	1,370	1,370
65	153	334	442	497	520	525	525	525	525	525
Total	12,493	30,725	47,133	61,926	75,047	86,280	95,441	102,475	107,512	110,749
	$H^{M(5)}$ Mortality Table—Interest $2\frac{3}{4}$ per-cent									
20	566	1,323	1,994	2,621	3,221	3,803	4,374	4,932	5,461	5,919
25	1,713	4,210	6,543	8,779	10,936	13,012	14,938	16,671	18,165	19,338
30	2,277	5,695	8,907	11,982	14,892	17,581	19,958	21,943	23,491	24,492
35	2,180	5,477	8,577	11,483	14,178	16,583	18,582	20,090	21,014	21,441
40	1,782	4,465	6,925	9,160	11,138	12,785	14,048	14,837	15,199	15,313
45	1,328	3,289	5,033	6,540	7,777	8,683	9,227	9,471	9,548	9,563
50	939	2,285	3,441	4,386	5,090	5,518	5,716	5,773	5,784	5,784
55	562	1,340	1,958	2,418	2,703	2,840	2,888	2,898	2,898	2,898
60	327	749	1,040	1,217	1,306	1,336	1,342	1,342	1,342	1,342
65	149	326	434	488	511	515	515	515	515	515
Total	11,823	29,159	44,852	59,074	71,752	82,656	91,588	98,472	103,417	106,605
	$H^{M(5)}$ Mortality Table—Interest 3 per-cent									
20	527	1,234	1,864	2,455	3,025	3,581	4,130	4,670	5,185	5,634
25	1,601	3,944	6,146	8,270	10,332	12,329	14,192	15,878	17,339	18,492
30	2,139	5,365	8,416	11,356	14,155	16,758	19,072	21,014	22,536	23,523
35	2,059	5,190	8,153	10,948	13,556	15,897	17,853	19,335	20,247	20,670
40	1,694	4,259	6,626	8,789	10,715	12,326	13,567	14,346	14,704	14,818
45	1,271	3,158	4,846	6,313	7,523	8,414	8,951	9,193	9,269	9,284
50	903	2,206	3,331	4,256	4,948	5,371	5,567	5,623	5,634	5,634
55	545	1,302	1,907	2,359	2,640	2,775	2,823	2,833	2,833	2,833
60	319	731	1,017	1,192	1,280	1,310	1,316	1,316	1,316	1,316
65	146	320	426	480	502	506	506	506	506	506
Total	11,204	27,709	42,732	56,418	68,676	79,267	87,977	94,714	99,569	102,710
	$H^{M(5)}$ Mortality Table—Interest $3\frac{1}{2}$ per-cent									
20	460	1,080	1,637	2,166	2,682	3,190	3,700	4,207	4,696	5,125
25	1,404	3,477	5,445	7,366	9,255	11,106	12,854	14,454	15,855	16,970
30	1,894	4,779	7,538	10,230	12,825	15,267	17,463	19,323	20,795	21,757
35	1,841	4,676	7,390	9,981	12,427	14,647	16,521	17,955	18,843	19,257
40	1,534	3,883	6,077	8,105	9,932	11,476	12,676	13,435	13,786	13,897
45	1,165	2,913	4,497	5,889	7,049	7,910	8,433	8,670	8,745	8,760
50	839	2,061	3,129	4,015	4,684	5,095	5,287	5,342	5,352	5,352
55	512	1,230	1,809	2,246	2,520	2,652	2,700	2,710	2,710	2,710
60	303	698	975	1,146	1,233	1,261	1,266	1,266	1,266	1,266
65	139	308	411	464	486	491	491	491	491	491
Total	10,094	25,105	38,908	51,608	63,093	73,095	81,391	87,853	92,539	95,585

Table XXIV.

Reserves for New Bonus in Model Office.

O^M Simple Bonus Office. O^M

Central Age at Entry	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	O^M Mortality Table—Interest $2\frac{1}{2}$ per-cent									
20	561	1,330	2,020	2,667	3,287	3,887	4,175	5,016	5,586	6,051
25	1,745	4,313	6,722	9,034	11,257	13,391	15,363	17,130	18,648	19,833
30	2,343	5,874	9,194	12,363	15,355	18,109	20,533	22,549	24,113	25,123
35	2,253	5,661	8,856	11,843	14,603	17,055	19,086	20,610	21,543	21,974
40	1,841	4,605	7,134	9,422	11,440	13,113	14,389	15,186	15,550	15,665
45	1,368	3,384	5,170	6,707	7,964	8,880	9,429	9,675	9,753	9,768
50	964	2,343	3,522	4,482	5,193	5,625	5,825	5,882	5,892	5,892
55	575	1,368	1,996	2,461	2,749	2,887	2,935	2,945	2,945	2,945
60	334	763	1,057	1,236	1,326	1,356	1,362	1,362	1,362	1,362
65	152	332	440	495	517	522	522	522	522	522
Total	12,136	29,973	46,111	60,710	73,691	84,825	93,919	100,907	105,914	109,135
	O^M Mortality Table—Interest $2\frac{3}{4}$ per-cent									
20	517	1,230	1,874	2,483	3,070	3,642	4,206	4,759	5,284	5,739
25	1,619	4,016	6,281	8,470	10,591	12,640	14,516	16,264	17,748	18,912
30	2,189	5,509	8,653	11,675	14,548	17,210	19,568	21,540	23,077	24,073
35	2,120	5,346	8,392	11,260	13,927	16,312	18,298	19,795	20,715	21,141
40	1,745	4,382	6,810	9,021	10,983	12,618	13,872	14,658	15,019	15,133
45	1,306	3,241	4,967	6,461	7,690	8,590	9,131	9,374	9,451	9,466
50	927	2,259	3,405	4,344	5,043	5,469	5,667	5,724	5,734	5,734
55	557	1,328	1,942	2,398	2,682	2,818	2,866	2,877	2,877	2,877
60	325	745	1,034	1,211	1,300	1,330	1,336	1,336	1,336	1,336
65	148	325	432	486	509	514	514	514	514	514
Total	11,453	28,381	43,790	57,809	70,343	81,143	90,004	96,841	101,755	104,925
	O^M Mortality Table—Interest 3 per-cent									
20	477	1,139	1,741	2,314	2,871	3,417	3,959	4,493	5,003	5,448
25	1,504	3,745	5,877	7,952	9,976	11,945	13,788	15,459	16,911	18,055
30	2,048	5,173	8,153	11,038	13,798	16,372	18,665	20,593	22,103	23,085
35	1,998	5,056	7,963	10,719	13,299	15,618	17,560	19,031	19,938	20,360
40	1,656	4,172	6,505	8,644	10,552	12,151	13,382	14,157	14,514	14,628
45	1,248	3,108	4,777	6,231	7,433	8,318	8,852	9,093	9,169	9,184
50	891	2,179	3,295	4,213	4,900	5,320	5,516	5,572	5,582	5,582
55	539	1,289	1,890	2,338	2,617	2,752	2,800	2,811	2,811	2,811
60	316	726	1,010	1,184	1,272	1,302	1,308	1,308	1,308	1,308
65	145	319	424	478	500	505	505	505	505	505
Total	10,822	26,906	41,635	55,111	67,218	77,700	86,335	93,022	97,844	100,966
	O^M Mortality Table—Interest $3\frac{1}{2}$ per-cent									
20	408	981	1,509	2,019	2,520	3,018	3,519	4,020	4,503	4,929
25	1,305	3,272	5,168	7,037	8,885	10,706	12,432	14,016	15,405	16,510
30	1,800	4,578	7,263	9,897	12,450	14,861	17,033	18,879	20,338	21,295
35	1,779	4,535	7,190	9,739	12,154	14,351	16,210	17,631	18,514	18,927
40	1,494	3,791	5,948	7,950	9,758	11,289	12,479	13,233	13,583	13,694
45	1,140	2,860	4,423	5,800	6,950	7,805	8,324	8,560	8,635	8,650
50	825	2,031	3,087	3,966	4,629	5,038	5,230	5,285	5,295	5,295
55	505	1,216	1,791	2,224	2,496	2,628	2,676	2,687	2,687	2,687
60	300	693	967	1,137	1,223	1,252	1,257	1,257	1,257	1,257
65	139	307	409	462	484	488	488	488	488	488
Total	9,695	24,264	37,755	50,231	61,549	71,436	79,648	86,056	90,705	93,732

TABLE XXV.

Reserves for New Bonus in Model Office.

$O^{M(5)}$ SIMPLE BONUS OFFICE. $O^{M(5)}$

Central Age at Entry	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	$O^{M(5)}$ Mortality Table—Interest $2\frac{1}{2}$ per-cent									
20	579	1,363	2,061	2,712	3,334	3,935	4,523	5,096	5,636	6,101
25	1,777	4,374	6,799	9,120	11,349	13,487	15,461	17,229	18,747	19,932
30	2,367	5,921	9,254	12,431	15,428	18,185	20,611	22,628	24,192	25,202
35	2,266	5,687	8,890	11,882	14,645	17,099	19,130	20,654	21,587	22,018
40	1,848	4,620	7,154	9,444	11,463	13,136	14,412	15,209	15,573	15,688
45	1,371	3,390	5,178	6,716	7,973	8,889	9,438	9,684	9,762	9,777
50	966	2,346	3,526	4,486	5,197	5,629	5,829	5,886	5,896	5,896
55	576	1,369	1,997	2,462	2,750	2,888	2,936	2,946	2,946	2,946
60	334	763	1,057	1,236	1,326	1,356	1,362	1,362	1,362	1,362
65	152	332	410	495	517	522	522	522	522	522
Total	12,236	30,165	46,356	60,984	73,982	85,126	94,224	101,216	106,223	109,444
	$O^{M(5)}$ Mortality Table—Interest $2\frac{3}{4}$ per-cent									
20	535	1,264	1,916	2,529	3,119	3,692	4,257	4,811	5,336	5,791
25	1,652	4,079	6,360	8,559	10,686	12,739	14,647	16,366	17,851	19,015
30	2,215	5,558	8,717	11,748	14,626	17,291	19,650	21,622	23,159	24,155
35	2,134	5,375	8,431	11,305	13,975	16,360	18,346	19,843	20,763	21,189
40	1,752	4,396	6,829	9,043	11,006	12,641	13,895	14,681	15,042	15,156
45	1,310	3,249	4,977	6,472	7,701	8,601	9,142	9,385	9,462	9,477
50	928	2,262	3,410	4,349	5,018	5,474	5,672	5,729	5,739	5,739
55	558	1,329	1,944	2,400	2,684	2,820	2,868	2,879	2,879	2,879
60	325	745	1,034	1,211	1,300	1,330	1,336	1,336	1,336	1,336
65	148	325	432	486	509	514	514	514	514	514
Total	11,557	28,582	44,050	58,102	70,654	81,462	90,327	97,166	102,081	105,251
	$O^{M(5)}$ Mortality Table—Interest 3 per-cent									
20	495	1,173	1,783	2,360	2,919	3,466	4,009	4,544	5,055	5,500
25	1,538	3,809	5,958	8,043	10,074	12,047	13,892	15,564	17,016	18,160
30	2,074	5,223	8,218	11,112	13,878	16,455	18,750	20,679	22,189	23,171
35	2,013	5,087	8,003	10,764	13,347	15,668	17,610	19,081	19,988	20,410
40	1,663	4,187	6,524	8,665	10,574	12,174	13,405	14,180	14,537	14,651
45	1,251	3,114	4,785	6,240	7,443	8,328	8,862	9,103	9,179	9,194
50	893	2,183	3,299	4,218	4,905	5,325	5,521	5,577	5,587	5,587
55	540	1,290	1,891	2,339	2,618	2,753	2,801	2,812	2,812	2,812
60	316	726	1,010	1,184	1,272	1,302	1,308	1,308	1,308	1,308
65	145	319	424	478	500	505	505	505	505	505
Total	10,928	27,111	41,895	55,403	67,530	78,023	86,663	93,353	98,176	101,298
	$O^{M(5)}$ Mortality Table—Interest $3\frac{1}{2}$ per-cent									
20	427	1,017	1,554	2,068	2,572	3,072	3,575	4,076	4,559	4,985
25	1,341	3,340	5,254	7,135	8,989	10,815	12,543	14,127	15,516	16,621
30	1,828	4,633	7,335	9,978	12,537	14,951	17,125	18,971	20,430	21,387
35	1,795	4,567	7,231	9,785	12,204	14,403	16,262	17,683	18,566	18,979
40	1,503	3,809	5,972	7,977	9,786	11,317	12,507	13,261	13,611	13,722
45	1,145	2,869	4,435	5,813	6,964	7,819	8,338	8,574	8,649	8,664
50	828	2,037	3,094	3,973	4,636	5,045	5,237	5,292	5,302	5,302
55	506	1,217	1,792	2,225	2,497	2,629	2,677	2,688	2,688	2,688
60	300	693	967	1,137	1,223	1,252	1,257	1,257	1,257	1,257
65	139	307	409	462	484	488	488	488	488	488
Total	9,812	24,489	38,043	50,553	61,892	71,791	80,009	86,417	91,066	94,093

Table XXVI.

Reserves for New Bonus in Model Office.

H^M Compound Bonus Office. H^M

Central Age at Entry	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	H^M Mortality Table—Interest $2\frac{1}{2}$ per-cent									
20	583	1,410	2,200	2,992	3,805	4,648	5,534	6,461	7,401	8,271
25	1,793	4,528	7,270	10,087	12,994	15,990	18,963	21,825	24,465	26,682
30	2,388	6,126	9,891	13,747	17,653	21,517	25,169	28,432	31,154	33,036
35	2,282	5,876	9,491	13,119	16,720	20,157	23,214	25,684	27,297	28,096
40	1,859	4,769	7,626	10,403	13,032	15,373	17,293	18,578	19,208	19,421
45	1,378	3,496	5,512	7,376	9,012	10,295	11,118	11,514	11,647	11,676
50	970	2,417	3,747	4,911	5,838	6,441	6,739	6,829	6,847	6,847
55	579	1,411	2,119	2,682	3,055	3,247	3,319	3,337	3,337	3,337
60	336	786	1,117	1,333	1,449	1,491	1,499	1,499	1,499	1,499
65	153	341	462	528	558	564	564	564	564	564
Total	12,321	31,160	49,435	67,178	84,116	99,723	113,412	124,723	133,419	139,429
	H^M Mortality Table—Interest $2\frac{3}{4}$ per-cent									
20	539	1,307	2,045	2,791	3,561	4,365	5,217	6,114	7,027	7,878
25	1,668	4,227	6,809	9,480	12,255	15,133	18,009	20,793	23,376	25,555
30	2,235	5,754	9,323	13,003	16,756	20,492	24,047	27,239	29,915	31,773
35	2,150	5,556	9,006	12,491	15,973	19,317	22,306	24,730	26,326	27,118
40	1,763	4,539	7,284	9,969	12,528	14,819	16,707	17,974	18,598	18,809
45	1,317	3,351	5,301	7,114	8,715	9,975	10,787	11,178	11,310	11,338
50	933	2,333	3,627	4,765	5,676	6,271	6,566	6,656	6,674	6,674
55	560	1,370	2,063	2,616	2,983	3,173	3,245	3,262	3,262	3,262
60	327	766	1,092	1,305	1,420	1,461	1,469	1,469	1,469	1,469
65	149	334	454	520	549	555	555	555	555	555
Total	11,641	29,537	47,004	64,054	80,416	95,561	108,908	119,970	128,512	134,431
	H^M Mortality Table—Interest 3 per-cent									
20	500	1,215	1,908	2,610	3,340	4,108	4,927	5,794	6,684	7,517
25	1,554	3,951	6,384	8,918	11,568	14,335	17,117	19,826	22,353	24,495
30	2,095	5,412	8,798	12,312	15,920	19,535	22,993	26,116	28,747	30,580
35	2,028	5,259	8,554	11,905	15,274	18,528	21,453	23,836	25,411	26,194
40	1,674	4,326	6,965	9,563	12,053	14,294	16,150	17,401	18,018	18,227
45	1,258	3,214	5,101	6,865	8,431	9,670	10,471	10,858	10,989	11,017
50	897	2,251	3,510	4,623	5,518	6,105	6,397	6,487	6,505	6,505
55	543	1,331	2,009	2,553	2,916	3,104	3,176	3,193	3,193	3,193
60	318	748	1,068	1,278	1,392	1,433	1,441	1,441	1,441	1,441
65	146	327	445	509	538	544	544	544	544	544
Total	11,013	28,034	44,742	61,136	76,950	91,656	104,669	115,496	123,885	129,713
	H^M Mortality Table—Interest $3\frac{1}{2}$ per-cent									
20	432	1,056	1,666	2,293	2,953	3,656	4,414	5,228	6,071	6,869
25	1,357	3,470	5,640	7,928	10,352	12,914	15,521	18,090	20,511	22,582
30	1,850	4,808	7,865	11,079	14,420	17,807	21,086	24,077	26,621	28,406
35	1,812	4,729	7,742	10,844	14,002	17,087	19,889	22,193	23,726	24,493
40	1,514	3,939	6,382	8,816	11,177	13,324	15,118	16,336	16,940	17,146
45	1,152	2,963	4,731	6,402	7,902	9,100	9,881	10,260	10,389	10,417
50	832	2,101	3,295	4,362	5,228	5,799	6,085	6,173	6,191	6,191
55	509	1,256	1,905	2,431	2,785	2,969	3,039	3,056	3,056	3,056
60	302	714	1,023	1,228	1,339	1,379	1,387	1,387	1,387	1,387
65	138	314	429	492	521	527	527	527	527	527
Total	9,898	25,350	40,678	55,875	70,679	84,562	96,947	107,327	115,419	121,074

Table XXVII.

Reserves for New Bonus in Model Office.

$H^{M(5)}$ Compound Bonus Office. $M(5)$

Central Age at Entry	Age of Office									
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	$H^{M(5)}$ Mortality Table—Interest $2\frac{1}{2}$ per-cent									
20	609	1,457	2,261	3,063	3,884	4,734	5,625	6,556	7,498	8,369
25	1,837	4,620	7,398	10,244	13,176	16,189	19,174	22,043	24,688	26,908
30	2,427	6,213	10,015	13,902	17,831	21,709	25,371	28,640	31,365	33,249
35	2,310	5,910	9,585	13,234	16,849	20,295	23,357	25,826	27,445	28,215
40	1,876	4,810	7,684	10,471	13,108	15,454	17,377	18,663	19,293	19,506
45	1,389	3,519	5,543	7,412	9,051	10,335	11,159	11,555	11,688	11,717
50	975	2,428	3,762	4,927	5,854	6,457	6,756	6,846	6,864	6,864
55	581	1,415	2,124	2,688	3,061	3,253	3,326	3,344	3,344	3,344
60	336	786	1,118	1,334	1,451	1,493	1,501	1,501	1,501	1,501
65	153	342	463	529	559	565	565	565	565	565
Total	12,493	31,530	49,953	67,804	84,824	100,484	114,211	125,539	134,251	140,268
	$H^{M(5)}$ Mortality Table—Interest $2\frac{3}{4}$ per-cent									
20	566	1,356	2,109	2,865	3,644	4,456	5,313	6,213	7,130	7,983
25	1,713	4,321	6,940	9,640	12,410	15,337	18,225	21,018	23,606	25,788
30	2,277	5,847	9,454	13,167	16,944	20,695	24,259	27,456	30,135	31,993
35	2,180	5,624	9,105	12,613	16,110	19,462	22,457	24,885	26,482	27,274
40	1,782	4,584	7,347	10,043	12,609	14,904	16,794	18,063	18,687	18,898
45	1,328	3,376	5,333	7,151	8,755	10,018	10,831	11,222	11,354	11,382
50	939	2,314	3,642	4,782	5,694	6,289	6,585	6,675	6,693	6,693
55	562	1,374	2,067	2,621	2,990	3,180	3,252	3,270	3,270	3,270
60	327	767	1,093	1,306	1,421	1,462	1,470	1,470	1,470	1,470
65	149	334	454	520	549	555	555	555	555	555
Total	11,823	29,927	47,544	64,708	81,156	96,358	109,741	120,827	129,382	135,306
	$H^{M(5)}$ Mortality Table—Interest 3 per-cent									
20	527	1,265	1,974	2,688	3,427	4,203	5,028	5,900	6,792	7,626
25	1,601	4,049	6,521	9,081	11,763	14,550	17,344	20,062	22,593	24,738
30	2,139	5,509	8,935	12,484	16,117	19,747	23,216	26,345	28,979	30,813
35	2,059	5,329	8,656	12,030	15,414	18,678	21,609	23,995	25,572	26,355
40	1,694	4,373	7,030	9,640	12,138	14,383	16,241	17,493	18,111	18,321
45	1,271	3,241	5,136	6,906	8,475	9,715	10,517	10,904	11,036	11,064
50	903	2,263	3,526	4,612	5,537	6,125	6,418	6,508	6,526	6,526
55	545	1,335	2,013	2,557	2,920	3,108	3,180	3,197	3,197	3,197
60	319	749	1,070	1,280	1,394	1,435	1,443	1,443	1,443	1,443
65	146	328	446	510	539	545	545	545	545	545
Total	11,204	28,441	45,307	61,818	77,724	92,489	105,541	116,392	124,794	130,628
	$H^{M(5)}$ Mortality Table—Interest $3\frac{1}{2}$ per-cent									
20	460	1,107	1,732	2,371	3,040	3,751	4,515	5,333	6,180	6,980
25	1,404	3,570	5,781	8,101	10,553	13,136	15,758	18,337	20,763	22,838
30	1,894	4,907	8,006	11,257	14,625	18,031	21,322	24,319	26,866	28,653
35	1,844	4,802	7,850	10,978	14,153	17,249	20,057	22,365	23,901	24,669
40	1,534	3,987	6,450	8,898	11,268	13,419	15,216	16,436	17,040	17,247
45	1,165	2,991	4,768	6,448	7,954	9,153	9,934	10,314	10,443	10,471
50	839	2,115	3,312	4,381	5,248	5,821	6,107	6,195	6,213	6,213
55	512	1,262	1,912	2,439	2,793	2,977	3,048	3,065	3,065	3,065
60	303	715	1,025	1,230	1,341	1,381	1,389	1,389	1,389	1,389
65	139	315	430	493	522	528	528	528	528	528
Total	10,094	25,771	41,266	56,596	71,497	85,446	97,874	108,281	116,388	122,053

Table XXVIII.

Reserves for New Bonus in Model Office.

O^M Compound Bonus Office. O^M

Central Age at Entry	Age of Office									
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	O^M Mortality Table—Interest $2\frac{1}{2}$ per-cent									
20	561	1,364	2,138	2,919	3,724	4,560	5,441	6,363	7,298	8,164
25	1,745	4,427	7,133	9,924	12,810	15,787	18,744	21,591	24,219	26,424
30	2,343	6,030	9,759	13,585	17,467	21,309	24,943	28,190	30,895	32,774
35	2,253	5,812	9,399	13,004	16,584	20,003	23,045	25,498	27,109	27,907
40	1,841	4,729	7,568	10,329	12,946	15,278	17,187	18,467	19,096	19,309
45	1,368	3,473	5,478	7,332	8,961	10,236	11,056	11,451	11,584	11,612
50	964	2,404	3,727	4,885	5,806	6,406	6,704	6,794	6,812	6,812
55	575	1,404	2,109	2,669	3,041	3,233	3,305	3,322	3,322	3,322
60	331	781	1,111	1,327	1,443	1,485	1,493	1,493	1,493	1,493
65	152	339	460	526	556	562	562	562	562	562
Total	12,136	30,763	48,882	66,500	83,338	98,859	112,480	123,731	132,393	138,379
	O^M Mortality Table—Interest $2\frac{3}{4}$ per-cent									
20	517	1,261	1,984	2,719	3,481	4,279	5,125	6,016	6,925	7,772
25	1,619	4,123	6,667	9,310	12,062	14,921	17,778	20,547	23,116	25,282
30	2,189	5,656	9,187	12,837	16,566	20,280	23,814	26,990	29,650	31,501
35	2,120	5,190	8,911	12,373	15,834	19,160	22,135	24,545	26,135	26,930
40	1,745	4,499	7,226	9,894	12,439	14,719	16,596	17,859	18,481	18,692
45	1,306	3,328	5,266	7,069	8,662	9,916	10,725	11,115	11,247	11,275
50	927	2,318	3,605	4,738	5,644	6,236	6,531	6,621	6,639	6,639
55	557	1,362	2,051	2,601	2,968	3,157	3,229	3,246	3,246	3,246
60	325	763	1,087	1,300	1,415	1,456	1,464	1,464	1,464	1,464
65	148	332	452	518	548	554	554	554	554	554
Total	11,453	29,132	46,436	63,359	79,619	94,678	107,951	118,957	127,457	133,355
	O^M Mortality Table—Interest 3 per-cent									
20	477	1,169	1,845	2,537	3,260	4,021	4,833	5,695	6,580	7,408
25	1,504	3,844	6,239	8,744	11,371	14,119	16,883	19,577	22,091	24,220
30	2,048	5,312	8,659	12,142	15,724	19,315	22,753	25,859	28,473	30,298
35	1,998	5,193	8,458	11,785	15,132	18,366	21,276	23,644	25,212	25,994
40	1,656	4,284	6,904	9,485	11,969	14,190	16,034	17,279	17,896	18,105
45	1,248	3,190	5,065	6,818	8,376	9,607	10,405	10,792	10,923	10,951
50	891	2,236	3,488	4,595	5,484	6,069	6,361	6,451	6,469	6,469
55	539	1,322	1,996	2,536	2,897	3,085	3,157	3,174	3,174	3,174
60	316	744	1,062	1,272	1,386	1,427	1,435	1,435	1,435	1,435
65	145	325	443	508	536	542	542	542	542	542
Total	10,822	27,619	44,159	60,422	76,126	90,741	103,679	114,448	122,795	128,596
	O^M Mortality Table—Interest $3\frac{1}{2}$ per-cent									
20	408	1,007	1,601	2,216	2,867	3,562	4,313	5,120	5,958	6,751
25	1,305	3,360	5,490	7,748	10,146	12,687	15,276	17,828	20,234	22,290
30	1,800	4,702	7,718	10,598	14,211	17,574	20,831	23,804	26,329	28,105
35	1,779	4,658	7,640	10,717	13,851	16,915	19,700	21,987	23,513	24,279
40	1,494	3,894	6,317	8,734	11,080	13,214	14,996	16,208	16,811	17,018
45	1,140	2,936	4,691	6,353	7,844	9,034	9,811	10,190	10,319	10,346
50	825	2,085	3,271	4,331	5,190	5,759	6,045	6,133	6,151	6,151
55	505	1,246	1,891	2,413	2,765	2,949	3,019	3,036	3,036	3,036
60	300	709	1,016	1,220	1,331	1,371	1,379	1,379	1,379	1,379
65	139	313	427	490	519	525	525	525	525	525
Total	9,695	24,910	40,062	55,120	69,804	83,590	95,895	106,210	114,255	119,880

TABLE XXIX.

Reserves for New Bonus in Model Office.

$O^{M(5)}$ COMPOUND BONUS OFFICE. $O^{M(5)}$

Central Age at Entry	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years
	$O^{M(5)}$ Mortality Table—Interest $2\frac{1}{2}$ per-cent									
20	579	1,398	2,181	2,967	3,775	4,614	5,496	6,419	7,355	8,221
25	1,777	4,489	7,213	10,015	12,908	15,890	18,850	21,699	24,329	26,534
30	2,367	6,078	9,822	13,659	17,548	21,394	25,030	28,279	30,987	32,863
35	2,266	5,839	9,436	13,048	16,633	20,055	23,098	25,551	27,162	27,960
40	1,848	4,743	7,587	10,351	12,969	15,301	17,210	18,490	19,119	19,332
45	1,371	3,480	5,487	7,342	8,972	10,247	11,067	11,462	11,595	11,623
50	966	2,407	3,731	4,889	5,810	6,410	6,708	6,798	6,816	6,816
55	576	1,405	2,110	2,670	3,042	3,234	3,306	3,323	3,323	3,323
60	334	781	1,111	1,327	1,443	1,485	1,493	1,493	1,493	1,493
65	152	339	460	526	556	562	562	562	562	562
Total	12,236	30,959	49,138	66,794	83,656	99,192	112,820	124,076	132,741	138,727
	$O^{M(5)}$ Mortality Table—Interest $2\frac{3}{4}$ per-cent									
20	535	1,295	2,027	2,767	3,532	4,332	5,180	6,072	6,983	7,830
25	1,652	4,185	6,747	9,402	12,162	15,027	17,888	20,658	23,229	25,395
30	2,215	5,707	9,254	12,914	16,649	20,366	23,901	27,078	29,738	31,589
35	2,134	5,520	8,951	12,419	15,884	19,211	22,187	24,597	26,187	26,982
40	1,752	4,514	7,245	9,917	12,462	14,742	16,619	17,882	18,504	18,715
45	1,310	3,334	5,274	7,078	8,671	9,925	10,734	11,124	11,256	11,284
50	928	2,321	3,608	4,741	5,647	6,239	6,534	6,624	6,642	6,642
55	558	1,363	2,052	2,602	2,969	3,158	3,230	3,247	3,247	3,247
60	325	763	1,087	1,300	1,415	1,456	1,464	1,464	1,464	1,464
65	148	332	452	518	548	554	554	554	554	554
Total	11,557	29,334	46,697	63,658	79,939	95,010	108,291	119,300	127,804	133,702
	$O^{M(5)}$ Mortality Table—Interest 3 per-cent									
20	495	1,203	1,888	2,585	3,311	4,075	4,889	5,752	6,637	7,466
25	1,538	3,911	6,325	8,843	11,478	14,231	16,998	19,693	22,207	24,336
30	2,074	5,364	8,727	12,221	15,810	19,405	22,845	25,952	28,566	30,391
35	2,013	5,223	8,498	11,831	15,182	18,418	21,328	23,696	25,264	26,046
40	1,663	4,300	6,925	9,509	11,986	14,216	16,060	17,305	17,922	18,131
45	1,251	3,196	5,072	6,827	8,386	9,617	10,415	10,802	10,933	10,961
50	893	2,240	3,493	4,601	5,490	6,075	6,367	6,457	6,475	6,475
55	540	1,323	1,997	2,537	2,898	3,086	3,158	3,175	3,175	3,175
60	316	744	1,062	1,272	1,386	1,427	1,435	1,435	1,435	1,435
65	145	325	443	508	536	542	542	542	542	542
Total	10,928	27,829	44,430	60,734	76,463	91,092	104,037	114,809	123,156	128,958
	$O^{M(5)}$ Mortality Table—Interest $3\frac{1}{2}$ per-cent									
20	427	1,043	1,646	2,267	2,922	3,620	4,374	5,182	6,021	6,815
25	1,341	3,429	5,579	7,850	10,257	12,804	15,396	17,949	20,356	22,412
30	1,828	4,758	7,792	10,984	14,305	17,672	20,930	23,904	26,429	28,205
35	1,795	4,691	7,683	10,767	13,906	16,972	19,758	22,045	23,571	24,337
40	1,503	3,912	6,341	8,762	11,108	13,243	15,025	16,237	16,840	17,047
45	1,145	2,945	4,703	6,365	7,857	9,047	9,824	10,203	10,332	10,359
50	828	2,090	3,277	4,337	5,196	5,765	6,051	6,139	6,157	6,157
55	506	1,248	1,893	2,415	2,767	2,951	3,021	3,038	3,038	3,038
60	300	709	1,016	1,220	1,331	1,371	1,379	1,379	1,379	1,379
65	139	313	427	490	519	525	525	525	525	525
Total	9,812	25,138	40,357	55,457	70,168	83,970	96,283	106,601	114,648	120,274

ABSTRACT OF THE DISCUSSION.

MR. H. J. BAKER, in opening the discussion, said that probably the most useful purpose to which model office tables could be put was to estimate the future effect of a present change in the valuation basis. He thought most of the members would be inclined to make a group valuation if they merely wished to find the immediate cost of such a change, but the model office in its complete form enabled them to do far more than that. They had, however, been prevented hitherto from making full use of the model office owing to the lack of information in respect of bonuses. They were, therefore, greatly indebted to Dr. Buchanan for the very useful and interesting paper which, in conjunction with Mr. King's well-known tables, put them in possession, so far as whole-life policies were concerned, of a complete working model of an average life office. Model office figures required, however, considerable caution and skill in their practical adaptation to the requirements of an individual company, and the author's tables probably required more careful handling in that respect than Mr. King's. As the author remarked, the assumptions made in the calculations differed considerably from what occurred in practice, and before using the tables, very careful investigation had necessarily to be made as to how far the circumstances of a particular company varied from those applicable to the model office. In that connection he would like to refer to the statement on p. 264 that the case of an office in which bonuses had been applied to a large extent in some other way than by addition to the sum assured would probably be met by selecting a model office younger than its calendar years would show. It would seem, however, that that was not necessarily so. If, as might well be the case, the lives taking their bonuses in the form of cash or reduction of premium were, on the whole, younger than those preferring the addition to the sum assured, the model office chosen should be, if anything, older than the actual company. In other words, the age distribution of the bonus additions was most important. Turning to the tables, he was glad to see that the author gave the net premiums corresponding to the business in force in the model office, and also the values of sums assured and net premiums separately. The absence of those tables had hitherto limited to some extent the practical usefulness of Mr. King's tables. It was true that Mr. King's ingenious formulas enabled one to arrive at these figures, but only at a cost of a considerable amount of labour. By means of Tables XX and XXI, giving the reserves for existing bonuses, it became possible to accurately estimate the effect of bonuses on a change in the valuation basis. It would be found that the cost of reducing the valuation rate of interest by 10*s*. per-cent, when bonuses were taken into account, was, roughly speaking, increased by $\frac{1}{2}$ per-cent of the reserves, while, on the other hand, the cost of changing from the H^M Table to the O^M Table was reduced by about $\frac{1}{2}$ per-cent of the reserves, it being assumed, of course, that the bonus additions were equivalent in amount and age distribution to those of the model office. He had had occasion to investigate the effect of a change of valuation basis in the case of an office distributing the whole of its surplus in the form of

permanent reduction of premium and valuing that reduced premium. He found that the cost of reducing the valuation rate of interest by 5*s.* per-cent was over 4 per-cent—nearly double the amount required in the case of an office making a net premium valuation and allotting reversionary bonuses. Perhaps the most interesting section of the paper was that which dealt with the analysis of profits, and some of the results brought out by the tables were very instructive. For example, although it was well known that the margin of loading was greater by the O^M Table than by the H^M, yet the precise extent of the difference between those two tables in that respect might not be so generally realized. Taking the case of a compound bonus office spending $12\frac{1}{2}$ per-cent of its premiums in expenses of management and commission, experiencing $O^{[M]}$ mortality and earning 4 per-cent interest, the results of an H^M 3 per-cent net premium valuation were, assuming that the office could be represented by the model office of 25 years' standing—Loading profit £21,427, interest profit £21,000, mortality profit £14,165; while the figures on an O^M 3 per-cent basis were—Loading profit £28,935, interest profit £21,513, mortality profit £5,480. The results for the 50 years' model office were H^M 3 per-cent—Loading profit £29,854, interest profit £50,960, mortality profit £16,390; O^M 3 per-cent—Loading profit £40,275, interest profit £51,070, mortality profit £5,550. In each case, the loading profit was increased by the substitution of the O^M Table for the H^M Table by no less than 35 per-cent, while the mortality profit was reduced in the younger office by 61 per-cent and in the older office by 66 per-cent. With regard to mortality profit it should be pointed out that Tables XI, XII and XIII related to one year only, and not, as in the case of the other tables, to a quinquennium. From those tables, taken in conjunction with the table on p. 199 of Vol. ii of the *Transactions of the Faculty of Actuaries*, it would be seen that in the model office experiencing $O^{[M]}$ mortality, the mortality profit, when reserves were based on the O^M table at 3 per-cent, increased from £529 in the first year to a maximum of £1,352 in the eighth year, and after the 15th year remained practically constant at about £1,000. The profit on the basis of H^M 3 per-cent reserves was £710 in the first year, and steadily increased with the age of the office, being £3,278 at 50 years, and nearly three times the corresponding O^M figure. He was disposed to think that the mortality experience of individual companies varied within wider limits than was generally supposed. For example, in the model office of 50 years' standing, experiencing $O^{[M]}$ mortality, and valuing by the H^M Table at 3 per-cent, the actual death strain was 91 per-cent of the expected, while in one of the companies which contributed its data to both the H^M and O^M experiences, which made each year a careful analysis of its claims, and whose business consisted almost entirely of whole-life policies, the actual death strain had been on the average only 74 per-cent of the expected by the H^M table during the last 29 years, and in no year had there been a loss from unfavourable mortality. The average profit from mortality had been greater than the average profit from interest. On the other hand, it was extremely probable that some offices experienced on the average a mortality less favourable than that shown by the table of mortality on which their

reserves were based. He believed that comparatively few offices systematically compared the expected death strain with their actual death strain, but unless that was done, no reliable estimate could be made of the mortality profit or loss. They still had no model office tables for endowment assurances, and having regard to the great and growing importance of that class of business, he was afraid that until they had such tables, the model office would only be of limited use in office work. It was becoming a growing practice for offices to classify in their Board of Trade returns their endowment assurances according to year of maturity, and it would be of great service if they would also state the mean valuation age of those policies maturing in the same year—a course which was already followed by at least one company. A very great amount of thought and labour must necessarily have been expended in the preparation of such tables as those before the Institute, and their thanks were consequently all the more due to the author for so willingly devoting his energies to the service of his profession.

Mr. G. King said he had a certain amount of hesitation in speaking, on account of the very frequent references the author had been good enough to make to his own work; and he had the feeling, which was probably altogether a mistaken one, that if he spoke very favourably of the paper it might be thought he was taking rather a prejudiced view, and looking with too much favour upon a paper which had a close relationship to his own work. It was a very valuable paper, which he thought would become more valuable when they had learnt how to use the tables. He thought it would have been a great advantage if the author had seen his way to give some practical examples of the uses of the tables. The author had left them to find these out for themselves, and busy men sometimes did not take that amount of trouble. The amount of labour that the author must have expended was enormous, and, on looking at the extent of the author's tables, and the intricate amount of calculations required to produce some of them, he thought that probably the author's arithmetical work was greater than that involved in the first paper which he (Mr. King) had read. The author had given them tools which they might apply to many purposes, and which they would find useful for purposes not apparent on the surface. The objects of the paper were stated on p. 260, namely, to enable a close estimate to be made of (1) what it will cost to make a reserve for existing bonus on a given basis; (2) what it will cost to declare a new bonus at a given rate. Speaking on the first point, he would like to make a remark upon the distribution of the existing bonuses. The subject had been already to a certain extent dealt with by Mr. Baker, but he (the speaker) had gone a little further. He had taken the figures from Table II for an office of 50 years' standing and compared it with a little table in his own paper (*J.I.A.*, xxxvii, 474) where he gave the existing bonuses in five large and typical companies. It was not possible to make an exact comparison, because the age groups differed slightly, but it was possible to get one close enough for practical purposes.

Taking total bonuses of 1,000, the following is the comparative distribution in four broad age groups.

King.		Buchanan.	
Below 42	64	Below 40	**41**
43–62	490	41–60	**400**
63–82	414	61–80	**525**
83 & over	32	81 & over	**34**
	1,000		**1,000**

It would thus seem that at the younger ages the proportionate amount of bonus in his (Mr. King's) table was considerably greater than in the author's, and at the older ages the reverse was the case. The result was that any estimates based upon the author's table would, he thought, rather exaggerate the cost of changing the valuation basis in reference to the bonus, and also the cost of new bonuses; and it would be necessary, as the author pointed out, to take an office of a younger age—to find out, in fact, which age of an office would give something approximating to the bonus distribution according to the conditions of the actual company to be tested. It would, of course, not be difficult to get that information. Probably the discrepancy arose from the fact that the author assumed that all bonuses remained in reversion, whereas his (Mr. King's) table was based upon the figures of actual companies, and a considerable amount of the bonuses had probably been surrendered: that would of course reduce the proportionate bonuses at the older ages. With regard to the valuation of simple bonuses, he would like to point out that it was rather anomalous to give tables for the valuation of simple bonuses at very low rates of interest. Taking Table XX, it would be found that the author gave the reserves for existing simple bonuses at $2\frac{1}{2}$ per-cent and $2\frac{3}{4}$ per-cent, and so on. He was of opinion that an office which valued its bonuses at those rates had no right to give simple bonuses; it was keeping back a portion of the profits and acting unfairly to its present members. The bonus did not require to earn a bonus, being a simple bonus, and therefore it should be valued at something nearer the practical rate of interest. He had occasion some little time since to go rather fully into the question of the reserves required for maintaining bonuses, both simple and compound. He found that in the case of a company with a practical rate of interest, which, at the present day might be estimated at $3\frac{1}{2}$ per-cent (that was probably a little below the actual rate, but not so much so as to leave more than the necessary margin of safety)—if they declared simple bonuses of 30*s*. per-cent per annum, and valued by the O^M Table, to maintain the bonus the sum assured should be valued at a little below $2\frac{3}{4}$ per-cent interest, the existing bonuses being valued at $3\frac{1}{2}$ per-cent; but if it was thought undesirable to value the bonus at a different rate of interest from the sum assured—although he did not see why that should be so—it would be sufficient if they valued both the sum assured and the bonus at 3 per-cent. Practically the same result was obtained by either method, and that would enable the company to maintain its bonus of about 30*s*. as a simple bonus. The method of comparison was arrived at by valuing the policies, taking into account future bonuses, and the premium to provide them, and valuing at the true rate of interest, $3\frac{1}{2}$ per-cent. If that was done,

it was practically the same as making a net premium valuation of the sum assured and existing bonuses at 3 per-cent. Therefore, it might be said that a margin of about 10*s*. per-cent in the rate of interest was sufficient in order to maintain a simple bonus of about 30*s*. Taking the O^M Table, rather different results were obtained from what came out by the H^M, and therefore it was necessary as time went on to revise their views on that point. It was a curious fact that the reserve for the sum assured, looked at by itself, if one wished to maintain a compound bonus, was rather less than if one wished to maintain a simple bonus. When that fact first came before him, he thought he must have made a mistake in the calculations, because it looked absurd on the face of it; and yet it was true. He went over the whole question again and worked out why it should be so, and found the reason; he was then very pleased to find in Mr. G. F. Hardy's paper upon discounted bonuses (*J.I.A.*, xxxi, 261) that the same fact was recorded there; and being thus confirmed, they might take it for granted it was true. The reason was that in valuing future bonuses and the original premiums necessary to produce them, as time went on the difference between the premium required for compound bonus and the premium required for simple bonus diminished, the consequence being that the compound bonus depended more on future premiums than the simple bonus, and thus required a smaller reserve for the sum assured; but, on the other hand, the reserve required for existing bonus where the bonuses were compound was much greater than that required for simple bonuses; the result being that when sum assured and bonus were taken together, they might, for compound bonuses, either value the sum assured, with provision for future bonuses, and the premium corresponding, all at the practical rate of interest, $3\frac{1}{2}$ per-cent, or they might value the whole by the net premium method at about $2\frac{3}{4}$ per-cent. It would therefore appear that for compound bonuses to be maintained at a rate of about 30*s*. per-cent, they required to keep 15*s*. per-cent margin of interest by the O^M Table, but it did not make much difference if they took the $O^{[M]}$ select. His figures were worked out before the select tables were published, and he just gave them as they stood; he had not had time to revise them with the most recent data, but he had tested them, and had found there was not any very material difference. Mr. Baker had also referred to the question of endowment assurances. He would like to add a word to what he said on the same subject in his last paper on the model office. He there gave (*J.I.A.*, xxxvii, 473) a certain rule for making allowance for endowment assurances. That rule was correct as far as it went, and was correct according to the example given, but he had confined his attention to a change in the rate of interest in the valuation. The rule had not gone far enough, and it did not apply when the table of mortality was changed. He did not exactly know what the effect would be of changing the table of mortality from the H^M to the O^M in valuing endowment assurances, but the rule in his paper would not apply; the difference would be much greater in the endowment assurance valuation than his remarks in that paper would lead one to think. He therefore would like to make, he would not call it a correction, but an addition to what he said about the

endowment assurance valuation and the effect of changes of valuation bases. He was afraid, however, that at the present day the vast mass of endowment assurances on the books of a company upset, to a large extent, the calculations made by the model office, and reduced very considerably the advantage which the model office gave them. When the model office was first brought out it gave remarkably close results, as many examples showed; but now-a-days it could not be trusted quite so fully. Especially in dealing with the cost of new bonuses, the endowment assurances would have a still greater disturbing influence than in regard to the sums assured, because the cost of bonuses to endowment assurances was very great compared with the cost of an equal reversionary bonus to a whole-life policy. He did not mean to imply that too much bonus was given to the endowment assurances, because he had shown on previous occasions that in practice an endowment assurance at the ordinary rates of premium current in the market was entitled to a reversionary bonus equal to the reversionary bonus on the whole-life policy. Other enquirers had come to the same conclusion; nevertheless, the cost of the bonus was heavy, and upset the calculations of the model office. He would conclude by thanking the author for the very great trouble he had taken to help the members by supplying them with such valuable tables, and by again expressing the hope that he would see his way to give some practical examples of how to apply them.

The President was sure the members would join with him in passing a very hearty vote of thanks to Dr. Buchanan for his admirable paper, which must have cost him a very great deal of labour in its preparation.

The resolution was then put and carried with acclamation.

Dr. Buchanan, in reply, thanked the members very cordially for the reception they had given to his paper. He was induced to take up the subject because it seemed to him to be an extremely promising one from an educative point of view. He felt that he might be able to put the working of a life office under a microscope, and he hoped the results of the investigation would be of some use to the profession generally. That was a question for the profession to determine, especially the senior members of it; but the reception which had been given to his paper would always make him look back with pleasure to his introduction to the meetings of the Institute. Mr. Baker had referred to the assumptions underlying the bonus tables. He did not pretend that these assumptions would ever be realized in practice, but the same might be said of the assumptions underlying the construction of the model office, and experience had shown that it gave wonderfully accurate results. Might not one, therefore, expect similar results with the bonus tables? He quite recognized that success must always depend largely on the skill and judgment of the actuary using them. The point upon which he had endeavoured to lay stress was that by means of the model office it should be possible to estimate the effect of a change of valuation basis on surplus accruing from certain sources with a degree of accuracy approximating to that with which they could estimate reserves. It might perhaps be thought he had devoted too much attention to the question of the incidence of surplus as between policies of different ages at entry;

but the question seemed to him to be one of more than academic interest. Fifty years ago a distinguished member of the Institute said that the offices which were most remarkable for large accumulations were those which charged a fairly adjusted scale of premiums, for it did not matter to those offices at what ages the business came in.* If that statement were true then, was it not *a fortiori* true at the present day, when business was so keenly canvassed, and when the public was probably more alive to the bonus merits of different policies than it was then? If an office gave too large a share of surplus to any particular class of policy, was it not likely to do the bulk of its business at those rates which were less profitable to itself and ultimately to all concerned? Reference had been made to the extension of the model office to endowment assurances. That extension would be one of much interest, but at present it did not seem to him there were data at their disposal on which to base such an office. The data of the new experience gave them no information in regard to the endowment term, and that seemed to him to be quite as important as the age at entry in the construction of a model office for endowment assurances. He again thanked the members for the reception they had given to his paper. He wished especially to thank Mr. Lidstone, whose suggestive letter prompted the paper, and Mr. King, for allowing him to work out a subject which was peculiarly his own.

[Dr. Buchanan has sent to us the following additional note for insertion in the *Journal*.—Ed. *J.I.A.*]

It has been suggested to me that it would be desirable to add a note explaining certain irregularities in the figures of Tables XII and XIII of the preceding paper. These are clearly shown in the following table, giving the amounts of mortality profit made in the Model Office, assuming $O^{[M]}$ Mortality to be experienced.

Expected less Actual Death Strain in Model Office experiencing $O^{[M]}$ Mortality.

Age of Office	Valuation Basis O^{M} Table throughout			
	$2\frac{1}{2}$ per-cent	$2\frac{3}{4}$ per-cent	3 per-cent	$3\frac{1}{2}$ per-cent
5 years	1,242	1,245	1,246	1,252
10 ,,	1,295	1,300	1,299	1,304
15 ,,	1,130	1,136	1,132	1,133
20 ,,	1,043	1,047	1,043	1,041
25 ,,	998	1,000	996	992
30 ,,	982	983	979	973
35 ,,	983	983	978	973
40 ,,	994	993	988	985
45 ,,	1,006	1,006	1,000	1,001
50 ,,	1,016	1,019	1,010	1,015

The point to which attention is drawn is the fact that the above figures do not throughout show, as they ought to do, a small increase in mortality profit corresponding to an increase in the valuation rate of interest.

* [The reference is to a paper on "The Value of Selection", by the late Mr. J. A. Higham, read before the Institute in 1850, and reprinted in vol. xx of the *Journal*. The particular reference is to the paragraph commencing at the foot of page 8 of that volume.—Ed. *J.I.A.*]

Using four-figure logarithms, the expected claims and their values were taken out to one decimal place for each age at entry and each duration separately; these were summed in quinquennial groups, and the differences of these totals, giving the expected death strain, were cut down to the nearest integer. This may introduce an error of $\pm\cdot5$; and as, even for the 5-year office, each of the figures quoted above depends on 20 such differences, involving 200 entries in a table of logarithms, it is evident that small irregularities may be expected. In the present case these irregularities are comparable with the changes due to a change in the valuation rate of interest.

A more regular progression would doubtless have been secured if the decimal places had been retained to the end of the work; but to do this seemed to be straining after accuracy in the last figure, which the tables do not claim to possess, and which model office estimates scarcely can possess. Practically, the figures show that, in the case of an office valuing by the O^{M} Table and experiencing $O^{[M]}$ mortality, the effect on mortality profit of a change in the valuation rate of interest is inappreciable.

This is due to the fact that this profit is earned chiefly in the first quinquennium, where the reserve values are small, and where a change in the valuation rate of interest only slightly affects the death strain, and the above statement would require modification in the case of an office whose mortality experience was throughout better than that of the valuation table. From the figures of Table XII some idea may be formed in such a case of the extent of the variation of mortality profit due to a change in the valuation rate of interest, by assuming the office to experience a hypothetical rate of mortality, such as a given percentage of that of the O^{M} Table.

On the Importance and Practicability of a Standard Classification of Impaired Lives. By S. W. Carruthers, M.D., *Medical Officer of the Mutual Life Insurance Company of New York.*

[Read before the Institute, 17 April 1905.]

I HAVE heard that a former distinguished President of this Institute once publicly stated that "Doctors rush in where Actuaries fear to tread." It is also said that "at forty a man is either a physician or a fool": but the presidential saying seems to suggest that a man may be both. With such a warning ringing in my ears, I yet venture to read before this learned Institute a paper on a thorny subject. I am doing so, first because that subject can be dealt with only by the united labour of actuaries and doctors; and second, because I am sure of the consideration and forbearance of the Fellows in any matters wherein I travel with halting gait.

By a "Standard Classification of Impaired Lives" I mean a system by which every office would be enabled to "pigeon-hole" (so to speak) lives with similar impairments in uniformly labelled

pigeon-holes: so that at any time a collective investigation could readily be made of the contents of one or more of these pigeon-holes, with a certainty that the material contained therein was at least sufficiently homogeneous to furnish a mortality experience of practical value.

On the importance of such a classification it is hardly necessary for me to enlarge before this Institute: I would merely like to emphasize the fact that it is deemed of prime importance by many physicians, more especially (I am bound with regret to admit) on the Continent, where perhaps medical directors as a class make a more scientific study of the medical side of insurance than is done in this island. That the need is not unfelt in Britain, however, is evident by the crisp description of it by Sir William Gowers, only a few months ago, in a paper read before the Life Assurance Medical Officers' Association. "I do "not know whether the effort has yet been made by this "Association, but it would be most desirable to make an attempt "to secure the actual tabulation and classification of the facts of "medical importance on a uniform system in every office, so that "they could be ascertained for a series of years without much "labour and without much time, and could be easily combined "and classified. Such a scheme would be worth much trouble. "It should be simple and not elaborate. But I must not pursue "this subject further."

The matter had been in my mind, in somewhat nebulous form, even before May 1902: but it first took definite shape at a meeting of the Life Assurance Medical Officers' Association in that month. Dr. Hingston Fox read a suggestive paper "On the Method of making Advances in Impaired Lives": his first two conclusions were as follows: "1. A strictly scientific method of "making advances in impaired lives is not at present possible. "2. An approximately true method is found by the medical "examiner estimating the extent to which the expectation period "is diminished by the defect present."

In the discussion which followed I ventured to make some remarks, from which I quote the following sentences: "It is "important to see whether any practical issue arises out of this "question. It seems to me that our radical difficulty as medical "men is that we do not treat impaired lives by the same method "as healthy lives. All insurance work can only be done by the "doctor and the actuary in co-operation. The work of insurance, "as far as it deals with healthy lives, is an absolutely mathematical "work, based on facts drawn up into mortality tables. The

"share of the doctor in producing these has been the humble "one of classification, separating the healthy from the impaired "lives. The doctor having done that, the actuary works out the "mortality tables. It seems to me that the only way in which "we can face the question of impaired lives is by a similar "division of the work; the doctors must classify impaired lives, "and then the actuaries work out the figures. . . . . We have to "remember that healthy lives are a single class, while impaired "lives consist of manifold classes, classes almost innumerable; "and an enormous mass of statistics is necessary to give reliable "results in regard to these impaired lives. . . . . Dr. Hingston "Fox says that 'an approximately true method is found by the "medical examiner estimating the extent to which the expectation "period is diminished by the defect at present.' . . . . It seems to "me that an *estimate* is not what is wanted; we want to "*ascertain* the extent to which the expectation is diminished by "the defect present. Only actuaries can do that, and they can "only do it after we doctors have classified the cases for them. "That leads me to the practical point. It has been said we "can never get the data in our time. That may be true. Long "experience is necessary. But the data will never be got unless "we begin. We must have enough disinterestedness to work for "posterity, and if this Association could devise some scheme for "classifying extra risks, I am sure the actuaries would be "very grateful; and that in England work could be done quite "as good as that which has been commenced [in the Specialized "Mortality Investigation] in America."

Pardon my quoting at such length what must be to you a mere bundle of truisms. The position is elementary, but it seems to me to be fundamental. There is but one thing I would alter. In criticizing Dr. Fox, I was entrapped by his use of the expression "Expectation Period" into using it myself, instead of speaking of mortality tables, to which I had already referred in the earlier part of my remarks. Of course what we want to ascertain is not the expectation period (so dear to many doctors), but the mortality tables for different classes of impaired lives.

My appeal did not meet with a response, which was not to be wondered at, for it came from a junior and inexperienced member of the Association. But the idea was too fascinating to forget; and, not succeeding in getting the Life Assurance Medical Officers' Association to tackle the difficulties of the labyrinth,

I proceeded in the happy confidence of youth and inexperience, to explore it alone. I am sanguine enough to hope that I have found an Ariadne clue; a clue which is, I think, thoroughly workable, though it will need to be perfected for practical use by the labour of wiser heads than my own.

Before briefly describing it to you, permit me to state the desiderata which any workable scheme must meet.

First and foremost, it must be reasonably easy of comprehension, and must lend itself to ready memorising of its main features, and of its most frequently used portions.

Secondly, it must possess a concise and legible notation, so that once a case has been classified it can be briefly docketed in such a way as to be readily picked out at any subsequent time. If these two requirements are fully met, it insures that a minimum of time will be spent in operating the system.

Thirdly, it must allow ample provision for the increase of our knowledge. Increased knowledge will necessitate the creation of new classes, and the subdivision of existing ones to a greater degree, or even along new lines. And it must be possible to do this without any wholesale recasting of the system.

Fourthly, while it must be up to the limit of our knowledge, it must recognise the fact that the outer fringe of our knowledge is not always of much practical value. To take an instance of what I mean: we are able at present to say definitely that certain pleurisies are tuberculous, and that certain others are not, but we cannot definitely place every pleurisy in one or other of these categories. While we must have a pigeon-hole for those which can be definitely included in the tuberculous class, and one for those which can be as definitely excluded, we must at present have one also (whose contents will be more than those of both the others put together) for pleurisies which we cannot definitely classify. And it must be easy to throw these three classes into one, with a view of getting out a reliable average mortality, which any company might use as a basis, loading more heavily (or declining if it saw fit) the undoubtedly tuberculous case, and granting more favourable terms to the case clear of all suspicion of tubercle.

Furthermore, there will ever be two great mental temperaments, known in the botanical world by the expressive names of "lumpers" and "splitters", and the classification should be such a one that a man of either temperament could use it, the "lumper" not being forced to split up his cases into endless varieties, nor the "splitter" compelled to place together cases which

he deems to fall into entirely different categories of risk. And if in any office (or group of offices making a common investigation) an Amurath does *not* an Amurath succeed, but a peaceful revolution takes place, then the "lumper" should be able, with the minimum of trouble, to neutralize the excessive splitting of his predecessor, or the "splitter" to parcel out what his forerunner had lumped together.

This is no mean programme. A scheme which passably meets the four requirements thus laid down will, I dare to say, find no actuary, and few medical men engaged in insurance work, to question its importance.

But what of its practicability? Are you inclined to think that I have sketched an outline too large to fill in? I hope to show you that this is not so. But I claim little credit, for the scheme which fills it is not mine, but is actually adopted in its entirety, and merely turned to a new purpose.

Two things I must say of it before entering into particulars. First, *all* the details are purely tentative; it is nothing but a first rough draft, whose faults are manifest. It is the *method* which I wish to commend to your consideration, the *details* would have to be most thoughtfully considered by a select body of experienced men. Secondly, it is no mere theoretical structure. For two whole years I have used it, imperfect as it is, and have docketed some 4,000 cases with it. I have proved its usefulness by the great rapidity with which it enables one to elaborate statistics on any given matter, thus saving in one single investigation a large proportion of the time spent in its original application. For instance, I recently investigated 3,000 applications in regard to family history of cancer, and by this method was enabled to ascertain in slightly over an hour and a half the precise incidence of cancer among the relatives of these 3,000 cases. I may also claim that it requires a minimum of time: after a short use of it, it becomes possible to docket cases once and for all in the average time of two-thirds of a minute to each (549 cases in 367 minutes), even if one has to take up the record specially for the purpose: whereas, of course, if the docketing were done at the time when the case is under the consideration of the medical director, and all its features are in his mind's eye, the time used would be even shorter. (It must also be remembered that I have intentionally been using the system with its fullest detail, so that a "lumper" would need even less expenditure of time in operating it.)

Those of you who are interested in library classification and cataloguing will immediately admit that my claims are justified when I say that the scheme which I advocate is merely an application of Melville Dewey's great plan of "Decimal Classification." To others, however, this conveys no meaning: and for them I must enter into details. Some of these details I cannot better give than in the words of the author of the system, modified (if necessary) so as to apply to the new use to which I suggest putting it.

The essential feature of the system is that it uses the Arabic numerals as indicators of classes, not using names at all; but it so uses the numerals that their arrangement recalls the subject. This is effected by dividing the matter to be classified (in the present instance the different sorts of impairments which may occur) into ten main classes. The ten main classes which I have been using are as follows: (Pardon my emphasizing again, before giving the detail, that the details are merely accidental and experimental, and must be subject, before any satisfactory standard could be agreed on, to the most searching criticism, and probably to many radical alterations; the important matter is the principle of the scheme, which can, however, only be elucidated by taking details as examples.)

000 Physique and Development.
100 Family Longevity.
200 Diatheses and Zymoses.
300 Tuberculosis.
400 Circulatory and Urinary.
500 Respiratory.
600 Alimentary.
700 Nervous System.
800 Surgical.
900 Miscellaneous.

It will be evident to all that this is not a scientific classification, for the last item at least has no scientific basis; and to the doctor's eye there are errors from a scientific point of view throughout. But this is in accord with the spirit of the system. "The impossibility of making a satisfactory classification "of all knowledge has been appreciated from the first, and "theoretical harmony and exactness have been repeatedly "sacrificed to practical requirements."

To give a further idea of the system we will take a single example of its smaller sub-divisions, the decade numbers, and the

unit numbers. The decade numbers are used (as a rule) for individual diseases or closely allied groups: the unit numbers for details in regard to these diseases. This can be best understood by taking an example. Thus the decades under the century 500 (Respiratory Diseases) are as follows:

510 Pneumonia.
520 Pleurisy.
530 Bronchitis.
540 Asthma.
550, 560, 570 [Blank].
580 Naso-pharynx.
590 Miscellaneous Respiratory.

Then 530 is further subdivided in unit numbers.

531 Family History. Subdivide thus:
- ·1 One grandparent.
- ·2 One collateral.
- ·3 One parent.
- ·4 Two collaterals.
- ·5 One collateral and one parent.
- ·6 Both parents.
- ·7 Other "two's."
- ·8 More than two cases.
- ·9 Doubtful cases.

532 One attack. Subdivide thus:
- ·1 Less than five years ago.
- ·2 Five to nine years.
- ·3 Ten to fourteen years.
- ·4 Fifteen to nineteen years.
- ·5 Twenty years or more.

533 Recurrent attacks.
- ·1 "Winter cough."

534 Chronic Bronchitis.
535 Emphysema.
536 Present (rhonchi).
- ·9 Harsh respiratory murmur (general).

537, 538, 539 [Blank].

It will at once be understood that many cases will have two or more numbers: thus, a man who in addition to a family

history of bronchitis (in mother's case) had also had an attack himself six years ago, would be docketed 531·3 and 532·2. It is perhaps as well to give an instance of a case with a number of flaws—in fact one so bad that it would be an impossible risk for acceptance.

> *Family History.*—Father died, *æt.* 53; apoplexy. Father's father died, *æt.* 60; gout.
>
> *Personal History.*—Has had three slight attacks of gout in toe; the last one about 18 months ago. Has occasional heartburn and acidity.
>
> *Present Condition.*—Weight 12 st. 12 lbs.; Height 5 ft. 5 in.; Chest 33 in., 32 in.: Abdomen 34 in.; Arterial tension somewhat increased, some accentuation of the Aortic second sound; slight Albuminuria (in two specimens): S.G. 1011, 1007.

This case would have a large series of numbers, as follows:—034, 053, 221·13, 224·1, 446·2, 463·92, 614, 721·3.

The meaning of these numbers is as follows:—

> 034: More than 25 per-cent over weight, 030; with abdomen more than expanded chest, 034.
>
> 053: Chest expansion less than 2 inches.
>
> 221·13: Family History of Gout, 221; grandparent certain, parent doubtful, 221·13.
>
> 224·1: Personal History of Gout, two or more attacks, 224; last attack less than 5 years ago, 224·1.
>
> 446·2: Pulse abnormalities, 440; high tension, 446; with accentuated second sound, 446·2.
>
> 463·92: Albuminuria, 460; slight but constant, 463; not known to have existed before, 463·9; with S.G. below 1012, 462·92.
>
> 614: Dyspepsia, 610; recurrent 614.
>
> 721·3: Apoplexy, 720; family history, 721; one parent, 721·3.

This seems very elaborate to read, but is most simple once the rudiments of the system have been mastered; and after but little practice this case could have been fully docketed with the eight numbers given above in well under two minutes. Whatever particular flaw was being investigated (whether overweight, albuminuria, gout, dyspepsia, or any of the others) this case would then be included in the investigation; and of course, in dividing either of these classes further, the other numbers would

be useful; thus, if investigating albuminuria cases, this would be at once put in a most important sub-class, albuminuria in the gouty; and so on.

It will also be seen that once a case is docketed, a junior clerk can be employed at a minimum expense to sort out the material for any investigation, being told simply what numbers he is to take out for the doctor or actuary.

"Wherever practicable, heads have been so arranged that each "subject is preceded and followed by the most nearly allied "subjects, and thus added convenience is secured." Thus 220 Gout comes between 210 Rheumatism and 230 Diabetes (or Glycosuria), to both of which it is allied; so also 510 Pneumonia is followed by 520 Pleurisy; and 530 Bronchitis by 540 Asthma.

The system has been made easy of recollection by using the same figures with the same meanings in many cases. Thus a final "1" almost always means Family History; for example, to to take the classes already referred to, 211 is Family History of Rheumatism, 221 of Gout, 231 of Diabetes, 511 of Pneumonia, 521 of Pleurisy, and so on. For those who care to subdivide further, a set of decimals is used to indicate the particular degree of consanguinity of the tainted relatives to the applicant, and thus the strength of the taint. (This set of decimals is, of course, uniform for all family histories.) Similarly, in applicants where serious disease has occurred in the past, and where the degree of impairment may be affected by the period which has elapsed since its occurrence, a simple arrangement of decimals allows this period to be recorded.

Decimals are often used for matters which many men may consider of minor importance, but which some may deem worth separating into different classes. Thus urines with sugar or albumen are divided according to whether the specific gravity is normal or abnormal (*i.e.*, high in the case of sugar, low in that of albumen).

While Dewey's original system utilizes every one of the numbers from 000 to 999 (in addition to a very elaborate, though unequally distributed, subdivision by means of decimals) I have not found it necessary to use the whole of these numbers. And I have not tried to fill up the blanks, for the numbers thus left vacant will find their use when additions or alterations are required by the progress of medical science, by enabling these to be made without recasting the whole classification. There are

even some of the "decade" numbers vacant besides very many of the "unit" numbers.

One matter may be referred to, which really belongs more to the sphere of the committee of experts whom I hope some day to see at work polishing and perfecting this scheme. This is the question—difficult at first sight—how much detail is desirable or permissible. I say difficult at first sight, for I believe it can be solved in one way only, by elaborating the fullest detail. For adopting this solution in my draft I had three reasons: first that I desired to find the weak points of the scheme, and to use it at the greatest disadvantage as to expenditure of time and trouble, to see whether it would break down under severe tests. Second, that my own mental temperament leans towards subdivision. Third, that one can never tell when an apparently unimportant class may become of great interest. Of this latter reason, let me give you an apposite illustration. When I made out my draft, if there was one point which above all others seemed to me to be a needless refinement it was allotting a section to Infectious Diseases, and differentiating the more important of them. Thus we have 281 Influenza, 282 Small-pox, 283 Scarlet Fever, 284 Diphtheria, 285 Typhoid, and so on. But during the two years which have elapsed, a suggestion has been made, on high authority, that typhoid fever predisposes to that class of arterial degenerations which, coming on in middle life from 45 or 50 onwards, prove so disastrous to insurance companies. Now, though it is on high authority, I am somewhat inclined to doubt this; but were this classification in general use, the suggestion, which is most important if true (not in the way of loading, but of putting examiners specially on their guard as to the state of the arteries in such cases) could have been definitely proved or disproved in six months by a comparatively inexpensive investigation of all cases docketed by the various companies as "285", to see whether there was an excessive mortality from arteriosclerosis and allied complaints.

The first of these reasons for using much detail applies to myself and to the preliminary investigation only; the second appeals to those who have a mental temperament similar to my own; but the third reason must appeal to all alike; and for that reason alone one strongly advocates wealth of detail. (The detail need not in the least confuse the use of the system, for a judicious employment of large and small print—if ever it reaches the stage of being printed—would secure entire clearness. This

has been most successfully done in Dewey's original classification, containing some 130 pages of detail.)

It may be objected that from many examination forms in use the details cannot be obtained; and I know that the present tendency in British circles is towards a simplification of the examination forms; but Britain is the exception, for the opinion on the Continent and in America, though certainly not unanimous, is strongly on the side of a detailed examination form. Companies where the detail is insufficient can always omit the decimals, or even treat (say) class 520 Pleurisy as comprising all varieties of Pleurisy (included under 522 to 527 in my classification, 528 and 529 being blank numbers). The numbers ending with 0 are kept free for precisely this purpose, all cases where details are deficient being classed under the general head.

This process, like many excellent processes, is far and away easier to carry out than to describe; I only ask that what I believe to be a thoroughly practical process may not be misjudged by you because of the difficulty of explaining it. I should like to conclude with a quotation from Dewey (for whose practical genius I have a very high admiration) in connection with one of the main objections that theoretical critics have made to his scheme.

"Theoretically, the division of every subject into just ten "heads is absurd. Practically, it is desirable that classification "be as minute as possible, without use of added figures; and the "decimal principle, on which our scheme hinges, allows ten "divisions as readily as a less number. The principle has proved "wholly satisfactory in practice, though apparently destroying "proper co-ordination in some places. The difficulty in such "cases is entirely obviated by the use of another figure, giving "nine sub-sections to any subject of sufficient importance to "warrant closer classification. . . . . We have not sacrificed utility "in order to force subjects on the decimal procrustean bed. "Decimals have been used as servants, not as masters. When "subjects have been separated or combined into just ten heads, it "has been from no necessity of the subject, but because it seemed "the most useful way, all things considered. . . . . There has "been a perverse misapprehension of this feature, and critics "oftenest stumble over 'procrustean ten.' In fact this is an "element of usefulness. A railroad also has the fault that it is "procrustean in its path and its times. It cannot come to your

"door, nor wait your convenience, as did the stage coach or "carriage; it cannot go to the fields for its loads of produce; "it cannot turn out of the way for obstacles; and it is simply "because it is so procrustean that it can do its work so much "better, and quicker, and cheaper. The parallel could fairly be "extended to many other cases, but any thoughtful mind will "recognize that the economy and ease of working of the decimal "system are dependent on its being procrustean."

We are all convinced of the importance of a standard classification of impaired lives; have I been able to give you a sufficient glimpse of its practicability to make you desire to look further into the matter? If so, I venture to say, I shall have justified your courtesy in permitting an individual entirely untrained in actuarial matters to occupy the time of this learned body.

Abstract of the Discussion.

Mr. E. A. Rusher, in opening the discussion, said that the Institute ought to be congratulated that Dr. Carruthers had brought before it the subject of impaired lives, which was of daily practical importance to actuaries, and yet which had received but scant notice at the Institute. In dealing with this subject, the actuary, perhaps, did not have quite the same object in view as the medical officer. He was concerned mainly, if not entirely, with the financial result of admitting lives subject to other than normal rates of mortality. Except as bearing upon that result, and as enabling him to estimate the susceptibility of a given life to such extra mortality, it might also be said that he did not care much whether the extra mortality was caused by hereditary predisposition, unfavourable environment, or by some accidental cause. What concerned him more particularly was the incidence of the impairment, that was to say, the way in which a number of lives whose impairment might be considered fairly homogeneous were subjected to an extra rate of mortality, at what age group or groups that extra mortality first began to show itself, and through what age period it existed. In short, actuaries required the curve of mortality, whence they could deduce the premium actually required for lives subject to the special risk. The doctor, on the other hand, had in mind to a large extent the resulting benefits to his profession in the alleviation, and it might even be the elimination, of causes that tended to increase the death roll. He would thus seek for some light to be thrown on the causes themselves, and on the period of life at which they exerted their most powerful influence. It probably was, however, the case that the effect upon the future mortality, and consequently the premium chargeable, was quite different in each of the three cases suggested. Thus, where the susceptibility arose from hereditary causes, it might manifest itself in much increased mortality rates at the younger ages, whilst that from environment might either show the same incidence

or might not exert an effect till later on in life; again, that from accidental causes might be fairly well spread over a long period of life. Thus, though from different standpoints, the doctor and the actuary came very much to the same general principle, that it was necessary to investigate at least the main classes of causes of impairment, and if possible some of the major portions of the sub-classes.

The paper might be said to consist of: (1) A general plea for a combination of the experience of the various offices on under-average lives, and (2) The particular adoption of the Melville Dewey scheme of Notation for classification. Taking the latter first, he would like to premise his observations by saying it was always easy to raise objections to a particular detail of any given scheme, and he hoped that Dr. Carruthers would not think it at all discourteous or wanting in due appreciation of the power of the Melville Dewey scheme if, for the particular purpose in hand, he did not quite follow him in his glowing eulogy, but rather put in a plea against minute sub-division in gathering material for such an investigation as the author contemplated. In the old Institute Experience, facts which related to impaired lives were tabulated in one heterogeneous mass, without any distinction whatever of the different causes for which they had been classified as impaired. Though the figures were sufficiently interesting to repay a careful analysis, it was evident that the absence of classification made them of very little practical use; but he could not help feeling that the sub-divisions suggested by Dr. Carruthers went too far in the opposite direction. The number of lives under observation being very limited, it would seem to serve no good purpose to adopt a system of classification so minute that it was quite conceivable that in some of the classes only one life might pass under observation, and they might with confidence look for a much more satisfactory result if they confined themselves to a few main groupings; although even then the facts might prove too few to enable them to base results upon them with any degree of confidence. Their American colleagues issued a short time ago their "Specialized Mortality Experience", in which the lives were divided up into 98 classes. Although they dealt with the whole body of assured lives, and not with that small fraction of them included under the term of "Impaired lives", it was obvious that in many of the classes the general results were practically valueless owing to the paucity of data. An extreme instance of that difficulty had come under his notice within the past few days, a brother actuary having informed him that upon making an investigation into the experience of his own office, having over 13,000 policies in force, he found that for the past six years he had an average of not more than 45 cases of impaired lives of all classes. Mr. Manly's experience, valuable as it was, dealt only with 945 cases, although Mr. Manly did not appear to have limited the investigation to impaired lives, but on the contrary, stated explicitly that only about half a dozen were rated up, and that they were mostly recent cases. It had been his (the speaker's) privilege recently to initiate an investigation into the risks on impaired lives in a large assurance company. In consultation with the medical officer, it was decided to limit the classification to 14 main heads, and 64 sub-

heads. The work was at present only in its initial stages, but it was already evident that even that classification was too minute, and that many of the sub-heads would have to be combined to obtain useful results. He would suggest to the author that he was much more likely to attain the end he had in view, and to meet with the sympathetic support and co-operation of the actuarial profession, it he confined his excellent plan within narrower limits. In the second place, he was not at all convinced that it would be advantageous to adopt Mr. Melville Dewey's system of numerical classification. The facts would, of course, be brought out in tabular form, and consist of large numbers of figures; and it seemed to him that it might save some confusion, and possibly be found preferable, to employ the letters of the alphabet for the main headings, with suffixes for the sub-classifications. The alphabetical plan was that which had been adopted in the experience to which he had referred. Thus, class a stood for phthisical taints, the sub-headings being a_1, "Father only died of phthisis", a_2, "Mother only died of phthisis", and so on. Class b similarly dealt with rheumatic fever and lesions of the circulatory system.

Turning to the other object of the paper, the plea for a combination of the experience of the various offices on impaired lives, that was a matter on which one could express the fullest and most unqualified approval. To have an experience of the kind would indeed be a boon and a blessing to the actuary, who at present had nothing but the most empirical means of even approximately gauging the extent to which a life should be rated up. There was at present the utmost diversity of practice in that respect. One office would rate a man up so many years and charge the premium accordingly, irrespective of the period at which the extra risk might become operative, or of the class of assurance. Another would take the same man at ordinary rates for an endowment assurance, whilst declining him altogether for a whole-life policy; and yet another would charge a diminishing debt against the sum assured. Even with the small knowledge that they already had, they were not always consistent, some offices dealing with all impaired lives on a similar plan, say, that of charging the premium for a higher age; whereas one would expect that each office would at least divide its impaired lives into classes according to the way in which they might be generally expected to be subject to extra mortality, adopting, say, the diminishing debt for entrants at young ages with a family history of consumption, and the extra premium for those that might be expected to have a tendency to gout. That confusion was due entirely to the absence of reliable information, one had almost said of any information at all, and it would be a great step forward if they could obtain an experience divided into a few main classes, possibly with sub-divisions. As suggested by Mr. Burn, in a valuable paper read last year before the Insurance Institute of Toronto, if there were such an experience the medical officers would inform the actuaries which class would most nearly approximate to their ideas of an impaired life that came before them, and from the experience tables the actuary could estimate the premium necessary to cover the risk. As already indicated, a start had been made

in that direction by one large company, though a long time must elapse, possibly years, before the results of such an elaborate investigation could be worked out to a practical conclusion. It would add materially to the value of the results if other actuaries would likewise obtain experience from the records at their disposal. It might even be suggested that now that the Institute had practically completed its laborious researches into the rates of mortality amongst healthy assured lives, it would not be an inappropriate task for them to enter upon a similar research into the comparatively untrodden ground of impaired lives.

Dr. Light thought that both actuaries and doctors must feel grateful to the author for his valuable paper on so important a subject, which especially called for the co-operation of the actuary and the doctor. The Melville Dewey scheme of notation and classification of under-average lives seemed to him a most excellent method if elaborate detail were needed, but from his experience in the classification of those lives he could confidently say that minute sub-divisions were not only unnecessary, but they were impossible for the purpose of an investigation such as the author suggested. What was required was a simpler system without elaboration, or the Melville Dewey scheme of notation much simplified. The one he had adopted, although by no means perfect, was that described by Mr. Rusher, in which the letters of the alphabet represented the main headings and the suffixes the classifications. It was simple, easily memorized, and rapidly worked. He considered that more than one letter on the subject of a heading would be liable to much confusion. For instance, there were five recognized reasons for rating up a life; (1) blemished family history; (2) blemished personal history; (3) distinct personal defects; (4) personal conditions of doubtful medical significance; (5) unfavourable conditions of life. He thought the great majority of those cases would fall under one working heading—some would fall under two, some would come under three—but in those compound cases the one heading would be of greater importance as compared with the others, and be the principal reason for rating up the life. He would suggest, to prevent confusion, that such a life be classed under its most important heading, and any less important flaws worthy of record entered somewhere else than under that heading. With regard to the first heading of blemished family history, he would like to point out that, apart from consumption and cancer, the other headings were of minor importance, and would not require any sub-divisions, such as those given by the author under the heading of family history of bronchitis. As far as the result of his own investigations went, it seemed to him that all lives rated up for personal blemish—for instance, bronchitis—would have to be placed under their main heading, irrespective of family history and other minor flaws, in order to get sufficient data to produce reliable results. For the same reason it would be found impracticable to split up the data for pleurisy; but even if it were practicable, he failed to see where the information was forthcoming to enable such a division to be made. What they must hope to obtain were reliable average mortality tables for the different classes

of impaired lives. The present mode of adjusting advances for impaired lives was highly unsatisfactory, and it was hoped that by the combined experience of the various offices a more scientific method might be obtained.

Mr. S. G. Warner was sure that, in the first place, they all very heartily welcomed the author, and looked upon it as an exceedingly pleasant and helpful incident that a member of the medical profession, with which the actuarial profession worked so closely, and to which it owed so much, should come and assist them in their deliberations on so important a subject. He suggested, in the second place, that the members might congratulate themselves on the fact that a paper of the kind which had been read was brought before them in the Spring, when, he believed (according to philosophers) human vitality was at its highest; otherwise, the list of diseases brought before them, and the unhappy experiences of the gentlemen selected as an illustration of the working of the scheme, might have had a distinctly depressing effect upon their spirits. The ideal of the paper was obviously that extra rating should be reduced to an exact science, which it was very far from being at the present time, and that actuaries should be able, scientifically, to estimate the mortality of each class of impaired lives. He did not profess to be in any way an expert upon the technique of a paper like the one under discussion. The particular method of classification suggested seemed, on the assumptions made, to be a very good and clear one, and he did not think he could profitably say anything further about it, except to concur in a general remark already made in regard to its excess of detail. He was afraid it was a counsel of perfection, and something which was almost hopeless, that actuaries should ever look forward to having so vast an amount of facts as would enable them to form sound scientific conclusions in regard to mortality on so minutely sub-divided a classification as that given by the author. It seemed to him that if anything of the sort were attempted in any scheme which approached in elaboration the details given in the paper, the investigation must necessarily, before anything like an adequate body of facts could be obtained, last so long that in the meantime social and medical conditions would substantially alter, so that many of the facts included in the earlier part of the investigation, and mixed with those which came at a later period, would be of differing values, and would go far to deprive the results of scientific value. It further appeared to him that many of the somewhat minute flaws provided for by the author's scheme of analysis would not be such as to bring a life into the class of impaired lives at all. It seemed to him, for instance, that the death of one grand-parent from respiratory disease would not, in the opinion of most of them, cause that life to be brought into the class of impaired lives; it would take its place with the general average lives that came before an office, and would be accepted at ordinary rates. Nevertheless, a class was provided in the paper for those whose only flaw appeared to be the loss of one grand-parent by respiratory disease. Further, there seemed to him to be a somewhat serious and fundamental difficulty when impaired lives were regarded strictly from the statistical point of view. It seemed to him that the difference, for such a purpose, between

those lives and the average lives which were accepted at ordinary rates was not altogether one of degree; it was almost a difference of kind; because if reliable mortality tables were to be obtained founded upon statistical information of that sort, followed out into elaborate detail, it would be found that the circumstances of almost each individual case differed widely. Variations in social environment, the medical assistance it was possible to procure, and similar features, made such wide distinctive differences as to upset the average, and rendered it very unlikely that reliable figures could be obtained enabling them to fix, as he supposed was the intention, scientific premiums for impaired lives on such a classification as the author's. Actuaries had to admit that the system at present in force was a very rough and ready one, but it was difficult to see what, for many years to come, was to take its place. After all, important as the subject was, it was a consolation to remember that the backbone of life assurance companies' business did not consist of impaired lives, but of lives which were considered to be in average health; and while the Institute welcomed any attempt to place more effectively at the disposal of its members the resources of science, enabling them exactly to estimate and precisely to deal with this smaller class, he was afraid, for the reasons he had ventured to suggest, that such a desirable consummation could only come at a distant date. Of course that need not in the least interfere with their gratitude to the author for bringing the problem before them, and for his heroic attempt at its solution; nor with their appreciation of the skill and ingenuity of that attempt.

The PRESIDENT, in proposing a hearty vote of thanks to the author for his paper, said he thought there was undoubtedly good work yet to be done in the direction indicated; but the classification to be attempted was very largely one which their medical friends must themselves determine. He would only ask the author—who he hoped would, with his colleagues, pursue the subject still further—to recollect that what actuaries and physicians should jointly aim at was something of which practical use could be made. For that purpose he thought any classification which was to be of everyday use must not be too elaborate. He did not wish, however, to discourage the author from the large amount of work which he appeared to be willing to undertake; and when he had elaborated something further which he considered a practical scheme, he thought the Life Offices would not withhold from him their collaboration in the direction of furnishing material.

The vote of thanks to Dr. Carruthers was then put and carried with acclamation.

Dr. CARRUTHERS, in reply, thanked the members most cordially for the very kind reception they had given to his somewhat crude paper. If they felt any gratitude to him for bringing the subject forward, he thought that this should be extended also to his valued colleague, Mr. Allin, on whose suggestion he brought the paper before the Institute, and who had given him valued advice at almost every step of the work. He was afraid, however, he had not made one point clear. The criticisms that

had been raised were all justified on two assumptions. With regard to impaired lives, his definition would not be the definition that seemed to be adopted by the members of the Institute. It seemed to him the speakers had supposed that the paper only had to do with rated lives. But it had to do with a much wider class than that. Mr. Warner had referred to the question of one grand-parent suffering from consumption. Of course that in itself was hardly enough to make an impaired life, certainly not to make a rated life, but it might have considerable weight in the general consideration of the case. He believed most doctors present would say that in the case of an applicant who was a little under weight, who had a poor chest expansion, and had a past pneumonia, even a grand-parent dying of consumption, would very likely turn the scale heavily against him. Doctors ought to be able to record every deficiency that occurred in an individual life. In the same way Dr. Light had stated that it was necessary to use the principal heading mainly. He tried to do that at first; it was his ideal; but he would like Dr. Light to tell him in a case where there was both albuminuria and gout which he would consider the principal heading. They might want to consider it among the gouty cases, or they might want to consider it among the albuminuric cases; and that was the reason why he laid aside the method which Dr. Light had adopted of principal headings without subordinate headings. The second matter that seemed to him to have been not clearly understood was that it was never in his mind that an actuarial investigation could possibly be made, at any rate at present, with all the minute sub-divisions he had suggested; he had been looking very far into the future. He tried to make it quite clear that, at the present time, a great many of the classes would have to be combined. Mr. Rusher, after referring to the smallness of the classes and the classification, answered himself by saying that, even with the larger classes his office were at present working with, they found they had to group them together. To his mind the principal beauty of the system he advocated was the absolute ease with which small classes that could not be dealt with separately might be grouped together. He assured the members that it was very little more trouble to docket a case with the smaller sub-divisions than it was with the larger ones. If the classes were not sub-divided originally, and it was afterwards found that the advances of science, or the increase in the number of lives at risk, would enable the actuary to investigate the matter with more minuteness afterwards, the whole work had to be done again from the beginning; whereas if they were sub-divided originally they could easily be combined for a collective investigation, and if science progressed further afterwards the sub-divisions were there already docketed, and could be used without further expense, either of money or of time. He was rather amused to find that, in the only instance Mr. Rusher gave of his classification, he had adopted more sub-divisions than he (the author) had. Mr. Rusher distinguished between a father and a mother dying of consumption, whereas he (Dr. Carruthers) did not distinguish between the two parents; they both came under class 313 in his classification. Mr. Warner admitted that the present system of dealing with impaired lives

was an unscientific one. His difficulty in dealing with the question throughout had been that unless the doctors made a start in the way of classification nothing could ever take its place. If the Institute of Actuaries and the Life Assurance Medical Officers' Association could see their way to take up the question of classification, not necessarily at all along the Dewey lines, but possibly along the lines used by Mr. Rusher and Dr. Light, he thought it would be a great step in advance.

LAW REPORT.

The judgment of Mr. Justice Buckley in the case of *Nelson & Company, Limited,* involves some important actuarial considerations, especially as to the interpretation of clauses 4 and 22 of the Life Assurance Companies Act, 1870, and the appended extract from the judgment (taken from the *Times* of 16 February 1905) will, we think, be of interest to our readers. The nature of the contract for the grant of a reversionary annuity is sufficiently set out in the appended extract from the Statement of Claim lodged by the Plaintiff.

The conclusion of Mr. Justice Buckley upon the points referred to may be summarized as under:

(1) The scheme of business carried on by the defendants was not consistent with Section 4 of the Life Assurance Companies Act, 1870, which requires that a separate account shall be kept of all receipts in respect of the annuity contracts, and that the receipts shall be carried to and form a separate fund absolutely the security of the annuity holders.

(2) The proposed scheme of reconstruction was not a reduction of contracts such as is provided by Section 22 of the above-named Act, on the ground (*a*) that the scheme provided for the present or prospective annuitants entering into a new contract with a new company, as the result of which they would be entitled to reduced contracts with the old company, instead of providing for a reduction of contracts directly between the annuitants and the old company; (*b*) that the payment made by way of premium under the scheme was not a separate sum capable of being set apart and appropriated, as required by Section 4 of the Act; (*c*) that the proposed reduction of contracts was one of inequality as between the several policy-holders; (*d*) that no material benefit was granted to the existing customers of the company, which was not also granted to its new customers.

HIGH COURT OF JUSTICE—Chancery Division.

(Companies Winding-up.)

Before Mr. Justice Buckley.

In *re* Nelson & Company, Limited.

Extract from Statement of Claim.

* * * * * * *

Statement of Claim.

4. The defendants, Nelson & Company, Limited (hereinafter referred to as "the defendant company") were incorporated under the Companies Acts, 1862 to 1900, on the 11th day of July, 1901, as a company limited by shares with a nominal capital of £101,000, divided into £1 shares. The primary object with which the company was incorporated was to acquire and carry on the business of a tea dealer theretofore carried on by one Rasmus Jensen, trading as "Nelson & Company," at Louth, in the county of Lincoln, and elsewhere.

5. The said Rasmus Jensen had in the year 1897 inaugurated as ancillary to the said business a certain life assurance scheme. By the said scheme the said Jensen in his trade name of Nelson & Company entered into contracts by means of cards to purchasers of his tea that he would pay every woman who should have become a widow since Christmas, 1897, and who since that date should have purchased not less than one half-pound of his tea per week for the last five consecutive weeks previously to her becoming a widow, 10*s*. per week as long as she remained a widow, and to every woman who became a widow previously to Christmas, 1897, or previously to her commencing to purchase Nelson & Company's tea, 10*s*. per week as long as she remained a widow, provided that she should have purchased half-a-pound of tea per week for ten years. Customers who required only a quarter-pound of tea weekly and purchased the same under the above conditions would receive 5*s*. per week instead of 10*s*.

6. The said scheme was carried on by the said Jensen down to the middle of the year 1901, and at that time customers to the number of between 150,000 and 200,000 were purchasing his tea under the aforesaid contracts. Widows to the number of 3,000 or thereabouts were then already in receipt of pensions.

7. Consequent upon the decision of the King's Bench Division in the year 1901 that the business so carried on by Nelson & Company was that of a Life Assurance Corporation within the meaning of the "Life Assurance Companies Act, 1870," it became necessary to provide the deposit of £20,000 required by the said Act. This necessity

Statement of Claim—*continued.*

and the fact that the business had assumed such large proportions led to the promotion and incorporation of the defendant company on the 11th of July, 1901. In the same month the sum of £20,000 was deposited in Court on behalf of the defendant company.

8. By its memorandum of association the defendant company was empowered to effect insurance for life, accident and sickness, to issue pensions and old age pensions, and generally to transact and carry on the business of insurance in all its branches, and by article 110 of the articles of association registered contemporaneously with the memorandum it was provided that the directors should set aside three-fourths of the profits earned by the company in each week as a fund to meet the liabilities of the company in respect of the members' (subsequently by special resolution altered to "customers'") cards issued or to be issued by the company, and the sum so set aside should be applied in discharge of the current liabilities thereunder, and in so far as in any week the whole amount should not be distributed the balance should be carried forward and might be applied at the discretion of the directors to make good any deficiency in any future week or weeks in respect of such pensions. But the company was not to be under any liability in respect of the said pensions beyond the amount of the three-fourths of the profits so to be earned by the company, and if in any week the three-fourths of the profits should be insufficient for the payment in full of the said weekly pensions the same might at the discretion of the directors be abated rateably or such deficiency might be made good out of any fund accumulated as thereinbefore directed.

Extract from Judgment.

Judgment.

This was a petition to wind up Nelson & Company, Limited, on the ground of its insolvency. The insolvency is not, and cannot be, disputed. The only defence has been that, in the interests of the unfortunate persons who are the policyholders of or entitled to the benefit of annuity contracts in the company, the Court should adopt, with or without modification, a scheme which has been put forward, and which is said to be a scheme for the reduction of the company's contracts under section 22 of the Life Assurance Companies Act, 1870. The business of Nelson & Company, Limited, may be described as that of attracting married women to become customers of the company and purchasers of the company's tea, at prices largely—say, 40 per-cent—above its fair market value, by the delusive and reckless promise of impossible pensions to be paid to them if and when they become widows. The pension scheme rested upon no actuarial basis of any kind. It ignored the age

Judgment—*continued.*

of the husband on whose death the pension would commence. It ignored the age of the wife during whose widowhood it would be payable. It ignored, with some exceptions, the health and expectation of life of the husband. The loading which was added to the price of the tea, and which may in a sense be regarded as the premium paid for the annuity contract, bore no actuarial relation whatever to the liability which the company was undertaking. Under no circumstances could it have justified a pension of anything approaching 10*s*. a week. If the customers had known that they were overcharged 8*d*. a pound for tea, and in the result might possibly during widowhood receive a pension of, say, 6*d*. a week, the number of the company's customers would probably have suffered considerable reduction. Even the 8*d*. a pound did not go to provide for the annuity contracts; it went into the general business; and, under the limited company's contracts, the policyholders could look only to 75 per-cent of the profits realised by its employment with the company's other funds in the business. The offer which the company made was a mere reckless promise of impossible pension with a view to induce persons to become customers * * * The number of customers increased rapidly, and ultimately reached something like 500,000 or 600,000. The number of widows who became entitled to pensions increased with like rapidity; and, at the date of this petition, there were about 12,696 widows entitled to 10*s*. a week, and 7,084 widows entitled to 5*s*. a week, making a total of upwards of 19,000 widows entitled to pensions, which aggregate to a sum of between £8,000 and £9,000 a week. There is actuarial evidence which is altogether uncontroverted that the reserve which this company should have had to meet the pensions thus payable and the company's prospective liability in respect of current contracts which might result in pensions is nearly £30,000,000 sterling. The figure is not accurate, and cannot be made accurate without knowing the ages of the lives, a detail which, as I have said, the company ignored. Making large deductions, as, for instance, a deduction on the footing that, say, 75 per-cent of the persons who are now buying tea will cease to buy, and thus cannot become entitled to pensions, and making certain other allowances, the reserve ought to be a sum measured at any rate, by millions—£10,000,000 would seem to be about the lowest sum. To meet this gigantic present and prospective liability, the company have available for the annuitants less than £20,000. There is the statutory deposit of £20,000, and, further, a sum of about £2,782 to the credit of the widows' pension fund. But from these has to be deducted a sum of £3,358, which the company says has been paid to widows

Judgment—*continued.*

over and above the share of profits to which they were entitled. The amount available is thus something under £20,000, a sum sufficient to pay, even in respect of the present current pensions alone, the amount due to them for little more than a fortnight. * * * In the interest of the unfortunate victims the Court is asked to consider a scheme of reduction of contracts under the Life Assurance Companies Acts. A scheme has been brought in. It is, in my opinion, open to insuperable objections upon several grounds. In the first place, it involves that this company shall continue business—a course to which, in the absence of overpowering reasons in the interest of the customers, I should not accede. Secondly, it proceeds upon a footing that a system of business shall be continued by which customers shall be attracted to become purchasers of tea at a price loaded with a sum to represent the premium on the insurance benefit, such sum, however, not being payable as a premium capable of being dealt with in manner required by Section 4 of the Life Assurance Companies Act, 1870, but included in a lump sum which is to be dealt with in manner indicated in the scheme. Stating it in figures, the matter would stand somewhat thus: The tea purchased by the company costs, it is said, 1*s.* 3*d.* to 1*s.* 5*d.* per pound, and would fairly be retailed at 1*s.* 8*d.* per pound. Sale of the tea was made, and would under the scheme continue to be made, at 2*s.* 4*d.* per pound, thus providing a loading of 8*d.* per pound to form the premium income to support the annuity contracts. The scheme involves that a new company called the Nelson Trading Company, Limited, shall sell the tea and shall pay 75 per-cent of the loading (6*d.* a pound) to this company (Nelson & Company, Limited), which shall, as between the two companies, be the insurer of what is called the insured customer of the Nelson Trading Company, Limited. A scheme of that kind is, to my mind, not a reduction of contracts of Nelson & Company, Limited. It is a scheme under which the customer, if he is minded to make a new contract with the Nelson Trading Company, Limited, is to be entitled to a reduced benefit out of his previously existing contract with Nelson & Company, Limited. This is not, in my opinion, within Section 22 of the Life Assurance Companies Act, 1870. That section authorizes a reduction of the contract between A and the company, but not a provision by which A shall enter into a new contract with a new company, and as the result of that shall be entitled to a reduced benefit from his contract with the old company. Again, Section 4 of the Life Assurance Companies Act, 1870, requires that a separate account shall be kept of all receipts in respect of the annuity contracts, and that the receipts shall be carried to and form a separate fund absolutely the security of

Judgment—*continued.*

the annuity-holders. The scheme of business which I am now asked to wind up, and the scheme of reduction which I am asked to approve, is one under which the payment which the assured makes by way of premium is not a separate sum capable of being kept separate and appropriated, as required by Section 4. The 2*s.* 4*d.* per lb. which she pays is what I may call a contributory sum for securing two benefits—the present receipt of one pound of tea, and the expectation of a future sum of money contingent upon widowhood. I do not think that this is consistent with the Act of Parliament. Another fatal objection is that, as counsel for the company conceded, the reduction of the contracts which they propose is a reduction of inequality as between the policy-holders. That which the Act allows is the reduction of all contracts to the relief of the common debtor, but not a reduction which operates in inequality as between the common creditors. I do not mean that absolute arithmetical equality must be ensured, but a scheme which proceeds upon a principle of inequality of reduction is not, I think, within the Act. Again, as is pointed out in the report made by Mr. Ackland to the Board of Trade, no sort of benefit is granted to the existing customers of the company which is not granted to new customers, unless it may be a trifling advantage in admitting the old customers to benefits within the first year. * * * Another objection which weighs strongly with me is that the effect of this scheme is to create what I may call a tied insurance business, under which customers are to pay too much for their tea upon the terms that the excess shall be taken by them, not to such insurance office as they like, to insure an annuity, but that they shall be tied to Nelson & Company, Limited, being such a company as I have described, as their insurers. I see no advantage, but every disadvantage, to the customer in this provision. It is said, on the other hand, that to reject any such scheme is to throw away the value of what is called—using a term singularly inappropriate to the facts of this case—the "goodwill" of the company, and that the widows, being confined to 75 per-cent of the profits, can get nothing in the winding-up beyond the £20,000. As to the latter, I am not sure that this is so; but I say no more upon the point, for it may well have to be considered in the course of the winding-up. I am clear, however, that, even if this be so, it is more to the interest of the policyholders to put an end at once to this concern, even if that which they get out of the wreck be but a small sum. For these reasons I think that the scheme for reduction of contracts is impossible, and cannot be approved, and I, therefore, at the conclusion of the argument, pronounced a winding-up order.

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Returns for 1904, published in 1905.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	267,358,146	25,341,156	292,699,302
Adjustments, for one new Return (+£35,807) and for balances transferred from Industrial to the Ordinary Branches (+£311,836 and −220)	+347,423	−311,616	+35,807
	267,705,569	25,029,540	292,735,109
Premiums	23,903,788	10,600,611	34,504,399
Consideration for Annuities	1,881,972	168,933	2,050,905
Interest and Dividends (less Tax)	9,995,582	846,655	10,842,237
Increase in value of Investments	70,520	169	70,689
Fines, Fees, &c.	13,914	1,490	15,404
Capital Paid-up	...	29,043	29,043
Customs Timber Measuring, &c.	3,720	...	3,720
Transfers from other Accounts	84,247	78,117	162,364
Miscellaneous	12,081	9,120	21,201
	303,671,393	36,763,678	340,435,071

OUTGO	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	16,727,304	3,944,927	20,672,231
Cash Bonuses and Reduction of Premiums	1,191,123	375	1,191,498
Surrenders	1,599,106	61,354	1,660,460
Annuities	2,030,031	140,742	2,170,773
Commission	1,245,945	2,667,517	3,913,462
Expenses of Management	2,031,476	1,934,592	3,966,068
Bad Debts	4,194	500	4,694
Decrease in value of Investments	426,373	9,249	435,622
Interest on Capital and Dividends and Bonuses to Shareholders	532,901	546,095	1,078,996
Transfers to other Accounts	336,494	347,289	683,783
Miscellaneous	9,232	...	9,232
Balance* at the end of the Year	277,537,214	27,111,038	304,648,252
	303,671,393	36,763,678	340,435,071

* This Balance includes the whole of the Life and Annuity Funds (£300,515,449), and, in addition, the Capital, &c., of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1904).

LIABILITIES	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances)	11,521,685	1,945,620	13,467,305
Life and Annuity Funds	274,240,114	26,275,335	300,515,449
Fire Funds of Companies transacting Life Business	13,582,612	...	13,582,612
Marine Funds of Companies transacting Life Business	899,573	...	899,573
Reserve Funds	4,497,941	1,380,225	5,878,166
Other Funds	3,088,460	279,639	3,368,099
Profit and Loss Balances	4,996,426	10,508	5,006,934
Depreciation and Investment Balances	2,374,496	43,307	2,417,803
Globe Annuitants (Liverpool and London)	1,654,200	...	1,654,200
Outstanding Claims	4,388,165	15,676	4,403,841
Outstanding Accounts	684,001	56,426	740,427
Temporary Loans	663,723	14,813	678,536
	322,591,396	30,021,549	352,612,945

ASSETS	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Mortgages	88,709,427	3,123,745	91,833,172
Loans on Policies	16,017,692	91,708	16,109,400
,, Rates	29,120,871	10,353,488	39,474,359
British Government Securities	7,543,498	2,129,551	9,673,049
Indian and Colonial Government Securities	19,212,644	500,144	19,712,788
Foreign Government Securities	10,104,179	402,877	10,507,056
Debentures	60,588,158	2,558,667	63,146,825
Shares and Stocks	39,180,305	719,579	39,899,884
Companies' own Shares	640,160	...	640,160
Land and House Property and Ground Rents	25,831,771	8,122,095	33,953,866
Life Interests and Reversions	8,982,173	2,353	8,984,526
Loans on Personal Security	1,825,886	4,167	1,830,053
Agents' Balances and Outstanding Premiums	5,928,998	672,011	6,601,009
Outstanding Interest	3,013,480	260,644	3,274,124
Cash, Deposits, Stamps, &c.	5,552,382	401,360	5,953,742
Customs Timber Measuring Balances, &c.	2,035	...	2,035
Deficiencies, Establishment Expenses, &c.	337,737	679,160	1,016,897
	322,591,396	30,021,549	352,612,945

Increase (+) *or* Decrease (−) *in the Chief Items of this Year's* Summary *as compared with the corresponding Items for the previous Year.*

	Ordinary Companies	Industrial Companies
Income.	£	£
Premiums	+ 905,200	+ 334,715
Consideration for Annuities	− 130,159	+ *165,513
Interest and Dividends (less Tax)	+ 512,885	+ 70,106
Net Result of Realization and Re-valuation of Investments	− 224,876	− 6,806
Outgo.		
Claims	+ 382,979	+ 28,536
Annuities	+ 88,884	+ *132,097
Surrenders	+ 206,363	+ 10,924
Commission	+ 55,301	+ 114,223
Expenses of Management	+ 79,541	+ 54,491
Liabilities.		
Paid-up Capital (including sundry Shareholders' Balances)	+ 118,844	+ 118,348
Life and Annuity Funds	+ 9,254,161	+ 2,125,156
Assets.		
Mortgages (including Loans on Rates)	+ 4,601,126	+ 720,225
Life Interests and Reversions	+ 593,412	+ 3
Loans on Policies	+ 1,134,934	+ 22,718
British Government Securities	− 164,895	+ 49,517
Indian and Colonial Government Securities	− 139,885	+ 36,422
Foreign Government Securities	− 331,530	+ 26,564
Debentures	+ 4,841,601	+ 167,242
Shares and Stocks	+ 1,205,206	+ 426,306
Companies' own Shares	+ 18,142	...
Land and House Property and Ground Rents	+ 1,358,357	+ 961,356
Loans on Personal Security	+ 289,092	+ 1,480

* Including the figures of "Nelson & Co."

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 96, of which 75 are classed as Ordinary, 12 as Industrial, and 9 appear in both Classes, the Returns of these Companies showing the Ordinary and Industrial business separately. The accounts of the Aberdeen Accountants, British Endowment Tea, British Widows' and Edinburgh Accountants are included for the first time.

During the year one name has been removed from the Official List of Companies, namely, British Empire; which Company has amalgamated with the Pelican. And two names have been added, namely, Manufacturers' Life Insurance Company, Limited; and the Popular Life Assurance Company, Limited; in which cases the Board of Trade have issued their Warrant under the provisions of Section 1 of "The Life Assurance Companies Act, 1872."

imary of the Assurances in Force, *as shown by the last Returns of the Companies.*

ORDINARY BUSINESS.

	With Profits		Without Profits		Total		Re-assurances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
Assurances.		£		£		£	£	£
ole Term of Life	789,070	377,252,662	149,172	71,040,801	938,242	448,293,463	27,983,894	420,309,569
ited number of 'remiums . .	51,666	32,970,814	12,379	5,159,200	64,045	38,130,014	2,252,628	35,877,386
	840,736	410,223,476	161,551	76,200,001	1,002,287	486,423,477	30,236,522	456,186,955
lowments . .	1,716	415,139	20,832	4,938,179	22,548	5,353,318	18,500	5,334,818
lowment Assur- aces . . .	1,079,796	177,219,129	77,895	21,633,546	1,157,691	198,852,675	3,020,980	195,831,695
it Lives . .	16,833	3,320,445	2,671	953,105	19,504	4,273,550	267,026	4,006,524
t Survivor . .	819	677,890	1,058	1,542,122	1,877	2,220,012	383,390	1,836,622
tingent . .	34	62,763	4,209	6,264,973	4,243	6,327,736	1,436,130	4,891,606
e . . .	11	25,250	1,519	5,453,771	1,530	5,479,021	1,586,744	3,892,277
cellaneous . .	3,784	1,856,341	21,101	14,393,188	24,885	16,249,529	1,901,852	14,347,677
	1,943,729	593,800,433	290,836	131,378,885	2,234,565	725,179,318	38,851,144	686,328,174
Annuities.								
mediate . .	...	...	...	...	37,707	1,990,251	52,931	1,937,320
erred . .	...	...	...	...	14,865	403,838	22,431	381,407
	...	...	...	...	52,572	2,394,089	75,362	2,318,727

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	With Profits		Without Profits		Total		Re-assurances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
Assurances.						£	£	£
ole Term of Life	252	16,381	21,244,372	206,837,210	21,244,624	206,853,591	1,180	206,852,411
ited number of emiums . .	...	...	323	6,127	323	6,127	...	6,127
	252	16,381	21,244,695	206,843,337	21,244,947	206,859,718	1,180	206,858,538
owments . .	140	42,475	1,639,620	15,582,940	1,639,760	15,625,415	...	15,625,415
owment Assur- ces . . .	45	2,599	517,997	5,179,187	518,042	5,181,786	88	5,181,698
Lives . .	...	...	408,182	6,549,345	408,182	6,549,345	...	6,549,345
ingent . .	...	...	4	1,910	4	1,910	400	1,510
ellaneous . .	...	...	2	2,100	2	2,100	1,000	1,100
	437	61,455	23,810,500	234,158,819	23,810,937	234,220,274	2,668	234,217,606
Annuities.								
mediate . .	...	...	...	...	65	2,034	...	2,034
erred . .	...	...	...	...	14	310	...	310
	...	...	...	...	79	2,344	...	2,344

e above figures are based on Returns deposited, for the most part, during the past five years, and erefore, merely an approximation to the amount of contracts in force at the present time. In the f two Companies, namely, the Customs Fund and the Northern, the amount of business at a more date has been included. The figures of the Colonial and Foreign Companies have been excluded, as Returns do not show separately the extent of business in the United Kingdom.

CORRESPONDENCE.

THE NOTATION OF PENSION FUND PROBLEMS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—It will probably not cause surprise if I dissent from the views on this subject expressed by Mr. Lidstone in his letter in the April number of the *Journal*. I quite think that it may be possible later on to simplify the notation relating to Pension Fund problems, but the time has not yet come, and I do not think any good purpose would be served by the Council of the Institute trying at present to prescribe an official notation. So far not much has been written on Pension Fund problems, and we have not yet had sufficient experience as to what would be the best notation. It was long before an official notation was adopted in other branches of actuarial enquiry, and the Council of the Institute took up the matter only when a general agreement, more or less complete, had already been arrived at. Similarly, in the case of Pension Funds, we must, I think, wait until further experience has been gained.

Mr. Lidstone's suggested symbols are an illustration of this point. He proposes to use six good symbols where I use only one, and with our short alphabet we cannot afford such extravagance. It is difficult enough now to find a sufficiency of symbols, and when the investigation of Pension Fund problems has further developed, the difficulty will be increased. Therefore, we must not be prodigal.

I cannot see where difficulty comes in with regard to the symbol F for the valuation factors, and my experience is that this symbol, with its affixes, is easily read, and the meaning of it in its various phases is instantly grasped. I may add that this is the view taken by many students and others, who have spoken to me on the subject, and who would deprecate a change. So far from Mr. Lidstone's proposal tending towards lucidity, a great many think it would only produce obscurity.

The case might be different if we could abolish the affixes altogether, but they are required for the commutation symbols, and I do not see how they can be dispensed with there. That being so, there is no harm in retaining them for the valuation factors, thus having a symbol that explains itself at sight.

I remain,

Your obedient Servant,

GEORGE KING.

15, *Walbrook, E.C.*

1 *June* 1905.

[Mr. Lidstone asks us to publish the following remarks with reference to the above letter.—ED. *J.I.A.*]

By the courtesy of Mr. King and the Editor of the *Journal*, I have had the opportunity of reading the above letter in proof, and of sending a few words in reply for insertion in the current number

of the *Journal*. It is, of course, not in the least surprising that Mr. King should personally prefer to adhere to the Notation which he himself devised and employs in practice, and which is therefore familiar to him, rather than to adopt any new Notation. The question is, however, one in which differences of opinion may well arise, and it will, no doubt, be eventually settled by the general body of professional opinion after independent consideration of the various Notations which may be suggested. The *Journal* is, perhaps, not a fitting place for a detailed discussion of the subject, and the comparative merits of the two Notations can be sufficiently appreciated by inspection of the symbols given in parallel columns in my letter on p. 209 of the current volume of the *Journal*. I would only add that Mr. King hardly seems entitled to claim that he has used only one symbol, as compared with six suggested by me, when the fact is that his one symbol, namely, F, is so little distinctive that it has to be supplemented by no less than seven subsidiary symbols—in addition to the letters representing ages, which appear equally in both systems.

G. J. L.

ON THE VALUATION OF POLICIES IN GROUPS.

[We have received the following interesting letter from our esteemed Corresponding Member, Herr Altenburger, as to the paper by Mr. F. Bell, published in the present volume of the *Journal*.—Ed. *J.I.A.*]

To the Editor of the Journal of the Institute of Actuaries.

Dear Sir,—I have read with very great interest the discussion which followed the reading of Mr. Fred. Bell's paper relative to the calculation of reserves in groups; and as my name, and the letters I had the honour to address in this matter to the *Journal* (xxxiv, p. 150; xxxv, p. 332) have been frequently mentioned, I think it necessary to explain my standpoint concerning the valuation of policies.

It is true, that the method dealt with in my letters is almost generally used in Germany and in Austria (not here in Hungary), but this fact is due to the circumstance that Actuaries in these countries have not liberty to make their valuations according to any principles which commend themselves to them, their companies being under the control of governments, which do not permit a free choice of the methods and bases of valuations. In Austria, for example, it is not permissible to use the method invented forty years ago by the late Dr. Zillmer (see *J.I.A.*, xv., p. 420), which is almost identical with the method explained by Dr. Sprague at the Brussels Congress (*Transactions of the First International Congress of Actuaries*, p. 186, *et seq.*), but it is compulsory to make the valuation on a net premium basis, although the rate of interest may be taken as 4 per-cent, even where the funds do not yield a higher rate.

The position of Actuaries is then:—Valuations have to be made annually, the basis of this work is invariable for a long term; is

it not natural that Managers and Actuaries search for means whereby their work can be simplified? These are the reasons of the origination and the wider adoption of the method in question.

Fortunately the outlook seems more promising. Governments and Directors begin to give more regard to the principles of actuarial science; and I think that in five or, perhaps, ten years, there will be no obstacle to the adoption of the ingenious method of Mr. Lidstone also in the countries referred to.

I am, Dear Sir, with the expression of my greatest respect,

Truly yours,

JULIUS ALTENBURGER.

V., *Mária Valéria-ut.*, 15*a.*, *Budapest*, *Hungary*.

14 *February* 1905.

BOARD OF TRADE COMMITTEE.

The President of the Board of Trade has appointed the following Committee—Mr. J. G. Butcher, K.C., M.P. (*Chairman*), Mr. Henry Cockburn (President of the Institute of Actuaries), Mr. Stuart Sim (Chief Registrar of Friendly Societies), and Mr. G. S. Barnes (Comptroller of the Companies' Department of the Board of Trade), with Mr. H. A. Payne as Secretary—to inquire as to the operations of Companies (not being life assurance companies) which collect periodical payments from the industrial classes in return for benefits promised in the future, and whether it is desirable that there should be any restrictions on such Companies, or any Government supervision of their transactions, and to report to the Board of Trade.

ERRATUM.

The last two lines on p. 141 of the present volume are misplaced, and should be transposed to the top of the same page. This error has been corrected in the later issues of the *Journal* for April 1905.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Social Conditions as affecting Widows' and Orphans' Pension Funds. *By* S. J. H. W. Allin, F.I.A., *of the Mutual Life Insurance Company of New York.*

[Read before the Institute, 17 April 1905.]

THE subject of Widows' and Orphans' Pension Funds occupies but a very small space in the pages of the *Journal* of our Institute. Indeed, until Mr. Manly's classic paper was written in 1903 (*J.I.A.*, xxxviii, pp. 101 *et seq.*) there had appeared no paper from which the student could glean any information on this branch of our work.

Mr. Manly dealt with the subject from a theoretical point of view, theoretical in fact, though practical in application. He showed the methods by which such Funds could be readily valued, and if he did not actually invent, at all events first brought to public notice the advantage of the collective method of tabulating facts. The tables exhibited in his paper, however, though voluminous, must be regarded as almost entirely theoretical; indeed, in his reply to the discussion that followed the reading of the paper, he himself states that they were not put forward as standard tables, but were only constructed to show the way in which an experience should be worked up.

It seems strange at first sight, in view of the comparatively large number of these Funds in existence, that Mr. Manly should have been so hampered in his preparation of tables for want of suitable statistics. This state of things is, however, probably

due to the fact that most of these Funds are of small size, and the experience to be derived from them as regards probabilities of marriage and issue would be so small as to be worthless.

If, however, we dealt with effects, and not causes, there is no reason why we should not, as suggested by Mr. Lidstone, go to the living instead of the dead for our figures, and by doing so we may learn much, even from Funds of limited dimensions. We could extend the principle even further and combine the experience of two or more Funds, where the circumstances and surroundings of the members in the different Societies are the same, or approximately so, but of course in doing this great caution would be necessary. If the experiences of all the Societies at present in existence were taken out, we might obtain a series of tables applicable to different classes of Funds.* With such tables in hand, which might be adjusted to meet special requirements, the Actuary would feel more confidence in the truth of his valuation results than he often does in the present day, with practically only two standard tables to fall back on, namely, those of Huie and Hewat.

Another advantage in applying the collective method to the living, to which I believe attention has not been drawn, is that we get an experience more "up to date", except in the case of very large funds where the data from deaths can be taken for the last few years only and yet be of sufficient size for a satisfactory experience. The decrease in the rate of mortality, which has been experienced not only among assured lives, but among the population at large, has only progressed slowly in accordance with improvement in medical science and hygiene, but in obtaining statistics for use in the valuation of Widows' and Orphans' Funds, we have other factors to deal with, namely, the marriage rate, the marriage age, and the probability of issue. As regards the last, we know that in some countries the birth rate has declined considerably, and it is probable that in this country families are smaller than was the case some years back. Under these circumstances it seems to be worth while to consider the matter, and see how far these conclusions are borne out by facts gleaned from actual experience, and to what extent they affect our valuation.

* It is not suggested that these Tables should be actually used for valuations, but that they would be of great use for the purpose of comparison where the Fund which is being valued is of small size and the data to be obtained therefrom consequently more or less unreliable.

Shortly after Mr. Manly's paper, to which I have referred, was read, I had submitted to me for valuation the Presbyterian Church of England Ministers' Widows' and Orphans' Fund, and took the opportunity of applying the collective method to the experience obtained therefrom, and the results seem to prove that further investigation into this field would not be fruitless.

A brief description of the Fund will probably be interesting, and is in some respects necessary before considering its experience.

The Presbyterian Church of England Ministers' Widows' and Orphans' Fund.

In 1870 it was decided by the Synod of this Church to establish a Fund to provide for the Widows and Orphans of the deceased ministers, and a nucleus of capital was obtained by means of donations and subscriptions. All ministers and professors holding office in the Church at the date of formation were eligible as members, and from the time of the adoption of the scheme everyone inducted to a ministerial charge was required to become a member of the Fund.

The Fund is maintained by contributions of members, and by annual subscriptions and donations.

Full information as to ages of members, their wives and children, are obtained on their joining the Fund, and enquiries are made annually in order to keep the facts up to date. The managers take the greatest care to see that these annual returns are obtained in every case.

Benefits.—An annuity of £30 to the widows that may be left by members, payable so long as they remain widows. An annuity of £10 to orphans until they attain the age of eighteen years. There are also one or two minor benefits.

Contributions.—All ministers holding employment in the Church must become members, and remain such until death or retirement, paying a yearly subscription of £5. In the latter event happening, the member may leave the fund without withdrawing any benefit, or he can continue a member by paying an increased contribution. Marriage and induction fees are also charged.

Age at entry.—As every minister in this Church has to undergo a six years' collegiate course, it follows that the average age at entry is comparatively high. In his report on the Fund, dated 31 December 1881, the late Mr. T. Y. Strachan, who was

the Consulting Actuary, states that the average age at ordination was twenty-nine, and there is reason to believe that there has been very little variation in this respect up to the present time.

A valuation of the Fund has been made every three years, and as, through the courtesy of the widow of the late actuary, all the valuation papers have been placed at my disposal, I am able to give the following table, which shows the number of members, their status, and the number of their children for each valuation year since 1880.

Table I.

Table *showing the status of Members for various years* (*i.e., whether Married, Widowers or Bachelors*), *together with the number of their Children under* 18 *years of age.*

Valuation Year (1)	No. of Members (2)	Married (3)	Per-cent (4)	Widowers (5)	Per-cent (6)	Bachelors (7)	Per-cent (8)	Children under age 18 (9)	No. of Children per family (10)
1881	263	214	81·4	8	3·0	41	15·6	571	2·67
1884	317	246	77·6	15	4·7	56	17·7	634	2·57
1887	331	265	80·1	16	4·8	50	15·1	601	2·26
1890	350	267	76·3	16	4·6	67	19·1	567	2·12
1893	372	298	80·1	13	3·5	61	16·4	561	1·88
1896	397	313	78·9	14	3·5	70	17·6	573	1·83
1899	427	340	79·6	10	2·4	77	18·0	575	1·69
1902	443	347	78·4	13	2·9	83	18·7	569	1·63

It will be noticed that the percentage of married men to total members has remained fairly constant throughout; moreover, when the size of the Fund is borne in mind, there does not appear to be more variation in the proportion of widowers and bachelors than might have been expected.

When, however, we come to consider the average number of children per family, it will be seen that there has been a persistent decline at each valuation period. From 2·67 per family in 1881 the number was reduced to 1·63 in 1902, or practically 38 per-cent. So large a reduction in the average size of the families must carry weight in a true valuation of the liabilities of a Fund, and it is desirable to ascertain if possible its origin. The three most probable causes are:—(1) increased age at entry, (2) increased age at marriage; this would practically be accounted for, but not necessarily, by cause (1) existing; (3) the personal limitation of the size of families.

The following table shows the numbers alive on the Fund below age 35 at four valuations, during the period 1881 to 1902.

TABLE II.

Age	Number of Members			
	Year 1881	Year 1887	Year 1896	Year 1902
(1)	(2)	(3)	(4)	(5)
Below 25	1	1	2	1
25–29	12	14	23	15
30–34	40	45	56	56

There seems to be nothing in these figures to indicate any substantial increase to the age at entry, and this is confirmed from information supplied by the Managers of the Fund.

In the late Mr. Strachan's report of December 1881, he states that the average age of the husband at marriage was 30·7 years, and the corresponding age of the wife 26·2. These statistics refer to first marriages only, but as the second marriages are few in number, their exclusion does not alter the figure to any appreciable extent.

An investigation on the same lines in 1902 discloses the fact that the husband's average age at date of marriage was 31·3, or an increase of roughly seven months, while the wife's average age was 27·3, or approximately thirteen months greater than was shown by the 1881 figures. These differences in ages are not sufficient by themselves to account for the large diminution in the size of the families, which is probably chiefly due to the third cause suggested.

The actual effect of the increased marriage age and the decline in the birth rate was next investigated.

The methods indicated by Mr. King were employed. (See *J.I.A.*, xxxviii, pp. 166–167.)

WIDOWS' PENSIONS.

The following tables show the number of married men living at each age, and the ages of their wives. The value of the widow's annuity is also given at each age of the husband, obtained by summing the annuities at the corresponding ages of the wives, and dividing by the number of husbands. The actual annuities used were Carlisle 3 per-cent with ·5 added to make them continuous. As the statistics at the older ages were insignificant, all ages above 64 in the 1881 observations, and 75 in the 1902 observations were excluded.

TABLE III.

TABLE *showing the number of Married Men at each age in* 1881, *with the corresponding ages of their Wives, and Widows' Annuities, by Carlisle* 3 *per-cent.*

Age of Husband (1)	Number of cases Married (2)	Ages of Wives (3)												Average value of Widows' Annuities (4)
25	1	22												21·80
26	1	23												21·60
27	...													...
28	2	25	25											21·16
29	1	29												20·26
30	2	30	37											19·74
31	6	25	31	32	32	26	29							20·26
32	4	23	31	33	40									19·63
33	6	25	25	26	31	31	34							20·36
34	8	22	29	30	32	32	34	35	45					19·48
35	4	25	28	29	32									20·43
36	7	25	26	29	36	37	37	42						19·29
37	8	27	31	32	34	35	37	37	38					19·42
38	7	28	31	34	36	37	38	38						18·99
39	11	24	28	29	29	34	35	36	36	37	42	42		19·14
40	11	24	26	29	30	31	33	34	35	40	41	41		19·31
41	6	31	32	34	35	36	42							18·90
42	12	26	25	31	32	32	33	34	37	38	39	41	38	19·13
43	8	36	37	39	41	44	44	46	47					17·21
44	7	31	31	32	41	45	46	48						17·82
45	9	35	36	38	42	43	43	45	46	46				17·29
46	6	34	42	45	45	46	47							16·83
47	9	31	37	38	43	43	45	47	48	48				17·06
48	3	47	48	49										15·48
49	8	41	42	44	45	45	48	49	50					16·15
50	6	38	41	42	43	43	53							16·69
51	3	49	51	52										14·55
52	4	42	44	45	47									16·48
53	3	47	48	51										15·24
54	9	40	40	47	48	50	50	51	52	57				15·20
55	7	41	46	49	50	53	54	62						14·40
56	7	42	45	47	49	51	51	51						15·39
57	2	54	55											13·11
58	4	44	52	55	59									13·71
59	4	39	44	48	54									15·83
60	4	47	52	53	61									13·30
61	...													...
62	5	55	56	57	57	59								12·19
63	3	45	52	56										14·31
64	2	57	58											12·42

These results are of course too small to successfully perform a graduation, and the observations were therefore gathered into quinary groups for ages 25–29, 30–34, &c., and the values of the widows' annuities were found as stated above.

The average annuity to the widow of a husband dying was

TABLE IV.

TABLE *showing the number of Married Men at each age in* 1902, *with the corresponding ages of their Wives, and Widows' Annuities, by Carlisle* 3 *per-cent.*

Age of Husband (1)	No. of cases Married (2)	Ages of Wives (3)															Average value of Widows' Annuities (4)
30	7	29	30	30	31	34	35	36									19·58
31	9	27	27	30	31	32	32	33	34	35							19·79
32	4	34	29	30	32												20·33
33	5	29	29	31	33	33											19·84
34	7	23	30	30	31	32	34	33									19·97
35	12	30	31	31	32	32	32	33	34	34	34	34	34				19·49
36	12	21	37	31	34	36	36	37	38	38	38	38	40				18·99
37	7	29	31	35	38	39	39	44									18·52
38	11	30	31	31	32	33	34	39	40	40	33	38					18·98
39	9	32	32	37	39	40	41	48	39	39							17·99
40	13	33	33	34	35	36	36	38	38	39	39	41	46	46			18·15
41	9	28	31	32	34	35	36	37	39	42							18·91
42	8	33	34	38	38	38	40	42	42								18·13
43	8	26	31	37	38	39	42	43	49								18·06
44	7	34	38	38	41	42	43	44									17·65
45	13	31	43	44	36	39	40	40	42	42	45	48	43	43			16·32
46	15	28	38	38	40	40	40	40	42	42	43	44	45	47	50	45	17·23
47	12	30	34	37	40	40	40	43	45	48	50	58	43				16·89
48	15	32	35	35	36	37	41	44	47	48	48	48	52	52	51	50	16·55
49	7	28	42	45	46	48	51	59									15·90
50	7	36	41	46	50	51	52	54									15·53
51	12	39	39	39	43	48	50	52	52	52	53	55	57				15·15
52	5	40	46	47	49	53											15·67
53	5	44	44	52	53	55											14·78
54	10	43	45	47	47	51	51	51	52	54	55						14·84
55	9	38	43	47	50	50	53	54	55	56							14·76
56	9	46	46	33	48	50	50	51	53	55							15·31
57	4	43	46	56	58												14·30
58	8	40	50	50	54	56	56	58	59								13·58
59	7	48	53	54	55	58	58	59									12·88
60	10	43	46	49	49	50	50	57	57	57	62						11·88
61	11	42	44	48	48	47	54	56	57	57	62	63					13·73
62	4	38	53	57	63												13·51
63	9	40	47	49	54	56	59	59	60	62							13·11
64	5	39	41	58	61	67											13·29
65	6	42	52	52	53	67	67										12·73
66	8	43	48	56	57	60	66	67	67								11·77
67	5	55	62	66	68	69											9·75
68	3	58	68	69													9·36
69	2	69	70														7·81
70	1	63															10·07
71	3	60	62	63													10·34
72	2	50	67														11·76
73	3	46	65	69													11·50
74	2	48	68														11·93
75	5	61	61	67	70	78											8·33

thus found for the central age of each group. These results were then plotted out on cross ruled paper and roughly graduated by

the graphical process. The graduations are exhibited in Diagrams I and II. The preliminary figures and the graduated annuities are given in Tables V and VI, and in order to show the effect of the graduation, the ungraduated annuity-values are reproduced in Table VI.

Table V.

Table *showing value of Widows' Annuities at Central Age. Carlisle 3 per-cent.*

1881				1902			
Age of Husband (1)	Number of Married Men (2)	Average Age of Wife (3)	Value of Annuity at Central Age (4)	Age of Husband (5)	Number of Married Men (6)	Average Age of Wife (7)	Value of Annuity at Central Age (8)
25–29	5	24·75	21·20				
30–34	26	31·12	19·91	30–34	32	30·7	19·27
35–39	37	32·44	19·34	35–39	51	35·33	18·86
40–44	44	36·78	18·58	40–44	45	37·66	17·95
45–49	35	44·16	16·74	45–49	62	42·68	16·66
50–54	25	44·14	15·69	50–54	39	48·72	15·16
55–59	24	50·04	14·70	55–59	37	51·24	14·23
60–64	14	54·6	12·99	60–64	39	52·92	13·32
65–69	...	...	...	65–74	35	61·83	11·05
				75	5	61·4	8·33

[For Table VI, *see* page 345.]

Attention may be drawn to the fact that by dealing with the numbers living, we are not hampered with having to treat first or second marriages separately, all married men being included in the above table, whether married for the first time or not.

The central age of the wife for each group is shown merely for the purpose of comparison, and was not used in obtaining the annuity-values.

The fact that the groups were taken for ages 25–29, 30–34, &c., may cause criticism, but it will be noticed that curiously the youngest married man in 1881 was age 25, and in 1902 was 30. In view of the small range of the experience it was thought well to include the younger ages instead of starting at ages 28 and 33 respectively. Very searching enquiries are also made by the managers of the Fund to verify ages, and certificates of birth have to be furnished when a claim arises. This fact being known to members, tends to correct any carelessness in giving statistics, so that probably no great error is involved.

A study of Diagrams I and II shows how slight the graduation has been, the general form of the curves being seen at a glance

TABLE VI.

TABLE *showing Ungraduated and Graduated values of continuous Annuities, payable from the death of a Husband to the Widow for remainder of her life. Carlisle 3 per-cent.*

	1881		1902	
Age of Husband (1)	Ungraduated Annuity-Value (2)	Graduated Annuity-Value (3)	Ungraduated Annuity-Value (4)	Graduated Annuity-Value (5)
25	21·80	21·3	...	.
26	21·60	21·2	...	...
27	—	21·1	...	...
28	21·16	21·0	...	...
29	20·26	20·8	...	...
30	19·74	20·7	19·58	19·6
31	20·26	20·5	19·79	19·5
32	19·63	20·2	20·33	19·4
33	20·36	20·0	19·84	19·2
34	19·48	19·9	19·97	19·1
35	20·43	19·7	19·49	19·0
36	19·29	19·5	18·99	18·9
37	19·42	19·3	18·52	18·8
38	18·99	19·0	18·98	18·6
39	19·14	18·8	17·99	18·5
40	19·31	18·6	18·15	18·3
41	18·90	18·3	18·91	18·1
42	19·13	18·1	18·13	17·8
43	17·21	17·9	18·06	17·5
44	17·82	17·7	17·65	17.2
45	17·29	17·4	16·32	17·0
46	16·83	17·1	17·23	16·8
47	17·06	16·9	16·89	16·6
48	15·48	16·7	16·55	16·3
49	16·15	16·4	15·90	16·1
50	16·69	16·1	15·53	15·9
51	14·55	15·9	15·15	15·6
52	16·48	15·6	15·67	15·3
53	15·24	15·4	14·78	15·1
54	15·20	15·1	14·84	14·9
55	14·40	15·0	14·76	14·6
56	15·39	14·8	15·31	14·4
57	13·11	14·5	14·30	14·2
58	13·71	14·3	13·58	14·0
59	15·83	14·0	12·88	13·8
60	13·30	13·8	11·88	13·6
61	—	13·5	13·73	13·4
62	12·19	13·1	13·51	13·2
63	14·31	12·9	13·11	13·0
64	12·42	12·5	13·29	12·8
65	...	...	12·73	12·5
66	...	...	11·77	12·2
67	...	..	9·75	12·0
68	...	.	9·36	11·7
69	...	...	7·81	11·5
70		...	10·07	11·2
71			10·34	10.9
72	..	...	11·76	10·5
73	...	. .	11·50	10·1
74	...	...	11·93	9·8
75	.	...	8·33	9·4

from the parallelograms for each group of five years, and it would, I think, be comparatively easy to trace the curve to higher ages without involving much error. It is submitted that the annuities brought out represent fairly closely the actual experience of the Fund.

Orphans' Annuities were dealt with in a similar manner. The annuities used for this purpose were calculated from the Table of Children's Mortality given by Mr. Manly on page 115 of Vol. xxxviii of the *J.I.A.*, with interest at 3 per-cent. The formula used for the annuity was that given by Mr. King in his paper on "Family Annuities" (*J.I.A.*, vol. xxx, p. 291), namely:

$$\bar{a}_{x+\frac{1}{2}\,\overline{18-x-\frac{1}{2}}|} = \tfrac{1}{2} + \tfrac{1}{4}\left\{a_{x\,\overline{18-x}|} + a_{x+1\,\overline{18-x-1}|} + a_{x\,\overline{18-x-1}|} + a_{x+1\,\overline{18-x-2}|}\right\}$$

The following table shows the values of the annuities used for the various ages.

TABLE VII.

Values of Temporary Annuities to Children ceasing at age 18. *Mr. Manly's Table of Children's Mortality at* 3 *per-cent* (*J.I.A.*, xxxviii, 115).

Year of Age (1)	Value of Annuity (2)
0– 1	12·525
1– 2	12·394
2– 3	11·913
3– 4	11·369
4– 5	10·791
5– 6	10·180
6– 7	9·536
7– 8	8·859
8– 9	8·149
9–10	7·410
10–11	6·646
11–12	5·831
12–13	5·036
13–14	4·135
14–15	3·304
15–16	2·395
16–17	1·459
17–18	·492

The number of children existing at each age arranged in quinquennial groups according to the age of the father, is given in Tables VIII and IX, and the value of the orphan's annuity at the central age is also shown.

TABLE VIII.

Fund 1881.

Table showing number of Children at all ages to 17 last birthday pertaining to Married Men, arranged in quinary groups, with corresponding Annuity-Values.

Age of Father	No. of Children	Age of Children: 17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	No. of Married Men	Annuity at Central Age (Mr. Manly's Table of Children's Mortality at 3 per-cent)
25–29	3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	1	1	5	7·36
30–34	40	...	...	...	...	...	...	...	...	1	1	1	1	3	3	8	6	6	10	26	17·65
35–39	90	...	...	...	...	2	2	1	4	6	3	6	6	5	12	9	11	12	11	37	24·47
40–44	133	1	1	4	3	6	5	9	12	11	8	12	10	8	12	8	7	7	9	44	25·83
45–49	148	5	5	6	6	8	10	10	15	8	11	9	7	11	10	6	13	2	6	35	32·09
50–54	77	6	6	7	4	8	5	6	7	6	4	2	4	4	2	3	2	1	0	25	17·51
55–59	55	6	7	4	5	8	3	4	4	1	2	4	1	2	1	0	1	0	2	21	11·39
60–64	25	5	3	2	1	2	2	1	3	2	...	2	...	1	...	1	...	...	...	14	8·04
65–69	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
70–74	...	...	...	...	...	...	...	...	...	...	...	...	...	...		...	...	...		...	...

TABLE IX.

Fund 1902.

Table showing number of Children at all ages to 17 last birthday pertaining to Married Men, arranged in quinary groups, with corresponding Annuity-Values.

Age of Member	No. of Children	Age of Children 17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	No. of Married Men	Annuity at Central Age (Mr. Manly's Table of Children's Mortality at 3 per-cent)
30–34	27	...	...	...	...	...	...	...	...	...	1	1	...	1	3	2	8	4	7	32	9·84
35–39	91	...	...	...	...	...	...	3	2	4	9	5	10	12	10	11	10	10	5	51	18·33
40–44	93	...	1	...	1	4	6	4	2	9	8	6	12	6	8	5	9	8	4	45	18·87
45–49	168	7	10	13	12	10	14	16	8	13	13	9	8	7	10	6	4	4	4	62	17·78
50–54	69	6	5	6	9	10	4	7	3	1	4	4	3	4	...	...	1	1	1	39	9·21
55–59	55	5	8	5	3	11	4	7	4	2	2	...	1	1	1	...	1	...	...	37	6·58
60–64	53	6	5	6	1	6	4	2	3	2	4	5	1	3	2	...	2	1	...	39	7·56
65–69	7	3	1	1	...	1	...	...	...	...	...	1	...	...	...	...	...	...	...	24	·77
70–74	6	1	1	1	...	1	1	...	...	1	...	...	...	...	...	...	...	...	...	11	1·91
75–79	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	7	...

These figures were then graduated in a manner similar to that employed for Widows' Annuities, and the results are shown in the following table, while the actual graduations are to be seen in Diagrams III and IV. I may say that I am not satisfied that these curves are the most satisfactory that could be obtained, but it is not suggested that the results should be used in practice, and I was anxious not to depart from the original data further than was necessary to obtain a smooth, continuous curve. It will be noticed that the general outlines of the curves are very similar.

TABLE X.

TABLE *showing the value on the death of a Husband of a Continuous Annuity to each surviving child, to be continued till age* 18 *is attained. (Mr. Manly's Table of Children's Mortality at* 3 *per-cent.)*

Age of Father	1881	1902	Age of Father	1881	1902
	Graduated Annuity-Value	Graduated Annuity-Value		Graduated Annuity-Value	Graduated Annuity-Value
(1)	(2)	(3)	(1)	(2)	(3)
23	·1	...	50	21·3	11·2
24	·9	...	51	19·3	9·9
25	2·2	...	52	17·5	9·0
26	4·6	...	53	15·8	8·4
27	7·0	·3	54	14·3	7·9
28	9·5	·9	55	13·1	7·4
29	12·0	2·0	56	12·0	6·9
30	14·0	3·7	57	11·1	6·5
31	16·0	5·8	58	10·3	6·1
32	17·8	8·9	59	9·6	5·8
33	19·5	12·4	60	8·9	5·4
34	21·1	14·9	61	8·3	5·0
35	22·7	16·5	62	7·8	4·7
36	24·1	17·7	63	7·3	4·4
37	25·4	18·7	64	6·9	4·1
38	26·6	19·2	65	...	3·8
39	27·7	19·7	66	...	3·5
40	28·7	20·0	67	...	3·3
41	29·5	20·1	68	...	3·0
42	30·3	20·0	69	...	2·8
43	30·8	19·9	70	...	2·5
44	30·7	19·6	71	...	2·3
45	30·1	19·2	72	...	2·2
46	29·2	18·4	73	...	2·0
47	27·9	17·3	74	...	1·8
48	26·1	15·6	75	...	...
49	23·7	13·2			

The differences in these annuity-values will be seen to be very considerable, and indicate the effect of some existing cause which is responsible for the decrease in the size of families.

So far only married men have been dealt with, and I now proceed to show the effect on the annuity-values when bachelors and widowers are included. Mr. Manly, in his paper on the subject, found the relative number of bachelors, widowers and married men living at each age, and adjusted his commutation columns accordingly. There is, however, no theoretical reason why the annuity-values should not be operated upon in the first instance, and special values of C_x and M_x, obtained therefrom. There appear also to be some advantages in the alternative method suggested as we can more readily see the effect of including the bachelors and widowers by comparing annuity-

Table XI.

Table *showing proportion of Married Men living at each age in quinary groups, and value, by Carlisle 3 per-cent, of Continuous Annuity at Central Age to Widows, deduced from the number of Married Men, and of all Members.*

Age of Member (1)	Number of Members (2)	Number of Married Men (3)	Ratio of Married Men to total Members (4)	Value of Annuity at Central Age deduced from Married Men (5)	Value of Annuity at Central Age deduced from all Members (6)
			1881 Experience		
20–24	1	0	·0	·0	·0
25–29	12	5	·417	21·20	8·83
30–34	40	26	·650	19·91	12·94
35–39	43	37	·860	19·34	16·64
40–44	51	44	·863	18·58	16·03
45–49	40	35	·875	16·74	14·65
50–54	29	25	·862	15·69	13·53
55–59	28	24	·857	14·70	12·60
60–64	14	14	1·000	12·99	12·99
			1902 Experience		
20–24	1	0	·0	·0	·0
25–29	15	0	·0	·0	·0
30–34	56	32	·571	19·27	11·01
35–39	66	51	·773	18·86	14·57
40–44	62	45	·726	17·95	13·03
45–49	69	62	·899	16·66	14·97
50–54	44	39	·886	15·16	13·44
55–59	40	37	·925	14·23	13·16
60–64	40	39	·975	13·32	12·99
65–69	29	24	·827	10·96	9·07
70–74	13	11	·846	10·65	9·47
75	8	7	·875	8·33	7·29

values than by comparing commutation columns. Also if some standard tables were prepared, and the ratios $\frac{\text{married men}}{\text{all members}}$ were given in addition to the annuity-values for both married men and all members, it would be in many cases comparatively easy to adjust such standard tables to meet the case of a Fund in which the average age at marriage and the birth rate agreed with the standard, but in which there was a larger or smaller proportion of married men. Of course adjustment must be made in respect of widowers' children, and for the purpose of children's annuities

TABLE XII.

TABLE *showing proportion of Married Men living at each age in quinary groups, and value of Continuous Annuity to each surviving child, to age* 18, *deduced from the number of Married Men, and of all Members.* (*Mr. Manly's Table of Children's Mortality, at* 3 *per-cent.*)

Age of Member (1)	No. of Members (2)	No. of Married Men (3)	Ratio of Married Men to total Members (4)	Value of Annuity at Central Age deduced from Married Men (5)	Value of Annuity at Central Age deduced from all Members (6)
			1881 Experience		
20–24	1	0	·0	·0	·0
25–29	12	5	·417	7·36	3·07
30–34	40	26	·650	17·65	11·47
35–39	43	37	·860	24·47	21·06
40–44	51	44	·863	25·83	22·28
45–49	40	35	·875	32·09	28·28
50–54	29	25	·862	17·51	15·09
55–59	28	24	·857	11·39	9·42
60–64	14	14	1·000	8·04	8·04
			1902 Experience		
20–24	1	0	·0	·0	·0
25–29	15	0	·0	·0	·0
30–34	56	32	·571	9·84	5·62
35–39	66	51	·773	18·33	14·16
40–44	62	45	·726	18·87	13·70
45–49	69	62	·899	17·78	15·98
50–54	44	39	·886	9·21	8·16
55–59	40	37	·925	6·58	6·09
60–64	40	39	·975	7·56	7·37
65–69	29	24	·827	·77	·64
70–74	13	11	·846	1·91	1·62
75	8	7	·875	·0	·0

TABLE XIII.

Value on the death of (1) *a Married Man, and* (2) *any Member, of a Continuous Annuity for the remainder of life to his Widow, by Carlisle* 3 *per-cent.*

Age at death (1)	1881 Experience: Annuity-Value applicable to Married Man (2)	1881 Experience: Annuity-Value applicable to any Member (3)	1902 Experience: Annuity-Value applicable to Married Man (4)	1902 Experience: Annuity-Value applicable to any Member (5)
23	...	...	...	...
24	...	...	...	...
25	21·3	2·6	...	...
26	21·2	4·2	..	...
27	21·1	6·1	...	·3
28	21·0	8·4	...	1·5
29	20·8	10·1	...	3·2
30	20·7	11·6	19·6	5·6
31	20·5	12·9	19·5	8·5
32	20·2	13·9	19·4	10·7
33	20·0	14·8	19·2	12·3
34	19·9	15·6	19·1	13·4
35	19·7	16·2	19·0	14·1
36	19·5	16·6	18·9	14·5
37	19·3	16·8	18·8	14·7
38	19·0	16·9	18·6	14·8
39	18·8	16·8	18·5	14·8
40	18·6	16·7	18·3	14·7
41	18·3	16·5	18·1	14·6
42	18·1	16·2	17·8	14·5
43	17·9	15·9	17·5	14·4
44	17·7	15·6	17·2	14·3
45	17·4	15·3	17·0	14·1
46	17·1	15·0	16·8	14·0
47	16·9	14·7	16·6	13·9
48	16·7	14·5	16·3	13·8
49	16·4	14·2	16·1	13·7
50	16·1	14·0	15·9	13·6
51	15·9	13·8	15·6	13·5
52	15·6	13·6	15·3	13·4
53	15·4	13·4	15·1	13·3
54	15·1	13·2	14·9	13·1
55	15·0	13·0	14·6	12·9
56	14·8	12·9	14·4	12·8
57	14·5	12·8	14·2	12·6
58	14·3	12·7	14·0	12·5
59	14·0	12·5	13·8	12·3
60	13·8	12·4	13·6	12·1
61	13·5	12·2	13·4	11·9
62	13·1	12·0	13·2	11·7
63	12·9	11·8	13·0	11·6
64	12·5	11·7	12·8	11·4
65	...	...	12·5	11·2
66	...	...	12·2	11·0
67	...	...	12·0	10·8
68	...	...	11·7	10·5
69	...	...	11·5	10·3
70	...	...	11·2	10·1
71	...	..	10·9	9·8
72	...	...	10·5	9·6
73	...	...	10·1	9·4
74	...	...	9·8	9·1
75	...	...	9·4	8·9

Diagram I.

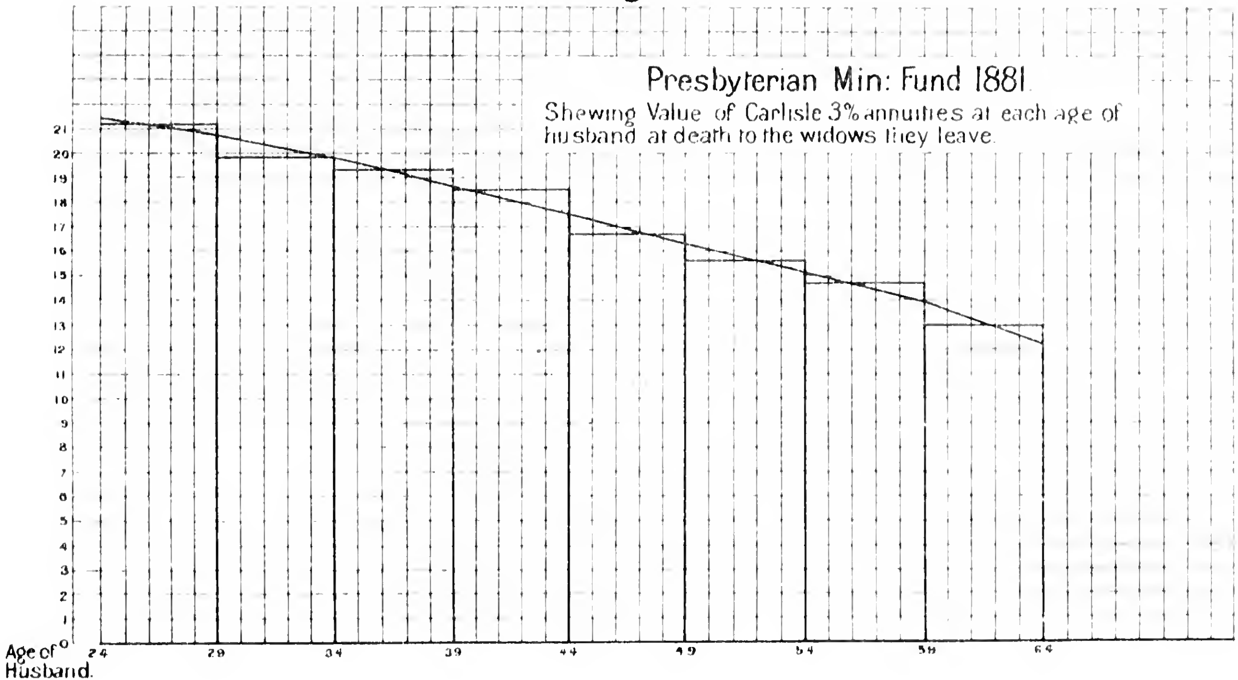
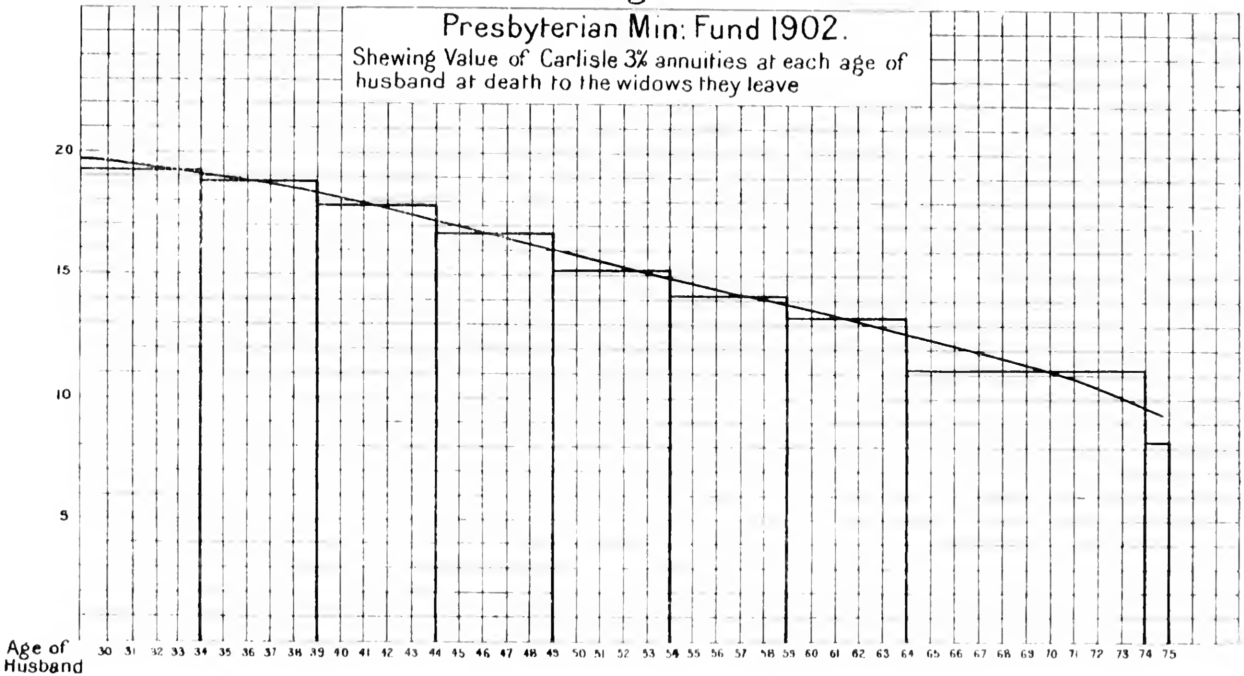

Presbyterian Min: Fund 1881.
Shewing Value of Carlisle 3% annuities at each age of husband at death to the widows they leave.
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
Age of Husband.
24
29
34
39
44
49
54
59
64

Diagram II.



Presbyterian Min: Fund 1902.
Shewing Value of Carlisle 3% annuities at each age of husband at death to the widows they leave
20
15
10
5
Age of Husband
30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75

Diagram III.

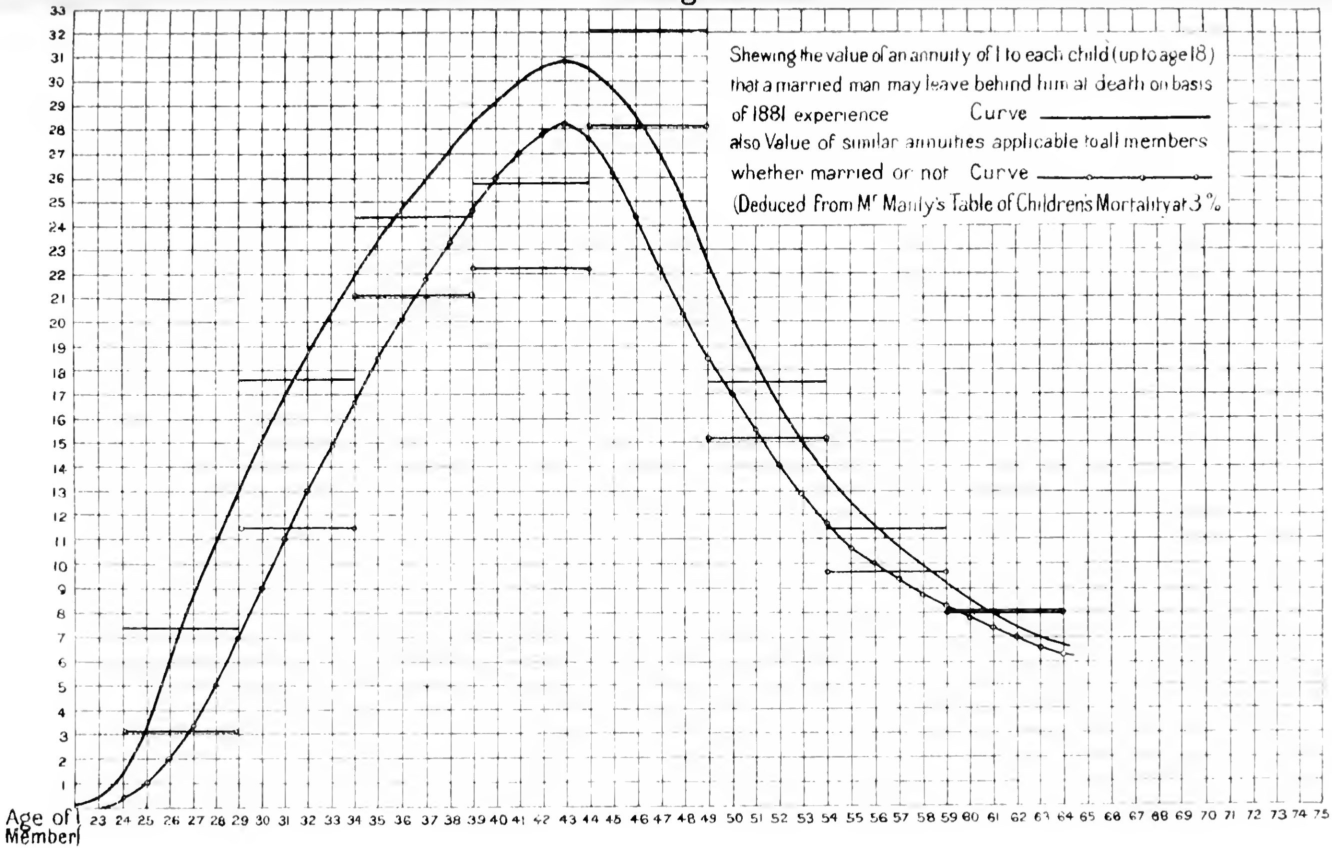

Shewing the value of an annuity of 1 to each child (up to age 18) that a married man may leave behind him at death on basis of 1881 experience Curve
also Value of similar annuities applicable to all members whether married or not Curve
(Deduced from Mr Manly's Table of Children's Mortality at 3 %
Age of Member
23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Diagram IV.

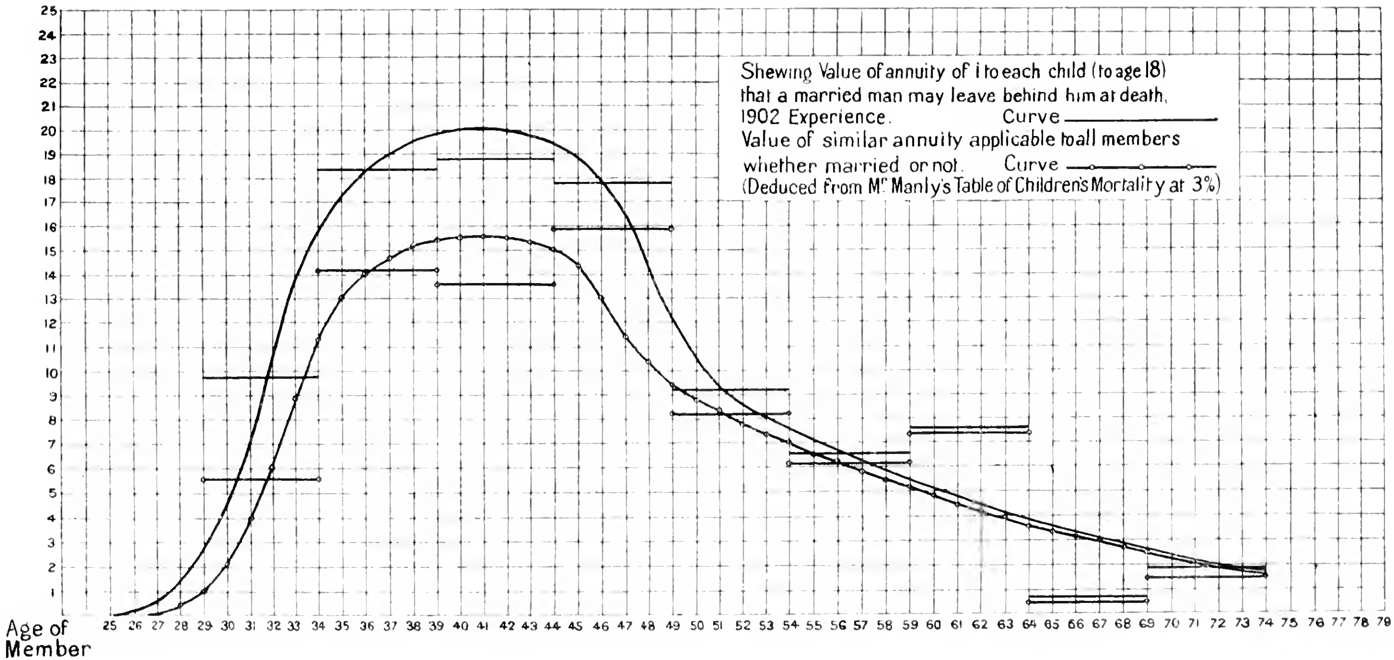

Shewing Value of annuity of 1 to each child (to age 18)
that a married man may leave behind him at death,
1902 Experience. Curve
Value of similar annuity applicable to all members
whether married or not. Curve
(Deduced from Mr. Manly's Table of Children's Mortality at 3%)
25
24
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
Age of Member
25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79

Diagram V.

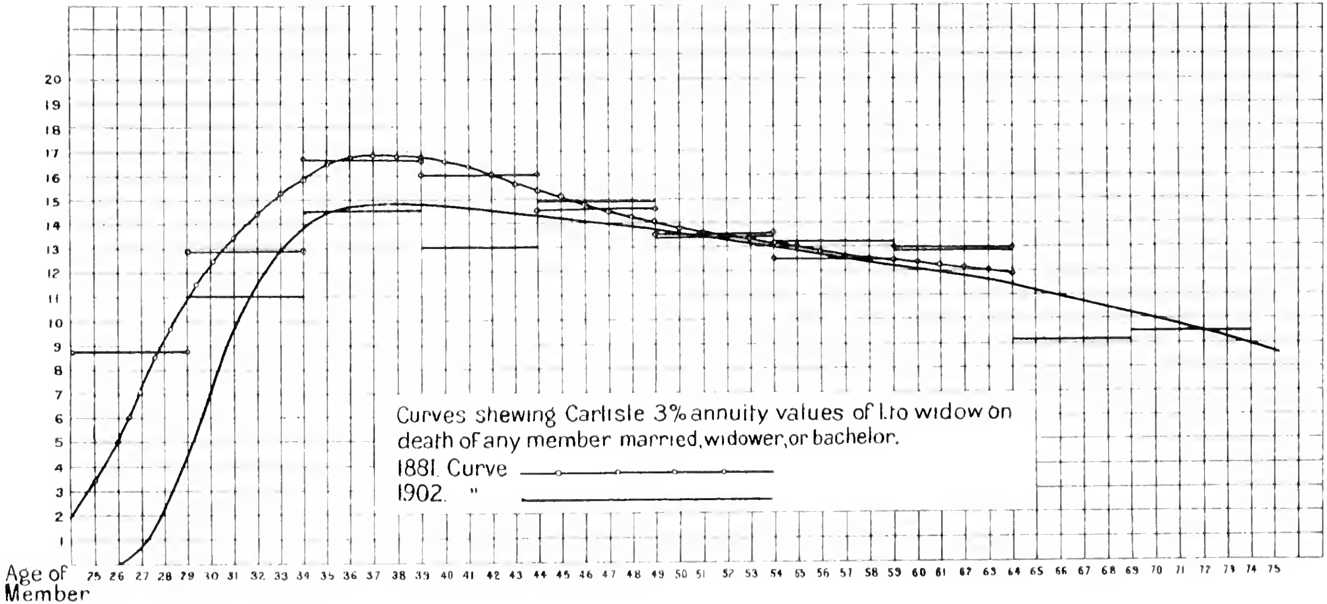

Curves shewing Carlisle 3% annuity values of 1. to widow on death of any member married, widower, or bachelor.
1881. Curve
1902. "
Age of Member
25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

TABLE XIV.

Value on the death of (1) *a Married Man, and* (2) *any Member, of Continuous Annuities to each of the children until they attain the age of* 18. (*Mr. Manly's Table of Children's Mortality at* 3 *per-cent.*)

Age at Death	1881 Experience		1902 Experience	
	Annuity-Value applicable to Married Man	Annuity-Value applicable to any Member	Annuity-Value applicable to Married Man	Annuity-Value applicable to any Member
(1)	(2)	(3)	(4)	(5)
23	·1	...	...	...
24	·9	·1	...	...
25	2·2	·6	...	...
26	4·6	1·5	...	...
27	7·0	2·6	·3	·1
28	9·5	4·2	·9	·2
29	12·0	6·0	2·0	·7
30	14·0	8·0	3·7	1·5
31	16·0	10·0	5·8	3·0
32	17·8	11·9	8·9	5·0
33	19·5	13·8	12·4	7·5
34	21·1	15·7	14·9	10·1
35	22·7	17·6	16·5	12·3
36	24·1	19·4	17·7	13·6
37	25·1	21·0	18·7	14·4
38	26·6	22·6	19·2	15·0
39	27·7	24·0	19·7	15·3
40	28·7	25·3	20·0	15·5
41	29·5	26·5	20·1	15·6
42	30·3	27·4	20·0	15·6
43	30·8	28·0	19·9	15·5
44	30·7	28·0	19·6	15·2
45	30·1	27·0	19·2	14·7
46	29·2	25·4	18·4	13·7
47	27·9	23·3	17·3	12·3
48	26·1	21·3	15·6	10·9
49	23·7	19·5	13·2	10·0
50	21·3	17·8	11·2	9·2
51	19·3	16·2	9·9	8·6
52	17·5	14·8	9·0	8·1
53	15·8	13·5	8·4	7·6
54	14·3	12·2	7·9	7·2
55	13·1	11·2	7·4	6·8
56	12·0	10·4	6·9	6·4
57	11·1	9·7	6·5	6·1
58	10·3	9·1	6·1	5·7
59	9·6	8·6	5·8	5·4
60	8·9	8·1	5·4	5·0
61	8·3	7·6	5·0	4·7
62	7·8	7·2	4·7	4·4
63	7·3	6·8	4·4	4·2
64	6·9	6·5	4·1	3·9
65	...	...	3·8	3·6
66	...	...	3·5	3·3
67	...	...	3·3	3·1
68	...	...	3·0	2·9
69	...	...	2·8	2·7
70	...	...	2·5	2·4
71	...	...	2·3	2·2
72	...	...	2·2	2·0
73	...	...	2·0	1·9
74	...	...	1·8	1·7

they must be treated as married men. The method suggested was employed in the present instance, and the foregoing Tables XI and XII, obtained from the figures in Tables V, VIII and IX, show the process in full.

The annuities for the central ages were then graduated by the graphic method (Diagrams III, IV and V) and the results are given in the foregoing Tables XIII and XIV. For the purpose of comparison the annuities applicable to the widows and progeny of married men are reproduced.

Probably owing to paucity of facts, there is considerable irregularity in the proportion of married men to total members, and more severe graduation had to be employed than in the case of the annuities applicable to married men only. The general form of the curve is however apparent.

The result of the investigation shows (1) that there has been an increase in the marriage age of the husband, and that with this increase there has been a correspondingly larger increase in the age of the wife; (2) that the number of children per family shows a considerable decline. Whether the increase in the marriage age would be shown by other funds seems doubtful, as there is no satisfactory cause for it forthcoming; but the decrease in value of the widows' annuities shows that even a slight change in the relative ages of wives and husbands will make itself shown in the valuation factors. When, therefore, a pension fund is valued by tables obtained from a totally different class of lives from those comprised in the fund, grave errors may be involved.

With regard to the fall in the number of children per family, this has been so persistent and so large that it cannot be regarded as due to paucity of facts. The class, too, that we have been dealing with is one where we should have least expected such a change; for it must be borne in mind that the emoluments enjoyed by the ministers in this church have not declined, as have those of the clergymen of the Church of England, but, on the other hand, have, if anything, increased. The decline shown by the above figures stands out, therefore, as a sign-post pointing to the tendency of the age. In the middle and lower middle classes, who contribute largely to the bank and commercial funds of this description, a still larger decrease in average family would probably be found; and strikes and depression in trade must surely have caused a corresponding effect even upon that improvident class represented by the British workman.

A valuation made on the basis of the 1902 figures for children's annuities would show a reduction of something like 10 per-cent on the 1881 figures. This difference in the case of many funds in a sound condition would not perhaps be considered of much practical importance, as the valuation on the basis of present tables would be on the safe side. In the fund under review there was a very satisfactory surplus, but I must confess that had there been a small deficiency I should have felt grave doubts as to whether I was justified in stating that the fund was insolvent. Even in the case of a strong fund it is very desirable to ascertain the exact state of affairs, and to know by how much our estimate of liabilities exceeds their true value.

The fund I have been dealing with is, of course, very small, probably too small for trustworthy deductions, but I venture to think that the results obtained are not very far from the truth. A study of the diagrams given shows a comparative amount of regularity in the increase and decrease of the ungraduated figures for each successive central age; indeed, as regards the widows' annuities, the regularity is more than might have been expected considering the paucity of facts, and leads one to hope that an experience of say five times the size of the one in question would be of practical use. I venture to suggest that even the results shown prove that there is a decided need for fresh tables for use in the valuation of widows' and pension funds, and that by going to the living instead of to the dead we could obtain valuable statistics from the experiences of funds which have hitherto been regarded as too small for consideration in this respect.

In conclusion I would draw attention to the fact that there is a large field at our very doors from which valuable data could be obtained. I refer to the insurance companies with which most of us have some small connection. It would be a very simple matter to obtain statistics showing the age of the officials and clerks, and the ages of their wives and children, and possibly even their salaries, though there might be some objections to the latter item being included.

In view of the large numbers employed by these companies a valuable experience could be obtained, which would probably be a representative table for the funds applying to the upper commercial class.

Abstract of Discussion.

Mr. E. C. Thomas was quite sure he was only voicing the collective opinion of the meeting in congratulating the author upon his labours in such a very fertile field of research, and in thanking him for the very suggestive paper he had submitted. It seemed to him to be valuable, as much for what it merely suggested, as for what it enlarged upon; and in any observations which he made he should merely be following out the train of thought suggested by the remarks in the paper. One's mind naturally turned to Mr. Manly's paper of April 1903, both on account of the references in the present paper, and because of the close association of ideas. The method to which Mr. Manly adhered in the main was that known as the collective method, but he indicated in his remarks at the meeting that by means of certain supplementary tables he saw his way to a possible satisfactory solution of the problems on individual lines. The collective method of constructing tables was so well described by Mr. Manly that it was hardly necessary to say anything further on the point. Briefly stated, it consisted in assessing the benefits payable to a particular class as a function of the number living, not of that particular class, but of all classes together. In applying the valuation factors, therefore, no weight was given to the distribution of the various classes in the particular fund at the moment of valuation. On the other hand, in tables of the individual type, as exemplified in Huie's book, the benefits were tabulated as a function of each particular class. Thus, there was the value of a benefit to the first wife of a member now a bachelor, or a husband, separately, with similar values for the second wives of bachelors, husbands, and widowers, taken separately. The question as to which method was the best for practical use seemed to him to be closely allied with the question of standard tables. There were many small funds from which it would be impossible to obtain sufficient data to construct a satisfactory table, and the difficulty was often enhanced by the fact that the records had been very imperfectly kept. If, therefore, it was decided to use some standard tables it would be doubtless felt that, apart from the possible deviations from the assumed future experience, there would be grave risk in using tables based on the collective principle, without due weight being given to the actual proportions of the various classes in the fund at the moment of valuation. They would therefore, be driven to use some table like Huie's. The objection to Huie's tables was that apart from the contingencies of third and subsequent marriages, there were sundry other problems, not dealt with by him, which seemed almost incapable of exact solution on individual lines. For instance, taking the case of an annuity to a widower with the reversion to the youngest child up to a specified age; in the case of a widower on the books with children under the age, there were three benefits to value, which, although distinct, were to a certain extent mutually dependent. There was the annuity to the present children of the widower, the annuity to a possible second wife, and the reversion to her children; and as the latter benefits were increased, so the first benefit, as Mr. Manly pointed out (*J.I.A.*, xxxviii, 136), was diminished. If there were some means

of applying the collective method, these awkward subsidiary problems could be eliminated. The suggestion made by Mr. Lidstone at the meeting already referred to (*J.I.A.*, xxxviii, p. 163), and elaborated by Mr. Allin in the present paper, of going to the living for the experience data instead of to the dead, provided a way out of the difficulty. If that idea were carried to its extreme limits, all difficulties would vanish, because they could then go to their own funds, even although they were quite small—a method which had been pursued by the writer of the paper, with apparently great success, in a fund consisting of only 300 or 400 members. The assumption involved was that the proportions of husbands, bachelors, widowers, the ages of the wives, and the numbers and ages of the children, for each particular age of the member would be the same among those dying as among those living at that age. Before proceeding further it would be desirable, he thought, to see what were the objections to that rather sweeping assumption. In the aforesaid discussion Mr. Lidstone remarked that there was no reason to suppose that the dying were anything else than an average sample of the living in those respects. Personally, he did not feel at all convinced that the experience of the two classes, living and dying, would be the same. Taking, for instance, the question of the number of children per family, it appeared to him that, in dealing with a professional or wage-earning class, it might very well be that the members dying in middle life would leave larger families, on the average, than were possessed by the survivors. The possession of a large family might act in a variety of ways to shorten a man's life. There was the incentive to greater exertions to attain success, in which he might ruin his own health. There were the additional domestic anxieties incidental to the upbringing of a large family, the greater financial strain, and the consequent inducement to deny himself the comforts, and, possibly, even the bare necessaries of life. He thought some considerable weight must be given to those considerations, and there might possibly be other considerations affecting in like manner the other elements of the problem which it was desirable should be faced, before making too hasty a generalization. When all was said, however, it would probably be admitted that the advantages of the method at least outweighed the disadvantages. If, then, it were conceded that this assumption could legitimately be made, the matter became simplicity itself; they did not require elaborate marriage and mortality tables, nor, as the author pointed out, need they distinguish between first, second, or nth marriages. They simply required as a fundamental basis of working columns of C_x and D_x for males, annuity-values for females, and term annuities for children; and for these purposes it would surely not be difficult to find some standard tables that would meet the requirements of the particular case. They then went to their own fund, and set out the ratios of the three classes, husbands, widowers and bachelors existing and on the fund at each age; secondly, the ages of the wives, including all marriages; thirdly, the numbers and ages of the children, both of husbands and widowers, and in respect of first or subsequent marriages. Then, for a payment on death to a husband, widower or bachelor, they took the C_x and multiplied by the appropriate ratio expressed as a decimal of unity, summed the column, and divided by D_x. For an

annuity to a widow, whether of a first or subsequent marriage, they took the average wife's annuity-value for age x of the husband, and multiplying that value into the C_x for the husband as already obtained, summed again and divided by D_x. A similar procedure would give the benefits for the children. That method, when fully carried out, was remarkable for its elasticity and simplicity. Varying annuities could be easily allowed for, by an additional factor applied to the C_x column. Mr. Manly gave an example in his paper of an annuity payable to the widow, depending on the amount of pension payable to the husband. To sum up, therefore, it seemed that, in the first place, standard tables based on the individual method were complex, and did not meet all requirements; secondly, that the use of standard tables of the collective type was open to the objection that no weight could be given to the existing facts; thirdly, that, if they went to the living instead of the dead, they could give full weight to the existing facts of the fund, and make a valuation of a comparatively simple character on the collective method. On the whole, it would appear that, except for purposes of comparison, standard tables might very well be dispensed with. He seemed in that respect to have arrived at a conclusion diametrically opposed to the author's, but he thought it would be seen that the author's ideas of standard tables were limited in extent. Mr. Allin suggested the tabulation of the ratio, "married men to all members," which seemed to be a very valuable suggestion; and he would add as an improvement that the ratios for husbands and widowers should be tabulated separately. The author did not say exactly how he would propose to make use of those ratios, but he would suggest that in constructing multipliers they might apply the ratios to the D_x of the denominator, getting a new denominator which might be called D_x^m; then, using those valuation factors, they could multiply them into the number of married men, and not into the number of all classes. For a payment on the death of a bachelor, a similar modification would be made in the appropriate multiplier, the ratio applied being that of bachelors to all members. This modified valuation factor would be multiplied into the number of bachelors on the fund at each age. That method would be similar in principle to the improvement introduced into the valuation of pension funds by Mr. Manly, of incorporating the salary in the denominator. It involved the assumption that given, for a particular fund, certain numbers existing as husbands, widowers or bachelors at the present time, those numbers would be subject to the same rates of increase or decrease in the future as in the experience adopted, and not that the present facts must be entirely in accord with that experience. The adoption of that device would go a long way towards removing the objections to standard tables of the collective type.

Mr. George King agreed with Mr. Thomas that, when such funds were dealt with by the collective method, smaller statistics could be used than if they were dealt with in the old ways. There was one particular point in the paper to which he thought some attention should be called, *i.e.*, the extraordinary apparent diminution in the number of children in a family. He used the word "apparent", because he did not think the case was proved from the figures in the paper. He could not help thinking that part of the phenomenon

was explained by two words on page 339, where they were told that, at the formation of the fund, the professors and others were *eligible* as members, and from the time of the adoption of the scheme all those who received appointments were *required* to become members. He could not help thinking that at the beginning there was a considerable selection—that only those who would derive immediate benefit from the fund would join, whereas those who did not see any immediate benefit to themselves would remain out, taking advantage of the option that was given to them: and he thought the tables in the paper rather indicated that that took place. In Tables III and IV the ages of the husbands and the number of cases for the two periods 1881 and 1902 were given. He had been at the trouble to test, by taking out in detail, the average age at both those points. The average age of the husbands in 1881 was 44·65, and in 1902 49·22—an increase of $4\frac{1}{2}$ years—and when it was remembered that the annuities to the children ceased at eighteen years of age, that made a considerable difference to the resulting values. It was not, however, so easy to explain the difference shown in the tables where the value of the annuity to the children according to the present age of the parent was given. It might be thought that there the selection would, to a certain extent, disappear, but he did not think it did so, because if an undue number of members joined with large families at the beginning, that would affect the figures very materially for some years afterwards. He could not help thinking, therefore, that the fall in the average number of the family was exaggerated. Probably the fund had now in that respect reached a normal state; it would probably be found in future that there would not be any very great change, and he thought that to a large extent what he had pointed out was due to the cause he had mentioned. Table XII, which was not very conveniently arranged for comparison, also threw a little light upon the subject. If they compared the ratio of married men to total members in the 1881 experience with that shown in the 1902 experience, it would be found that the ratio was very much greater in 1881 at the younger ages, but that at the older ages it was greater in 1902. That, he thought, was further evidence of the selection to which he had referred. At the same time he should hesitate to express a very strong opinion on that point without being able to examine the actual details of the experience. The paper was a very interesting one, which he hoped to study and make use of.

[The following notes by Mr. Vyvyan Marr were handed in and read at the meeting, Mr. Marr being unable to attend personally.]

Mr. Vyvyan Marr: I recently investigated the collective, or varying assurance, method of valuation of widows' funds, comparing the valuation factors as deduced from two widely different experiences. For this purpose I used the statistics relating to 681 bankers who died and 822 ministers who died, and the results are published in the *Transactions of the Faculty of Actuaries*, vol. ii, p. 307. The statistics formed part of Mr. Archibald Hewat's "Investigations of the Marriage and Mortality Experiences of Scottish Bankers and Scottish Ministers", and were kindly placed at my disposal by him. The results pointed to the fact that in valuing a fund great care

must be exercised in making use of a set of figures deduced from the experience of another fund, as so much depends upon the various probabilities, and the relative marriage status of the members. Mr. Allin confirms this view, and from his Tables XIII and XIV I think that the method would be even more untrustworthy when statistics relating to "existing" are used in computing the valuation factors, in place of statistics relating to "deaths." If we compare the amount to be set aside on the death of each member, to provide the "Widow's Annuity" that may then become payable, as given in Mr. Allin's Table XIV, with mine, it will be observed that Mr. Allin's values are considerably higher than those deduced by me in connection with Scottish ministers, who, no doubt, occupy a similar social position to the ministers investigated by him. I think this may be attributed to two causes: (i) That in using the statistics relating to existing members, elements of selection are omitted, which would be taken into account either in calculating probabilities of marriage for use in a valuation by the reversionary annuity method, or in deducing tables, based on deaths alone, for use in a valuation by the varying assurance method; and (ii) taking the relative ages of wives at the moment they become widows as being the same as, in place of higher than, the relative ages of wives whose husbands are still alive. As regards the first point, it will be observed from Mr. Allin's Tables I and II that the relative number of bachelors, married men, and widowers varied considerably from one period to another. The probabilities of marriage, and the numbers who die leaving widows, will not necessarily fluctuate so much. This, I think, is shown by comparing the relative numbers dying as bachelors, husbands, and widowers in Mr. Hewat's Scottish Ministers' experience, which I made use of, with the relative numbers in existence at the close of his observations, as exhibited in the following table:

Ages	Dying							
	Bachelors		Husbands		Widowers		Total	
	No.	Per-cent	No.	Per-cent	No.	Per-cent	No.	Per-cent
25–29	7	1	1	...	...	...	8	1
30–34	14	2	15	2	...	...	29	4
35–39	19	2	24	3	...	...	43	5
40–44	9	1	34	4	3	...	46	5
45–49	10	1	33	4	7	1	50	6
50–54	7	1	49	6	8	1	64	8
55–59	11	1	59	7	6	1	76	9
60–64	17	2	73	9	11	1	101	12
65–69	7	1	92	11	21	3	120	15
70–74	19	2	64	8	29	3	112	13
75–79	11	2	52	6	31	4	94	12
80–84	2	...	24	3	25	3	51	6
85–	2	1	9	1	17	2	28	4
	135	17	529	64	158	19	822	100

Ages	Existing at Close of Observation							
	Bachelors		Husbands		Widowers		Total	
	No.	Per-cent	No.	Per-cent	No.	Per-cent	No.	Per-cent
25–29	16	1	5	1	...	...	21	2
30–34	63	5	62	5	1	...	126	10
35–39	44	4	118	9	1	...	163	13
40–44	44	4	151	11	4	...	199	15
45–49	23	2	137	11	4	...	164	13
50–54	15	1	100	8	4	...	119	9
55–59	24	2	125	9	9	1	158	12
60–64	14	1	108	8	9	1	131	10
65–69	5	...	53	4	8	1	66	5
70–74	5	...	55	4	9	1	69	5
75–79	2	...	27	2	9	1	38	3
80–84	2	...	12	1	13	1	27	2
85–	...	...	6	1	6	...	12	1
	257	20	959	74	77	6	1,293	100

In regard to the widows' ages, influences, such as the very high rates of mortality among married women at younger ages, have to be taken into account, and the relative ages of wives will not, so far as I can see, be the same as the relative ages of widows at the moment of entry into widowhood. I cannot say what the difference may be, but the following table will show that one does exist:

Husband's Age at Marriage	Average Age of Wife		Husband's Age at Death	Widow's Age at Death of Husband	
	Bankers	Ministers		Bankers	Ministers
25	24	25	25	25	26
30	26	27	30	29	30
35	27	29	35	32	33
40	29	29	40	36	38
45	29	33	45	40	41

The average ages at marriage are those deduced by Mr. Hewat, while the ages relating to the date of widowhood are those found by me in the investigation referred to. In conclusion, I would point out that as Mr. Allin excludes all children over age 18, his tables exaggerate the decrease in the number of children per family. This arises through all the children included in the 1881 experience having passed out of observation by 1902, while the surviving parents remain.

Mr. H. W. Manly, after thanking the author for his valuable contribution, said it was extremely desirable that a good many papers of the type under discussion should be written, in order that the Institute could obtain statistics relating to widows' and orphans'

funds, particularly such figures as the author had produced, showing the changes which occurred from time to time. There were two curious changes in the social condition of the members of this fund, their wives, widows and children. It would be noticed that, during the later years of the period of observation, the members of the fund had married women more of their own age. That was rather a striking point which required explanation, because it did not occur to him that men of the present day were inclined to take wives nearer their own age than they were years ago. The effect of the change came out very strikingly in Table VI, where the difference in the value of an annuity to the widow on the death of the husband was roughly an average of one year's purchase less up to nearly fifty years of age. Both Mr. Thomas and Mr. King had referred to the reduction in the number of children in a family. Whether any, or what, checks were put upon the production of families now-a-days, compared with what existed previously, he could not say. Mr. King had certainly given a reasonable explanation why it was possible that, in the early years, the members of the particular church referred to should have made a selection, *i.e.*, that those with children came in, and those without children remained out of the fund. Later on, when membership was compulsory, the average for the whole of the members of the church was obtained. That, he thought, was an extremely reasonable explanation, but, even taking that into account, there seemed to be a reduction in the number. He thought it was very desirable that these changes should be watched very carefully, and that it would be most instructive and interesting if other members who had widows' and orphans' funds to value would give their experience in similar form.

Mr. A. W. Watson desired to associate himself with the previous speakers in thanking the author for his most interesting paper. As Mr. King truly said, although the experience was not a large one, it would prove exceedingly useful when they desired to corroborate or correct a particular case with which they happened to be dealing at the moment. He was not at all satisfied that the reason of the apparent diminution in the number of children per family was that suggested by Mr. King; he was rather inclined to think there was something in the author's view that it was the tendency of the age. His opinion was based upon the fact that some ten years ago he had to prepare a set of valuation tables for widows' and orphans' funds, based upon the experience of a large number of funds during the previous ten years; and in using those tables at the present time he found he had to adjust the results in almost every case, because the liability brought out was far greater than the present experience showed to be necessary. The number of children per deceased member in the funds to which he referred, which belonged generally speaking to the industrial class in all parts of the country, had very distinctly fallen; so much so, that when the fall in the number of children was combined with the fall in the death rate of the members themselves, the liability in respect of children's benefits had become almost non-existent. When he said non-existent, he referred, of course, to lump sum benefits, not to children's annuities, but the value of the lump sum payments had certainly become very insignificant. He found it was now almost

sufficient to average them at about two children per member for the members dying under 50 years of age, without going into very great elaboration as to the incidence at each age of deceased members. He could have wished, in regard to the financial aspect of the paper, that the author had gone a little further, and investigated the decline in the liability when to the change in the children's experience was added the fall in the death rate amongst the members themselves. The first element in determining the liability for any benefit payable on the death of a member was the rate of mortality prevalent amongst the members themselves; and when it was remarked how greatly that rate had decreased during the past twenty years or longer, as evidenced by a comparison of the H^M and O^M Life Tables, or the results of the two Manchester Unity investigations, it would be seen that the declining size of the family, combined with the decline in the death rate amongst the members, might be expected materially to affect the resulting liability. He would like to suggest to the author that it would be very interesting to see how the liability was affected by combining the annuities with the reversionary values of a life table applicable to the circumstances of 1881, and one applicable to the circumstances of the present day. The variation of those values ought to be scrutinized very closely, because, if a relatively true measure of the present liabilities was not obtained, the results in the valuation of a widows' and orphans' fund were apt to be very far from the truth.

Mr. H. W. Andras had hoped, from the title of the paper, that the author would have given a comparison with the results of other investigations into the experience of widows' funds, and especially with those of several Scottish actuaries. Mr. Thomas had described very clearly the collective method and the individual method, and he thought it would be instructive if some one would compare the effect of applying the two methods to the same statistics of marriage and mortality. He had recently had experience of two widows' funds, too small to form the basis of a paper, or for a contribution to valuable statistics, but he thought lessons might be learned from the circumstances of even small societies. In one of them, the special risks of future marriages to bachelors and widowers were practically eliminated by the rules as to contributions and benefits. The bachelors were merely required to subscribe a small sum per annum, which, accumulated at compound interest to the date of marriage—if they married—was applied in reduction of the contribution for a reversionary annuity to the wife after the death of her husband. Similarly, when a married man became a widower, if he elected to continue his membership, he paid a small subscription which, accumulated at compound interest, reduced his contribution for a reversionary annuity at his attained age should he re-marry. In that way complicated benefits were eliminated. He thought it was very useful that orphan benefits had, by the ingenuity of Mr. Manly and other actuaries, been brought within the range of satisfactory actuarial computation. With regard to the proposal to construct general standard tables, he quite agreed with several previous speakers that every society was more or less a law unto itself, and, therefore, it was almost impracticable to carry out the suggestion. At the same time, the publication by workers in the

same field of their methods and results was of inestimable value to the actuarial profession, and would be the means of establishing a series of standards of comparison, by which to test approximately the results of enquiries as to the experience of communities similarly circumstanced.

The resolution of thanks to Mr. Allin was then put, and carried with acclamation.

Mr. Allin, in reply, thanked the members for the kind way in which they had received his paper, and for the small amount of adverse criticism passed on it. While Mr. Thomas's remarks were most interesting, he did not altogether agree with his view that husbands who died must have more children than those who did not. Mr. Thomas also seemed to think that he (the author) was advocating standard tables for general use. He only did so for the purposes of comparison and to fill up gaps in the actual experience of the fund being valued. As one or two members had said, they must go to the funds themselves for the experience. At the same time, something was wanted to check those funds, and he thought a better standard might be obtained than Huie's and Hewat's tables, which were the only ones available at the present time. He believed several private tables were in existence, but so far as he knew they had never been published. With regard to Mr. King's remark on the subject of selection, he did not think the selection was important. He could not obtain full details in regard to the original numbers, but the figures available seemed to show a very large proportion, not only of bachelors but of widowers, and also a very large number of married men who had no children. He thought a great portion of any selection had worn off. The fund was started in 1870, and the first valuation was made in 1881, so that, as the average age at marriage was about 30, it would mean that one would not expect much reduction in the number of children to men under age 40 or 45. There actually was a great deal of difference in the value of the children's annuities below age 45, so that he thought there must have been something beyond any little selection to cause that; and Mr. Manly and Mr. Watson seemed to agree in this view. In reply to Mr. Watson's observations with regard to using the experience of the fund, he could not help thinking that something was wanted to check the figures a little more than the experience of the fund itself. He did not go into the subject of the valuation of the liabilities, because the experience of the fund was small, even though the figures brought out seemed to be approximately correct.

ADDENDUM.

[Mr. G. J. Lidstone, who was prevented by the lateness of the hour from joining in the discussion, sends us the following remarks for publication.—Ed. *J.I.A.*]

The central idea of Mr. Allin's paper is (as it was happily expressed by Mr. Andras) that each fund is a law to itself, and further, that the law may change from time to time, so that it is essential to base the valuation—either explicitly or by adjustment—

on data derived from the recent experience of the fund. While these principles are recognized as fundamental by all actuaries of experience in this branch of work, they cannot be too strongly emphasized or illustrated, for experience shows that even when there is much in common between the occupations and social positions of the members of two different funds, it is unsafe to assume that the same basis of valuation will apply to both. This is well illustrated by the following Table showing for quinquennial ages the adjusted ratios of married officials to total officials at various ages in the staff of a large English Joint Stock Bank, as compared with the similar ratios derived from Mr. Hewat's Tables based on the Experience of the Scottish Banks:

Ratios of Married* to Total Officials

Age	Hewat's Tables	English Bank Officials	Excess of English Bank Officials		Age
			Actual	Proportion of (2)	
(1)	(2)	(3)	(4)	(5)	(6)
20	...	...	...	...	20
25	·075	·087	·012	·16	25
30	·291	·332	·041	·14	30
35	·507	·574	·067	·13	35
40	·638	·718	·080	·13	40
45	·712	·798	·086	·12	45
50	·755	·842	·087	·11	50
55	·780	·867	·087	·11	55
60	·795	·881	·086	·11	60
65	·805	·888	·083	·10	65

* The "Married" here include Widowers.

The Table shows that the proportion of married in the English Bank is greater by from 10 to 16 per-cent than that shown by the Scottish Bankers' Experience, and hence, if that experience were adopted in the valuation, the value of the Widows' Annuities would (*cæteris paribus*) be seriously under-estimated.

With a small membership it is impossible in any moderate period of observation to obtain sufficient data from which to calculate the proportionate number of bachelors, husbands and widowers *dying* at each age, and it is therefore not only convenient but necessary to base these ratios on the numbers *living*. For example, in a fund of 1,000 members, the annual deaths would probably not exceed 15 or 20 per annum, so that at least 50 years' experience of the deaths would be required to secure observations equal in extent to those derivable from a single count of the total existing membership, and at the younger and middle ages the period would be even longer.

In dealing with the living instead of the dying, it is, of course, tacitly assumed that the rate of mortality amongst bachelors and married is identical, though this is not necessarily the case, as Mr. Thomas pointed out in the discussion. In practice, however, it

would not appear probable that the differences in the rates of mortality would be sufficient to disturb seriously the resulting ratios of bachelors, married and widowers. It is usually found that the married men show, if anything, a *lighter* rate of mortality than the bachelors, and when this is the case the proportion of married derived from the living will be *greater* than that which would be derived from the dying, so that the use of the former will *pro tanto* supply a very desirable margin on the side of safety.

It may be of interest to show that the series of values of the ratio Bachelors : Total members, say, k_x^B, implicitly involves the marriage rates, and that these (provided they have been constant over a series of years) can therefore be determined from k_x^B. Let l_x equal the number of bachelors at age x, and λ_x the number of husbands and widowers at age x. Let μ_x be the force of mortality for the total body of members $l_x+\lambda_x$; and let $\mu_x+\theta_x$ be the force of mortality for the bachelors taken alone; further, let ν_x be the force of marriage (first marriages only). Then l_x is subject to the two decremental forces $(\mu_x+\theta_x)$ and ν_x, but $l_x+\lambda_x$ is subject to the decremental force μ_x only. Thus we have—

$$k_x^B=\frac{l_x}{l_x+\lambda_x}$$

$$\log k_x^B=\log l_x-\log (l_x+\lambda_x)$$

$$-\frac{d}{dx}\log k_x^B=-\frac{1}{k_x^B}\frac{d}{dx}k_x^B=(\mu_x+\theta_x+\nu_x)-\mu_x$$

$$=\nu_x+\theta_x.$$

Putting $x+\frac{1}{2}$ for x, we have

$$-\frac{1}{k_{x+\frac{1}{2}}^B}\frac{d}{dx}k_{x+\frac{1}{2}}^B=\nu_{x+\frac{1}{2}}+\theta_{x+\frac{1}{2}}=\frac{k_x^B-k_{x+1}^B}{k_{x+\frac{1}{2}}^B}\text{ nearly.}$$

If $\theta_x=0$, *i.e.*, if the rate of mortality be the same for bachelors and married, the expression $\frac{k_x^B-k_{x+1}^B}{k_{x+\frac{1}{2}}^B}$ gives the central marriage rate, that is, approximately, the force of marriage in the middle of the year x to $x+1$; while if the bachelors show the higher mortality, θ_x is positive, and the above expression gives a quantity slightly *higher* than the true force of marriage.

If l_x and λ_x are subject to diminution by secessions as well as deaths, θ_x must be taken to represent the difference between the forces of *mortality and secession combined*, for bachelors and married respectively.

When the values of k_x^B have been obtained and graduated, it is generally desirable to deduce the values of ν and set them out graphically. If they show a marked departure from the well-defined general type, this suggests either a faulty graduation of the values of k, or some artificial disturbance of those values, calling for further investigation and correction. Such a disturbance is specially likely to arise in a recently-established fund in which membership is optional at the commencement, though compulsory upon later entrants, as, for example, with a bank or commercial institution,

which—after the fund has once been started—requires all officials subsequently appointed to join the fund as a condition of service. It will generally be found that the members joining voluntarily at the commencement consist largely of married men, or of those who have more than the average prospect of early marriage, and hence for many years the proportion of married men will be greater than normal, except at ages under (say) $20+n$, when n is the number of years from the foundation of the fund. In these circumstances, values of k deduced from the members of the fund only would not represent the values which would be likely to obtain when the membership had reached a normal distribution, and values of ν deduced from such abnormal values of k would be entirely misleading, and might even be negative. In such a case it is necessary to base the values of k and ν on the whole staff, members and non-members combined.

Even with a normal set of values of k, it would, of course, be unsafe to regard the deduced values of ν as giving more than a general idea of the rates of marriage; they are nevertheless sufficient (as it is hoped to show at length in the *Journal*) to enable the actuary to value separately the benefits to bachelors and married men, by formulæ which are strictly consistent with those of the collective method when the distribution is normal, but which avoid the inaccuracy which that method introduces when the distribution is abnormal.

In the foregoing investigation, the "married" have been taken to include widowers, as in Mr. Hewat's Tables, but for practical work it is often desirable to subdivide the "married" into husbands and widowers.

[Mr. Allin has sent us the following additional remarks for publication.—Ed. *J.I.A.*]

Since making my remarks in reply to the discussion on this paper, I have had an opportunity of perusing Mr. Marr's notes. The question of selection I have already dealt with. As regards the proportions of bachelors, married men, and widowers, it is quite true that these will vary from time to time, especially in a small fund, and it is necessary to see, when constructing tables by the collective method, that any temporary changes in the constituents of the fund are allowed for. This is one of the objections to the collective method, but is an objection which is not insuperable, and the advantages of the method seem to outweigh its disadvantages. Mr. Marr's table of percentages based upon the *total members of all ages* seems to be somewhat misleading. What is really required is a table such as the following, showing *for each separate age, or each quinary group of ages*, the percentage of married men to total members, for it is these percentages only that enter into a valuation by the collective method. The appended table, arranged from the tables given by Mr. Marr, does not seem to throw much light on the subject, and the fact that there are a larger number of husbands among the existing is quite possibly due to chronological changes.

Dying				Existing			
Ages	Total Number	No. of Husbands	Per-cent	Ages	Total Number	No. of Husbands	Per-cent
25–29	8	1	12	25–29	21	5	24
30–34	29	15	52	30–34	126	62	49
35–39	43	24	56	35–39	163	118	72
40–44	46	34	74	40–44	199	151	76
45–49	50	33	66	45–49	164	137	84
50–54	64	49	77	50–54	119	100	84
55–59	76	59	78	55–59	158	125	79
60–64	101	73	72	60–64	131	108	82
65–69	120	92	77	65–69	66	53	80
70–74	112	64	57	70–74	69	55	80
75–79	94	52	55	75–79	38	27	71
80–84	51	24	47	80–84	27	12	44
85–	28	9	32	85–	12	6	50

Mr. Marr also makes the statement that "the relative ages of wives will not, as far as I can see, be the same as the relative ages of widows at the moment of entry into widowhood." Here I am afraid I must join issue with him. If the ages are not very closely the same, it is due to some chronological change. The table Mr. Marr gives, to my mind, proves nothing, as he has compared the average age of the wife of a man who *marries* at age x with the average age of the widows of men *dying* at age x, and of course a considerable difference is shown, especially at the older ages. In using the collective method, we are not concerned with the age at marriage. All we have to consider is the condition and status of members and their wives at a fixed date, and if a comparison were made between the average age of the wives of all men now living at any particular age (*whenever* married) and the average age of the widows of men dying at that particular age, I do not think we should find much difference, unless there had been social changes.

With regard to the exclusion of children over age 18, I think that, if Mr. Marr will reconsider his statement, he will see there is no exaggeration in the decrease of children, as I have compared like facts at different periods. Children over 18 years of age were excluded from both experiences.

[Mr. Marr, having had an opportunity of perusing Mr. Allin's remarks, asks us to publish the following supplementary notes.—Ed. *J.I.A.*]

As Mr. Allin points out, when my table is used to ascertain whether the relative number of married men alive at any age may be taken as an indication of the number who die leaving widows at that age, it is necessary to compare the number of husbands at any age with the total number of members at that age. I am obliged to him for giving a table of the percentages. These percentages can, of course, be obtained roughly by dividing the figures in column 5 of my table by those in column 9. There is a considerable difference at

some ages between the percentages deduced from the statistics relating to "existing" and those relating to "died", and valuation factors based on one of the tables would differ considerably from valuation factors based on the other.

Since perusing Mr. Allin's remarks, I have looked further into the question of the relative average ages of husbands and wives, and the average ages of widows at the moment of entry into widowhood, as shown by the statistics of the Scottish Ministers' Experience. I find there is an average difference of 5·6 years between the ages of the 959 existing married ministers and their wives, while the 529 widows were, on the average, 8·5 years younger than their husbands were when they died.

The following figures show this difference analyzed:

EXISTING.

Age of Husbands	No. of Cases	Wives' ages less on average by
27–39	186	1·8 years
40–49	296	4·4 ,,
50–59	221	6·4 ,,
60–69	157	8·5 ,,
70–79	81	10·7 ,,
80–	18	10·3 ,,

DIED.

Husbands dying between ages	No. of Cases	Widows' ages at date of Husbands' deaths less on average by
27–39	40	2·2 years
40–49	67	5·1 ,,
50–59	108	7·3 ,,
60–69	165	8·5 ,,
70–79	116	11·1 ,,
80–	33	17·6 ,,

The fund in question had been, at the date of the investigation, fifty years in existence. While, possibly, in the interval changes in the social condition have taken place, no very great selection could have been exercised against the fund. Membership has always been compulsory and is confined to ministers of the Church. Even at the inception of the fund all the ministers then existing had to join. Very complete records have always been kept, and the statistics can be considered reliable.

In taking exception to Mr. Allin's remarks about the decrease in the size of families, I had in view that the statistics for the year 1881 were possibly influenced by a selection which might have been exercised, when the fund was started, by membership not being compulsory.

ACTUARIAL NOTE.

On the Calculation of Contingent Assurance Premiums, when Makeham's law holds. By A. S. Hume, *and* W. Stott, *of the Royal Insurance Company, Liverpool.*

On looking into Mr. Colenso's paper on "Contingent Assurances" (*J.I.A.*, xxxi, p. 337), the following modifications and accompanying tables appeared to us to be useful for practical purposes, and we venture to submit them, in the hope that they will be found of sufficient interest to the profession.

In preparing a table based on the H^M Experience, we had in view the importance of having a complete set of tables based upon a single mortality table, and suitable for the student of the *Text-Book*. We do not desire it to be inferred that the H^M Table should be used in practically computing premium rates where survivorship contingencies are involved.

We, however, give a similar table (with the addition of a column of μ_x) based on the Carlisle Table, graduated according to Makeham's formula, as this is more likely to meet with acceptance in actual practice. The tables of 3 per-cent annuity-values given by Mr. Colenso, $\bar{a}_x$, $\bar{a}_{xx}$, $\bar{a}_{xxx}$, and $\bar{a}_{xxxx}$, have been here collected in a convenient form for the determination of annual premiums.

Problem. To find the single premium for an assurance of 1 payable at the death of x, provided he dies first of n lives $x, y, z \ldots$

From the general formula (41), page 233 *Text-Book* II, we have

$$\bar{A}^1_{xyz\ldots} = \frac{c^x}{c^x+c^y+c^z+\ldots}\bar{A}_{xyz\ldots} - \frac{c^y+c^z+\ldots-(n-1)c^x}{c^x+c^y+c^z+\ldots}\lambda s.\bar{a}_{xyz\ldots}$$

where λs represents the Napierian logarithm of Makeham's constant s; or, writing

nc^r for $c^x+c^y+c^z+\ldots$, and $\bar{a}_{rrr\ldots}$ for $\bar{a}_{xyz\ldots}$,

$$\bar{A}^1_{xyz\ldots} = \frac{c^x}{nc^r}\bar{A}_{rrr\ldots} - \left(1-\frac{nc^x}{nc^r}\right)\lambda s\bar{a}_{rrr\ldots}$$

$$= \frac{c^x}{nc^r}(\bar{A}_{rrr\ldots} + n\lambda s\bar{a}_{rrr\ldots}) - \lambda s\bar{a}_{rrr\ldots}$$

$$\therefore \bar{A}^1_{xyz\ldots} = c^x\frac{\frac{1}{n}\bar{A}_{rrr\ldots}+\lambda s\bar{a}_{rrr\ldots}}{c^r} - \lambda s\bar{a}_{rrr\ldots}$$

As particular cases we have—

$$\bar{A}^1_{xy} = c^x\frac{\frac{1}{2}\bar{A}_{rr}+\lambda s\bar{a}_{rr}}{c^r} - \lambda s\bar{a}_{rr}$$

$$\bar{A}^1_{xyz} = c^x\frac{\frac{1}{3}\bar{A}_{rrr}+\lambda s\bar{a}_{rrr}}{c^r} - \lambda s\bar{a}_{rrr}$$

$$\bar{A}^1_{xyzw} = c^x\frac{\frac{1}{4}\bar{A}_{rrrr}+\lambda s\bar{a}_{rrrr}}{c^r} - \lambda s\bar{a}_{rrrr}$$

or, in more concise notation,

$$\bar{A}^1_{xy} = c^x \times f_1(rr) + f_2(rr)$$
$$\bar{A}^1_{xyz} = c^x \times f_1(rrr) + f_2(rrr)$$
$$\bar{A}^1_{xyzw} = c^x \times f_1(rrrr) + f_2(rrrr).$$

By tabulating $\log c^x$ for all required values of x, and $\log f_1(\quad)$, and $f_2(\quad)$, for two, three and four lives, for all necessary values of r, we have a simple and direct means of making the calculation, the operation, excepting the interpolation, being (as $-\lambda s$ is a positive quantity) entirely positive. With the *Text-Book* at hand, which enables us readily to determine values of r, a single page will contain all the tabular matter necessary for the three particular cases considered.

EXAMPLE I. *To find* $\bar{A}^1_{32:40}$. H^M (*Text-Book*) 3 *per-cent.*

By uniform seniority table, or μ column, $r = 36{\cdot}72$.

$$\begin{array}{rl@{\qquad}rl}
\log c^{32} = & 1{\cdot}26902 & \log c^{32} = & 1{\cdot}26902 \\
\log f_1(36, 36) = & \bar{3}{\cdot}82223 & \log f_1(37, 37) = & \bar{3}{\cdot}79667 \\
& \bar{1}{\cdot}09125 & & \bar{1}{\cdot}06569 \\
\log^{-1} = & {\cdot}12338 & \log^{-1} = & {\cdot}11633 \\
f_2(36, 36) = & {\cdot}09514 & f_2(37, 37) = & {\cdot}09341 \\
& {\cdot}21852 & & {\cdot}20974 \\
& {\cdot}20974 & & \\
& {\cdot}00878 \times {\cdot}72 & & \\
= & {\cdot}00632 & & \\
\therefore \bar{A}^1_{32:40} = & {\cdot}21220 & &
\end{array}$$

EXAMPLE II. *To find* $\bar{A}^1_{30:35:45}$. H^M (*Text-Book*) 3 *per-cent.*

By uniform seniority table, or μ column, $r = 38{\cdot}49$.

$$\begin{array}{rl@{\qquad}rl}
\log c^{30} = & 1{\cdot}18971 & \log c^{30} = & 1{\cdot}18971 \\
\log f_1(38, 38, 38) = & \bar{3}{\cdot}60755 & \log f_1(39, 39, 39) = & \bar{3}{\cdot}58236 \\
& \bar{2}{\cdot}79726 & & \bar{2}{\cdot}77207 \\
\log^{-1} = & {\cdot}06270 & \log^{-1} = & {\cdot}05917 \\
f_2(38, 38, 38) = & {\cdot}07841 & f_2(39, 39, 39) = & {\cdot}07670 \\
& {\cdot}14111 & & {\cdot}13587 \\
& {\cdot}13587 & & \\
& {\cdot}00524 \times {\cdot}49 & & \\
= & {\cdot}00257 & & \\
\therefore \bar{A}^1_{30:35:45} = & {\cdot}13854 & &
\end{array}$$

[Editorial Note.—In his well-known paper on this subject, Mr. Colenso suggests (*J.I.A.*, vol. xxxi, p. 341) a very similar method of dealing with the problem here under notice, based on the formula—

$$\bar{\mathrm{A}}^{1}_{xyz\ldots}=\frac{1}{n}c^{x-r}-\left\{c^{x-r}\left(\frac{\delta}{n}-\lambda s\right)+\lambda s\right\}\bar{a}_{xyz\ldots}$$

Writing this in slightly modified form, we have—

$$\bar{\mathrm{A}}^{1}_{xyz\ldots}=c^{x}\left[\frac{\dfrac{1-\delta\bar{a}_{xyz\ldots}}{n}+\lambda s\,.\,\bar{a}_{xyz\ldots}}{c^{r}}\right]-\lambda s\,.\,\bar{a}_{xyz\ldots}$$

The identity of this expression with that forming the basis of Messrs. Hume and Stott's Tables is at once apparent.

It will be remembered that the tables appended to Mr. Colenso's paper (*J.I.A.*, xxxi, pp. 354–6), and also those subsequently published by that gentleman (xxxii, pp. 187–193), were based upon methods different from the above, involving the calculation of annuity-values on joint lives of equal ages at a special rate of interest.

It should perhaps be noted that the present writers follow Mr. Colenso in adopting, for the Carlisle Table, the graduation by Messrs. G. King and G. F. Hardy's "first curve", deduced by their "aggregate" method, from the data in four groups of twenty ages each, between the limits of age 15 and 94, with a single set of constants throughout (see *J.I.A.*, vol. xxii, p. 201). The value of $\log_{10}c$ adopted for this first curve was ·0396671. As the value adopted for the H^{M} (*Text-Book*) Table was ·03965686, which differs by slightly more than ·00001 from the above, the tables of uniform seniority given in the *Text-Book*, Part II (second edition), can (excepting in extreme cases of difference between the individual ages) be used without appreciable error in determining the equivalent age, where it is desired to employ the Carlisle Tables, as here given, for the determination of the premium.]

Factors for Contingent Assurances.

H^M (*Text-Book Graduation*) **3** *per-cent* ($\log_{10} c =$ ·03965686).

x	$\log c^x$	r	$\log f_1(rr)$	$f_2(rr)$	$\log f_1(rrr)$	$f_2(rrr)$	$\log f_1(rrrr)$	$f_2(rrrr)$
20	·79314	20	2·20298	·11837	2·01270	·10396	$\bar{3}$·86649	·09316
21	·83279	21	·18316	·11700	3·99639	·10260	·85403	·09181
22	·87245	22	·16210	·11566	·97807	·10129	·83868	·09054
23	·91211	23	·13978	·11436	·95812	·10002	·82093	·08932
24	·95176	24	·11687	·11305	·93715	·09876	·80167	·08814
25	·99142	25	·09341	·11171	·91529	·09751	·78141	·08696
26	1·03108	26	·06957	·11041	·89299	·09624	·76047	·08577
27	1·07074	27	·04550	·10905	·87032	·09495	·73902	·08456
28	1·11039	28	·02132	·10765	·84746	·09362	·71743	·08331
29	1·15005	29	$\bar{3}$·99691	·10622	·82423	·09227	·69536	·08204
30	1·18971	30	·97238	·10475	·80094	·09087	·67308	·08074
31	1·22936	31	·94777	·10324	·77749	·08944	·65077	·07939
32	1·26902	32	·92293	·10170	·75378	·08797	·62806	·07801
33	1·30868	33	·89801	·10011	·72998	·08646	·60519	·07659
34	1·34833	34	·87290	·09849	·70594	·08491	·58206	·07514
35	1·38799	35	·84766	·09683	·68161	·08334	·55866	·07366
36	1·42765	36	·82223	·09514	·65711	·08173	·53496	·07215
37	1·46730	37	·79667	·09341	·63238	·08009	·51111	·07060
38	1·50696	38	·77092	·09165	·60755	·07841	·48696	·06902
39	1·54662	39	·74504	·08981	·58236	·07670	·46262	·06741
40	1·58627	40	·71889	·08802	·55696	·07497	·43781	·06579
41	1·62593	41	·69261	·08615	·53135	·07321	·41285	·06413
42	1·66559	42	·66616	·08425	·50555	·07141	·38764	·06244
43	1·70524	43	·63947	·08233	·47937	·06961	·36202	·06075
44	1·74490	44	·61260	·08038	·45301	·06777	·33612	·05904
45	1·78456	45	·58547	·07841	·42637	·06592	·30991	·05731
46	1·82422	46	·55821	·07641	·39956	·06403	·28346	·05556
47	1·86387	47	·53070	·07439	·37239	·06215	·25664	·05381
48	1·90353	48	·50296	·07235	·34491	·06026	·22950	·05205
49	1·94319	49	·47496	·07030	·31717	·05836	·20207	·05028
50	1·98284	50	·44673	·06824	·28919	·05644	·17431	·04851
51	2·02250	51	·41828	·06616	·26094	·05452	·14620	·04675
52	2·06216	52	·38958	·06408	·23228	·05262	·11774	·04499
53	2·10181	53	·36068	·06199	·20341	·05071	·08903	·04323
54	2·14147	54	·33146	·05991	·17421	·04881	·05987	·04150
55	2·18113	55	·30199	·05782	·14471	·04692	·03034	·03979
56	2·22078	56	·27225	·05575	·11488	·04505	·00056	·03808
57	2·26044	57	·24226	·05368	·08478	·04319	$\bar{4}$·97042	·03640
58	2·30010	58	·21202	·05162	·05435	·04135	·93992	·03475
59	2·33975	59	·18149	·04958	·02362	·03954	·90908	·03312
60	2·37941	60	·15069	·04756	$\bar{4}$·99258	·03776	·87788	·03153
61	2·41907	61	·11962	·04556	·96120	·03601	·84638	·02997
62	2·45873	62	·08827	·04359	·92953	·03429	·81455	·02844
63	2·49838	63	·05664	·04165	·89758	·03260	·78236	·02696
64	2·53804	64	·02470	·03975	·86527	·03096	·74987	·02551
65	2·57770	65	$\bar{4}$·99254	·03787	·83267	·02936	·71703	·02411
66	2·61735	66	·96005	·03605	·79977	·02780	·68385	·02276
67	2·65701	67	·92733	·03425	·76660	·02629	·65038	·02145
68	2·69667	68	·89428	·03251	·73308	·02483	·61661	·02019
69	2·73632	69	·86100	·03081	·69933	·02341	·58256	·01897
70	2·77598	70	·82746	·02915	·66528	·02204	·54822	·01780
71	2·81564	71	·79360	·02756	·63091	·02073	·51353	·01669
72	2·85529	72	·75952	·02601	·59633	·01947	·47860	·01563
73	2·89495	73	·72515	·02452	·56145	·01826	·44343	·01461
74	2·93461	74	·69056	·02307	·52632	·01710	·40797	·01364
75	2·97426	75	·65571	·02169	·49093	·01600	·37229	·01272
76	3·01392	76	·62061	·02036	·45531	·01495	·33635	·01185
77	3·05358	77	·58526	·01909	·41944	·01395	·30016	·01103
78	3·09324	78	·54967	·01788	·38336	·01300	·26378	·01025
79	3·13289	79	·51389	·01672	·34706	·01211	·22721	·00951
80	3·17255	80	·47786	·01562	·31055	·01126	·19040	·00882

Factors for Contingent Assurances.
Carlisle Table (Makeham Graduation) 3 *per-cent* ($\log_{10}c =$ ·0396671).

x	μ_x	$\log c^x$	r	Two Lives $\log f_1(rr)$	Two Lives $f_2(rr)$	Three Lives $\log f_1(rrr)$	Three Lives $f_2(rrr)$	Four Lives $\log f_1(rrrr)$	Four Lives $f_2(rrrr)$	r
20	·00871	·79334	20	$\bar{2}$·11718	·14992	$\bar{3}$·91055	·12883	$\bar{3}$·75333	·11328	20
21	·00879	·83301	21	·09518	·14871	·89077	·12775	·73554	·11232	21
22	·00884	·87268	22	·07318	·14745	·87083	·12663	·71749	·11132	22
23	·00890	·91234	23	·05107	·14615	·85086	·12546	·69928	·11028	23
24	·00896	·95201	24	·02881	·14480	·83068	·12425	·68075	·10920	24
25	·00903	·99168	25	·00610	·14341	·81034	·12299	·66211	·10808	25
26	·00910	1·03134	26	$\bar{3}$·98402	·14196	·78978	·12169	·64334	·10691	26
27	·00919	1·07101	27	·96137	·14048	·76916	·12034	·62447	·10569	27
28	·00928	1·11068	28	·93869	·13894	·74828	·11894	·60524	·10443	28
29	·00938	1·15035	29	·91583	·13736	·72726	·11750	·58565	·10313	29
30	·00948	1·19001	30	·89288	·13572	·70602	·11601	·56609	·10177	30
31	·00960	1·22968	31	·86983	·13403	·68461	·11447	·54615	·10037	31
32	·00973	1·26935	32	·84653	·13230	·66307	·11288	·52604	·09892	32
33	·00987	1·30901	33	·82319	·13051	·64126	·11124	·50564	·09743	33
34	·01003	1·34868	34	·79958	·12868	·61929	·10955	·48498	·09589	34
35	·01020	1·38835	35	·77588	·12679	·59712	·10781	·46413	·09430	35
36	·01039	1·42802	36	·75205	·12485	·57462	·10603	·44297	·09266	36
37	·01059	1·46768	37	·72800	·12287	·55198	·10420	·42157	·09098	37
38	·01082	1·50735	38	·70380	·12083	·52912	·10232	·39979	·08926	38
39	·01106	1·54702	39	·67943	·11874	·50594	·10040	·37783	·08749	39
40	·01133	1·58668	40	·65487	·11661	·48262	·09843	·35553	·08568	40
41	·01162	1·62635	41	·63008	·11443	·45901	·09641	·33293	·08383	41
42	·01195	1·66602	42	·60512	·11220	·43513	·09436	·31001	·08194	42
43	·01230	1·70569	43	·57991	·10994	·41102	·09226	·28682	·08001	43
44	·01269	1·74535	44	·55453	·10763	·38662	·09013	·26329	·07805	44
45	·01311	1·78502	45	·52897	·10527	·36194	·08796	·23938	·07606	45
46	·01358	1·82468	46	·50314	·10288	·33698	·08576	·21516	·07404	46
47	·01409	1·86435	47	·47710	·10045	·31174	·08353	·19065	·07199	47
48	·01465	1·90402	48	·45084	·09799	·28617	·08127	·16572	·06992	48
49	·01526	1·94369	49	·42435	·09549	·26032	·07899	·14053	·06782	49
50	·01593	1·98335	50	·39756	·09298	·23421	·07668	·11488	·06572	50
51	·01667	2·02302	51	·37056	·09043	·20776	·07436	·08891	·06360	51
52	·01747	2·06269	52	·34332	·08785	·18099	·07202	·06263	·06147	52
53	·01836	2·10236	53	·31580	·08527	·15388	·06968	·03594	·05933	53
54	·01932	2·14202	54	·28801	·08267	·12644	·06733	·00886	·05720	54
55	·02038	2·18169	55	·25996	·08005	·09873	·06497	$\bar{4}$·98141	·05507	55
56	·02154	2·22136	56	·23164	·07743	·07065	·06262	·95357	·05295	56
57	·02281	2·26102	57	·20300	·07482	·04224	·06028	·92537	·05084	57
58	·02421	2·30069	58	·17414	·07219	·01349	·05795	·89684	·04874	58
59	·02573	2·34036	59	·14495	·06958	$\bar{4}$·98440	·05563	·86784	·04667	59
60	·02741	2·38003	60	·11547	·06698	·95497	·05333	·83852	·04462	60
61	·02924	2·41969	61	·08572	·06440	·92521	·05106	·80881	·04259	61
62	·03125	2·45936	62	·05566	·06182	·89509	·04882	·77871	·04060	62
63	·03345	2·49913	63	·02530	·05929	·86462	·04661	·74823	·03865	63
64	·03586	2·53869	64	$\bar{4}$·99468	·05677	·83384	·04444	·71740	·03674	64
65	·03850	2·57836	65	·96372	·05429	·80270	·04231	·68617	·03487	65
66	·04139	2·61803	66	·93247	·05185	·77121	·04022	·65458	·03305	66
67	·04456	2·65770	67	·90090	·04945	·73939	·03819	·62263	·03128	67
68	·04804	2·69736	68	·86905	·04710	·70727	·03620	·59033	·02956	68
69	·05184	2·73703	69	·83690	·04480	·67477	·03427	·55766	·02790	69
70	·05601	2·77670	70	·80444	·04255	·64200	·03239	·52470	·02628	70
71	·06058	2·81636	71	·77170	·04036	·60888	·03058	·49137	·02473	71
72	·06558	2·85603	72	·73867	·03822	·57546	·02882	·45771	·02324	72
73	·07107	2·89570	73	·70534	·03615	·54172	·02713	·42370	·02181	73
74	·07707	2·93537	74	·67171	·03415	·50766	·02551	·38941	·02044	74
75	·08366	2·97503	75	·63782	·03221	·47334	·02394	·35484	·01913	75
76	·09087	3·01470	76	·60365	·03034	·43871	·02245	·31995	·01788	76
77	·09877	3·05437	77	·56920	·02854	·40381	·02102	·28478	·01669	77
78	·10743	3·09403	78	·53450	·02681	·36867	·01965	·24938	·01556	78
79	·11692	3·13370	79	·49955	·02515	·33324	·01835	·21369	·01449	79
80	·12731	3·17737	80	·46434	·02356	·29757	·01712	·17773	·01348	80

Annuity-Values.
Carlisle Table (Makeham Graduation)—3 *per-cent.*

x	$\bar{a}_x$	$\bar{a}_{xx}$	$\bar{a}_{xxx}$	$\bar{a}_{xxxx}$
20	22·023	18·181	15·623	13·737
21	21·864	18·034	15·492	13·621
22	21·700	17·881	15·356	13·500
23	21·530	17·723	15·214	13·374
24	21·356	17·560	15·067	13·243
25	21·175	17·391	14·915	13·107
26	20·990	17·216	14·757	12·965
27	20·798	17·036	14·593	12·817
28	20·601	16·849	14·424	12·664
29	20·399	16·657	14·249	12·506
30	20·190	16·459	14·068	12·342
31	19·975	16·254	13·881	12·172
32	19·754	16·044	13·688	11·996
33	19·527	15·827	13·490	11·815
34	19·294	15·605	13·285	11·628
35	19·055	15·376	13·074	11·435
36	18·809	15·141	12·858	11·237
37	18·558	14·900	12·636	11·033
38	18·300	14·653	12·408	10·824
39	18·035	14·400	12·175	10·610
40	17·765	14·141	11·936	10·390
41	17·488	13·877	11·692	10·166
42	17·205	13·607	11·443	9·937
43	16·917	13·332	11·189	9·703
44	16·622	13·052	10·930	9·465
45	16·321	12·766	10·667	9·224
46	16·015	12·476	10·400	8·979
47	15·703	12·182	10·129	8·730
48	15·386	11·883	9·856	8·479
49	15·063	11·580	9·579	8·225
50	14·736	11·275	9·299	7·970
51	14·404	10·966	9·017	7·713
52	14·068	10·654	8·734	7·454
53	13·728	10·340	8·450	7·195
54	13·384	10·025	8·165	6·937
55	13·036	9·708	7·879	6·678
56	12·686	9·390	7·594	6·421
57	12·333	9·073	7·310	6·165
58	11·978	8·755	7·027	5·910
59	11·621	8·438	6·746	5·659
60	11·263	8·123	6·468	5·410
61	10·904	7·809	6·192	5·165
62	10·546	7·498	5·920	4·924
63	10·187	7·190	5·653	4·687
64	9·829	6·885	5·389	4·455
65	9·473	6·584	5·131	4·229
66	9·119	6·288	4·878	4·008
67	8·768	5·997	4·631	3·793
68	8·419	5·712	4·390	3·585
69	8·074	5·433	4·156	3·383
70	7·734	5·160	3·928	3·187
71	7·398	4·894	3·708	2·999
72	7·068	4·635	3·495	2·818
73	6·743	4·384	3·290	2·645
74	6·425	4·141	3·093	2·479
75	6·113	3·906	2·904	2·320
76	5·809	3·679	2·722	2·168
77	5·513	3·461	2·549	2·024
78	5·224	3·251	2·383	1·887
79	4·944	3·050	2·226	1·757
80	4·673	2·857	2·076	1·635

REVIEWS.

*On fitting curves to a series of measurements or observations.**

If the form of an expression having n unknown constants be given, and it has been ascertained that in t cases the expression assumed certain values, we can obtain the n constants exactly when $n=t$ provided the equations are capable of solution; but if $t>n$, and the values assumed are only known approximately, the choice of a good method of finding the unknown constants becomes a more complicated matter. The problem is of considerable statistical importance, and the particular case of Makeham's hypothesis has often been attacked by actuaries who have required to find values of a, β and c in the equation $\text{colog}\, p_x = a+\beta c^x$, when approximate values of $\text{colog}\, p_x$ have been found for several values of x. Dr. Cantelli, in the memoir before us, attempts a solution of the general problem; he first remarks that the "method of least squares", though giving an excellent result, is of small practical value, because it often leads to equations which are incapable of solution; and then, on lines almost identical with those followed by Professor Pearson in "Biometrika" (vol. i, part 3), he shows that the method of moments, and a new method which is the real subject of the paper, give results almost as satisfactory as the method of least squares from the point of view of close agreement, and are considerably better as general methods for practical work. Dr. Cantelli's method consists in dividing the statistics into groups, and equating the figures to corresponding areas deduced from the theoretical curve, just as Mr. King and Mr. G. F. Hardy applied Makeham's hypothesis by the summation method which is given in Part II of the *Text-Book*; he uses, however, not the sum, but the integral, which is much simpler on the theoretical side, and for nearly all statistical purposes more accurate. For instance, if it were desired to find values for a_0, a_1, a_2, in the expression $y=a_0+a_1x+a_2x^2$, in order to graduate a series of exposed to risk from age 20 to age 97, Dr. Cantelli would first divide the series of exposed into three groups (because there are three constants, and therefore three equations are required) and obtain $\sum_{x=20}^{x=45} \mathrm{E}_x$, $\sum_{x=46}^{x=71} \mathrm{E}_x$, and $\sum_{x=72}^{x=97} \mathrm{E}_x$ from the ungraduated statistics.

Since the exposed at age x gives all those exposed between ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$, the values of $\sum_{x=n}^{x=m} \mathrm{E}_x$ can be equated to $\int_{n-\frac{1}{2}}^{m+\frac{1}{2}} y\,dx$, and the following equations are formed, from which the constants are obtained—

$$\int_{19\cdot5}^{45\cdot5} (a_0+a_1x+a_2x^2)\,dx = 26a_0+845a_1+28927a_2 = \sum_{x=20}^{x=45} \mathrm{E}_x$$

$$\int_{45\cdot5}^{71\cdot5} (a_0+a_1x+a_2x^2)\,dx = 26a_0+1521a_1+90443a_2 = \sum_{x=46}^{x=71} \mathrm{E}_x$$

$$\int_{71\cdot5}^{97\cdot5} (a_0+a_1x+a_2x^2)\,dx = 26a_0+2197a_1+187111a_2 = \sum_{x=72}^{x=97} \mathrm{E}_x$$

* "Sull' adattamento delle curve ad una serie di misure o di osservazioni." By Dr. Francesco P. Cantelli. Rome, 1905.

Dr. Cantelli's practical examples are interesting, especially as in one case he gets a closer agreement than that obtained by the method of moments, but, as is almost necessarily the case when an alternative solution of a well-known problem is suggested, we feel that some objections to the method require more consideration than Dr. Cantelli has given them in his present memoir. Messrs. King and Hardy pointed out that they made one or two trials before deciding on the best groups for their summation method, and this trial work, to which Dr. Cantelli also refers, is a strong objection to his "method of areas" as it is conveniently termed. Another objection is, that the method assumes that we can integrate the theoretical expression between any required limits, which is not always possible (for example, $e^{-c(x-h)^2}$ between a and b); and in this respect the method compares unfavourably with the method of moments, where integration has only to be performed between the extreme limits assumed by the curve, which is more often possible (for example, $e^{-c(x-h)^2}$ between $-\infty$ and $+\infty$); and a solution might sometimes be obtained, even when integration is impossible, by the use of a reduction formula connecting the successive moments.

We think, therefore, that the method of areas is unlikely to be sufficiently general to replace Professor Pearson's method, but Dr. Cantelli's interesting paper, even if it does nothing else, serves to remind us that it may yet be possible to find new methods of solving statistical problems, as general as those in present use, and that research in theoretical statistics is worthy of the energy which some investigators are bestowing on it.

W. P. E.

---

*The Simplification of Calculations by Mechanical and Graphical Processes.**

The great extent of modern financial institutions, and the growing volume of the calculations required in their administration, necessitate recourse to labour-saving devices, to cope with the mass of work which could not be accomplished by ordinary methods. Logarithms and other mathematical tables, the slide rule and the Arithmometer, have long been familiar instruments for the purpose, and more recently the Comptometer and the Cash Register have been widely adopted in banking and business houses. M. d'Ocagne, in his account of calculating machinery and devices, describes all these, and others not so widely employed.

He divides appliances for calculation into the six following groups:

1. Arithmetical instruments,
2. Arithmetical machines,
3. Logarithmic instruments and machines,
4. Numerical tables,
5. Graphic tracings,
6. Graphic tables (the abacus or nomogram),

* "Le Calcul Simplifié par les Procédés Mecaniques et Graphiques." By Maurice d'Ocagne, Engineer of Roads and Bridges; Professor at the School of Roads and Bridges; Lecturer at the Polytechnic School. Second Edition. Paris, 1905.

traces their development historically, and indicates their mechanism and scope, the text being supplemented by diagrams and illustrations of the machines.

To Blaise Pascal apparently belongs the distinction of having invented the first calculating machine. This he did in 1642 at the age of 18, with the object of simplifying the work of his father, who held a Government post in Normandy. The invention opened a new field for mechanicians, and found numerous imitators, but the labours of many of these are now merely of historical interest. Latterly, Americans have been foremost in the production of practical appliances for calculation. Among other machines, the author describes the Burroughs Comptometer, which adds several columns of figures at once, and the Comptograph of Felt and Tarrant, which not only adds, but also prints the columns and their totals. The Cash Register, invented by Messrs. Patterson, of Dayton, Ohio, is also used in many business houses. In 1820, M. Thomas (of Colmar) invented his Arithmometer, which adds and subtracts, and by continued operations multiplies and divides. M. d'Oeagne describes its ingenious mechanism at some length. The smaller German machine known as the Brunsviga, originally invented by a Russian named Odhner, also adds and subtracts, and multiplies and divides by repeated addition and subtraction. The essential feature of most of these machines is a system of wheels with a variable number of teeth. This feature is adopted in various German machines, among others those invented by Büttner (1888), Esser (1892), and Küttner (1894), of which the Büttner machine resembles Thomas's Arithmometer in appearance.

Other machines are based upon the principles underlying the calculus of finite differences, with the object of effecting summations which permit of passing from the constant value of a fourth (or other) difference to the value of third differences, from these to second and first differences, and finally to the values of an expression of the fourth degree. Such a machine was invented by George Scheutz* with the aid of his son Edward. He conceived the original idea in 1834, but did not finally realize it until 1853, and the machine was exhibited at the French Exhibition of 1855. It was purchased for the Dudley Observatory at Albany in the United States, and has been used for the calculation of tables of logarithms, sines, and logarithmic sines. It prints as well as calculates the tables. Under the head of Analytical and Algebraical Machines, M. d'Ocagne also describes Babbage's invention, the completion of which was interrupted by his death.

Following an explanation of the principles of logarithms and logarithmic calculations, M. d'Ocagne gives a description of various forms of slide rule, including the spiral slide rule invented by Professor George Fuller. This compact instrument is equivalent to a straight rule 83 feet 4 inches long, or a circular rule

* One of these machines was, as is well known, purchased by the British Government, and employed in the General Register Office for the calculation and printing of some of the tabular results deduced from the English Life Table No. 3. (*See* English Life Table, 1864, Introduction, page xiii; also Mr. Peter Gray's reference in *J.I.A.*, vol. xvii, page 252, footnote.)—Ed. *J.I.A.*

13 feet 3 inches in diameter, and gives results approximating within 1 in 10,000. Mention is also made of M. Torres' machine, by the aid of which may be solved equations of the form $x^9+Ax^8=B$, or $x^9+Ax^7=B$, and the appendix contains a full description of Tchebichef's arithmetical machine, which adds, multiplies, subtracts, or divides.

Of special interest, on account of its less general employment in this country, is the instrument invented by M. Leon Bollée, to whom belongs the honour of devising the first machine for performing the operation of multiplication directly, and not by repeated additions. Like Pascal, he did this when eighteen years of age, and the machine was first publicly exhibited at the Exposition Universelle of 1889. To quote M. d'Ocagne, "in 1892 M. Bollée constructed a new model of his machine, in which are realized perfections of the highest interest. Thus, a general clutch system is arranged in such a manner that the machine refuses, not only to perform a calculation which is impossible or false, but also to make any false movement, even against the inclination of the operator. The advantage of this arrangement is not apparent for multiplication, but it is felt in division, and in a remarkable manner in the extraction of a square root, which, in the first model of the Bollée machine, as in preceding machines, demanded sustained attention on the part of the operator, but which in the new model is effected quite automatically, an achievement one would have been tempted to consider *a priori* as almost impossible of realization."

The concluding portion of the book deals with computation by graphic methods, applicable mainly to approximate calculations, and therefore of interest to the engineer rather than the actuary. The work forms a valuable reference to the field of mechanical aids to computation, a field seemingly wide enough to be bewildering, but, as M. d'Ocagne says in his closing paragraph, "In the arsenal of tools he is permitted to explore, a good workman is not at a loss to select that best suited to his task."

W. R. S.

THE INSTITUTE OF ACTUARIES.

ROYAL PATRIOTIC FUND CORPORATION.*

By the "Act to reorganize the administration of the Patriotic Fund" (3 Edw. VII, c. 20), which was passed 11 August 1903, the Patriotic Commission was dissolved as from 31 December 1903, and a body corporate was established, under the title of the "Royal Patriotic Fund Corporation", to which the property, rights, duties, and liabilities of the Commission were transferred.

* For previous references to the *Patriotic Fund* in the *Journal*, see vol. xxxvi, pp. 85, 400: vol. xxxvii, p. 446; and vol. xxxviii, p. 378; also Messrs. Schooling and Rusher's paper on "The Mortality Experience of the Imperial Forces during the War in South Africa, 1899–1902" (vol. xxxvii, pp. 545-616); and Messrs Burn and McDonald's paper on "The Rates of Re-marriage and Mortality amongst Widows in Receipt of Relief from the Patriotic (Russian War) Fund" (vol. xxxviii, pp. 433-494.).

By the first schedule of the Act, the Corporation is constituted as follows: (*a*) twelve "appointed members" nominated by His Majesty for three years, but eligible for re-appointment; (*b*) the Lord Lieutenants of Counties; (*c*) the Chairman of every County Council; (*d*) the Lord Mayors, and Mayors of County Boroughs in England, Wales, and Ireland; (*e*) the Lord Provosts, and Provosts of Burghs in Scotland having a population of more than 50,000 at the last Census; and (*f*) not more than seven co-opted members. The above constitute the Council, and provisions are added, for the appointment from that body of an Executive Committee; for deposit of all securities, and the interest thereon, at the Bank of England; for audit of the accounts of the Corporation; and for annual reports to His Majesty.

The Forty-second (and final) Report of the Commissioners, for the year 1903, has been published as a Blue Book, and includes a Table of the Beneficiaries on the various Funds under their administration; also a statement of the balances in hand on each such Fund, and detailed Capital Accounts and Income Accounts to 31 December 1903, in respect of each Fund.

The following letter was addressed to the Council of the Institute of Actuaries, in February 1904, by Colonel J. S. Young, Secretary of the Corporation:

53, Charing Cross,
London, S.W.
20 *February* 1904.

Dear Sirs,—You are, it is believed, aware that the Royal Commission of the Patriotic Fund was dissolved as from 31 December last by Act of Parliament passed last year, and that under the same Act the property, rights, duties and liabilities of the Commission have been transferred as from the 1st January to the Royal Patriotic Fund Corporation.

The Executive Committee of the new Corporation have been considering the financial position of the various Funds for the administration of which the Corporation will be responsible henceforth.

The position of the Transvaal War Fund, as set forth in the Report of your Council to the late Patriotic Fund Commission, dated 18 May 1903, has been especially considered in view of your estimate that no less a sum than £231,000 would be required, in addition to the amount of the Transvaal War Fund in the hands of the Patriotic Fund Commissioners at 31 December 1902, to enable the widows on the books of the Fund to be placed upon the same scale of allowances as enjoyed by widows on previous War Funds administered by the Patriotic Fund Commissioners.

Contributions to the Fund during 1903 have been insignificant in

aggregate amount, while, on the other hand, the roll of widows and orphans has increased, and experience of former wars shows that the roll will go on increasing for some time to come by the deaths of men from wounds or disease contracted in the Transvaal War.

His Royal Highness the Duke of Connaught, President of the Royal Patriotic Fund Corporation, and the Executive Committee, are desirous of appealing for further contributions from the public to ameliorate, if possible, the pecuniary position of the Transvaal War widows. But before doing so, they feel they ought to have further actuarial advice.

In these circumstances I am directed by His Royal Highness the Duke of Connaught, to ask if your Council would be pleased to extend to the Corporation the great public services they rendered in actuarially valuing the Funds under the administration of the late Patriotic Fund Commission, and transferred to the new Royal Patriotic Fund Corporation.

In any case, I am directed by His Royal Highness to ask as a favour that your President and any other representative of your Council would confer with the Finance Committee of the Corporation upon this subject.

I am, Dear Sirs,

Yours faithfully,

(Signed) J. S. YOUNG, Colonel,

Secretary.

The Honorary Secretaries,
Institute of Actuaries,
Staple Inn Hall, W.C.

A conference took place on 25 February 1904, between Messrs. Hughes (President), Higham (Ex-President), and Schooling (Hon. Secretary), as representing the Council of the Institute, and the Finance Committee of the Corporation, consequent on which the following letter was addressed by Colonel Young to the Hon. Secretaries of the Institute:

53, Charing Cross,
London, S.W.
12 *April* 1904.

Dear Sirs,—I now transmit herewith the usual statistical information in regard to the Funds in respect of which my Committee desire such information as your Council feel they can afford.

The information desired is as follows:

1st. In regard to the undermentioned Funds which were last actuarially re-valued at the dates specified below, it is desired to be informed, approximately, how the surplus in each Fund of assets over liabilities, then declared, has been altered by the changes which have since occurred.

Fund last valued at 31 *December* 1900.
Ashantee War Fund.

Funds last valued at 31 *December* 1901.
"Captain" Fund.
"Eurydice" Relief Fund.
Zulu War Fund.
"Atalanta" Relief Fund.
Light Brigade (Balaclava) Relief Fund.
"Victoria" Relief Fund.

Funds last valued at 31 *December* 1902.*
Patriotic (Russian War) Fund.
Rodriguez Fund.
Royal Naval Relief Fund.
Soldiers' Effects Fund.
Transvaal War Fund.
Indian Mutiny Relief Fund.

2nd. Information is desired what increases of weekly allowances, that is to say, shillings per week, of an equal rate of increase for all ranks, could be given to the widows on

"Captain" Fund . . .	£13,484	19	8
"Eurydice" Fund . .	4,800	0	0
Zulu War Fund . . .	7,945	7	9
"Atalanta" Fund . .	1,200	0	0

if the amounts stated opposite each Fund were added to the capital of each Fund. These were transferred from each Fund as surpluses to the Patriotic (General) Fund under the provisions of section 4, Patriotic Fund Act, 1881.

3rd. In regard to the Transvaal War Fund, special information is desired, in view of the fact, explained to you at the conference held here on 25 February, that, before an appeal for further contributions from the public to the Fund is made, my Committee feel it is necessary to have further actuarial data than the data as at 31 December 1902, kindly furnished by your Council to the late Patriotic Fund Commission in Report dated 18 May 1903, and contained in four schedules which accompanied that Report.

I have, therefore, been directed to submit to you the following tables, prepared exactly as the tables which were prepared of statistics of the Fund as at 31 December 1902.

These are lettered as follows: A, 1 and 2; B, 1 and 2; C, 1 and 2; D, 1 and 2; E; F; G, 1 and 2.

In connection with these the information desired is:

1st. How far, approximately, the surplus of £69,698 of assets over liabilities as at 31 December 1902, and shown in Schedule 3 to your Report, dated 18 May 1903, has altered by the changes during the year 1903.

* The Patriotic (General) Fund was subsequently included in this list.

2nd. How far, approximately, the deficiency, £231,389, required to provide the benefits shown in Schedule 2 of your Report, dated 18 May 1903, has been altered by the changes which have occurred in 1903.

I am also directed to transmit fresh tables, B, 3 and 4; C, 3 and 4; and to ask what sum of money would be required to provide the allowances at the rates therein shown, for the numbers given as at 31 December 1903, *in addition to* the rates and according to the numbers given in Tables A, 1 and 2; E; and F.

I am to add that my Committee feel greatly indebted to your Council, and are very reluctant indeed to trouble them at all for any fresh valuations, after the Reports furnished to the late Patriotic Fund Commission up to 31 December 1902. But, as explained to you at the conference which took place with you on 25 February last, my Committee feel themselves in a position of great difficulty, as a new statutory body entrusted with great responsibilities, involving an appeal for additional contributions from the public in respect of the Transvaal War widows and children.

They therefore desire to be assured of their position actuarially, as referred to herein, to enable them to act with due regard to all the public interests concerned.

I am, yours faithfully,
(Signed) J. S. YOUNG, Colonel,
Secretary.

THE PRESIDENT AND HONORARY SECRETARIES,
Institute of Actuaries.
Staple Inn Hall, W.C.

The following is the text of the Report made to the Corporation, in response to the above letter, by the Council of the Institute, showing the results of a valuation of each of the several Funds as at 31 December 1903. A detailed Valuation Balance Sheet in respect of each Fund is appended to the Report; also additional schedules, as to the beneficiaries under the Transvaal War Fund, and the values of the existing and proposed additional benefits. The results of these Valuation Balance Sheets are here given in summary form; also (on p. 386) the detailed Valuation Balance Sheet of the Transvaal War Fund.

INSTITUTE OF ACTUARIES,
STAPLE INN HALL,
LONDON, W.C.
17 *June* 1904.

ROYAL PATRIOTIC FUND CORPORATION.

SIR,—In furtherance of the desire of the Committee of the Royal Patriotic Fund Corporation to be supplied with an estimate of the

surpluses at their disposal on taking over the various Funds lately administered by the Commissioners of the Royal Patriotic Fund, we now submit a complete account of the assets and liabilities of Fourteen Funds (including the Transvaal War Fund) as at 31 December 1903, founded on the statistics supplied with the Secretary's letter of 14 April.

It was understood by the members of the Council of the Institute who had an interview with the Finance Committee of the Corporation on 25 February, that it was not intended to provide for allowances on further widowhood, or for increases at certain ages, but as a letter from the Secretary to the Corporation of 12 April asks for the information, and supplies the necessary statistics, the Council, in the circumstances, have had the valuations made, and the Corporation therefore have before them the fullest particulars.

The request made to the Council was merely for estimates of the effect of the changes which had taken place in the membership, since its last valuation of the various Funds. On looking into the matter, however, it was found desirable to make a full valuation of each Fund as at 31 December 1903, as an estimate based upon the changes only in the number of beneficiaries would not have been completely satisfactory in every case.

In the case of four of the Funds, namely, The "Atalanta", "Captain", "Eurydice", and Zulu War, the Council was asked what increases of weekly allowances could be given now in respect of the surpluses transferred from those Funds to the Patriotic (General) Fund. To enable the Corporation to estimate what additional allowances could be granted, we have appended to the balance sheet of each of these Funds, and also to that of the Transvaal War Fund, a statement of the cost of an extra shilling per week to each of the widows now receiving assistance from the respective Funds.

With regard to the Transvaal War Fund, we would point out that, if the whole of the benefits detailed in Schedules 1 and 2 are to be granted, a further sum of £231,815 will have to be found to supplement the existing assets of the Fund, but if no addition be made to the scale of allowances now granted to existing beneficiaries, there is a surplus available to the Fund of £86,657, as shown in Schedule 3, in addition to which a further sum of about £20,000 could be relied upon in respect of the estimate made for future expenses, which have been assessed on the basis of the complete scheme of benefits set forth in Schedule 2.

In the existing circumstances of a recently completed change in the administration of the Funds, the Council have once more willingly supplied the Corporation with the fullest details possible, in order to assist their labours. It desires, however, to express the opinion that a re-valuation of the Funds will be quite unnecessary for a few years, as the Funds can be safely controlled by means of quinquennial re-valuations.

The Council has in previous Reports called the attention of the Commissioners to the important advantages of establishing a Consolidated Fund; and it feels bound to urge that, the interests of each particular class being safeguarded by the regulations imposed

by the Corporation, such a consolidation of the assets would greatly facilitate the general administration of the Funds, and will in time become almost a necessity.

We are, Sir,

Your obedient Servants,

W. HUGHES,
F. SCHOOLING,
GEO. TODD,
On behalf of the Council of the Institute of Actuaries.

To Colonel J. S. Young,
Secretary,
Royal Patriotic Fund Corporation.

The following letter has been received by the President from Colonel J. S. Young, in acknowledgment of the services rendered by the Council of the Institute in the valuation of the several Funds:

53, Charing Cross,
London, S.W.
28 *June* 1904.

Sir,—I have laid before His Royal Highness the Duke of Connaught, President of this Corporation, your letter to me of 17th inst., transmitting a complete account of the assets and liabilities of fourteen Funds (including the Transvaal War Fund) as at 31 December 1903.

His Royal Highness is deeply sensible of the very great public services you and your colleagues on the Council of the Institute of Actuaries, and the Honorary Secretaries of the Institute, have rendered in an honorary capacity, in affording this Corporation such valuable information and assistance as is afforded in your Report and the statements which accompanied it.

The information in respect of the Transvaal War Fund is particularly valuable, and will enable the Corporation to base an appeal for further contributions to that Fund upon the authoritative data you have so kindly furnished.

Appreciating highly the labours involved in furnishing such exhaustive Reports, His Royal Highness begs you to accept for yourself, your colleagues on the Council, and the Honorary Secretaries of the Institute, the grateful thanks of His Royal Highness and the Corporation.

I have the honour to be, Sir,

Your obedient Servant,

(Signed) J. S. YOUNG, Colonel,
Secretary,
Royal Patriotic Fund Corporation.

The President of the Institute of Actuaries,
Staple Inn Hall, W.C.

TRANSVAAL WAR FUND, *as at* 31 *December* 1903.

LIABILITIES.

NON-COMMISSIONED OFFICERS AND MEN—		
£8,284 per annum to 516 Widows not in receipt of other Allowances	£162,211	
Less Allowance for Re-marriage	41,676	
		£120,535
£1,678 per annum to 532 Widows, supplementary to Army Pensions	£33,124	
Less Allowance for Re-marriage	8,621	
		24,503
£781 per annum to 69 dependent Relatives		8,822
£11,647 per annum to 3,211 Orphans		83,044
Outfits for 3,211 Orphans		11,900
OFFICERS—		
£913 per annum to 25 Widows	£16,962	
Less Allowance for Re-marriage	3,514	
		13,448
£310 per annum Temporary Allowances to 30 Orphans		2,674
Outfits for 30 Orphans		512
Funeral Allowances		2,895
Management Expenses and Commission on Post Office Orders		40,000*
Total		£308,333
Surplus		86,657
		£394,990

ASSETS.

	Nominal £	s.	d.	Interest	@	£
Birmingham Corporation Stock (1946)	9,100	0	0	3½	105	9,555
Cardiff Corporation Stock (1914–54)	8,962	16	8	3	90	8,067
Croydon Corporation Stock (1940–60)	8,726	14	9	3	90	7,854
Devonport Corporation Stock (1920–60)	9,019	18	11	3	90	8,118
Hastings Corporation Stock (1915–54)	10,500	0	0	3	90	9,450
Hertford County Stock (1920–40)	8,000	0	0	3	90	7,200
Huddersfield Corporation Stock (1920–40)	8,914	16	1	3	90	8,023
Local Loans	50,000	0	0	3	98	49,000
London County Council Stock (1920)	10,000	0	0	3	92	9,200
London County Council Bills due 7 May 1904	6,500	0	0	...	98½	6,403
Metropolitan Consolidated Stock (1929)	736	6	2	3½	105	773
National War Loan	170,542	14	7	2¾	95½	162,868
Newport (Mon.) Corporation Stock (1915–55)	10,400	0	0	3	90	9,360
Portsmouth Corporation Stock (1913–33)	2,528	11	5	3	90	2,276
Canada Bonds (1908)	5,000	0	0	4	100	5,000
Cape of Good Hope Bonds (Annual drawings 1 per-cent Accumulative)	4,000	0	0	4½	100	4,000
Ceylon Inscribed Stock	564	0	10	4	110	620
New South Wales Stock (1918)	9,400	0	0	3½	100	9,400
,, ,, (1924)	10,000	0	0	3½	100	10,000
,, ,, (1910)	20,000	0	0	4	100	20,000
New Zealand Inscribed Stock (1940)	1,228	18	6	3½	100	1,229
Bristol Water Works Debenture Stock	5,000	0	0	3½	100	5,000
Great Western Railway Debenture Stock	24,216	16	6	4	127	30,755
London and India Docks Co. Preferred Ordinary Stock	1,839	0	0	4	100	1,839
Midland Railway Preference Stock	12,000	0	0	2½	75	9,000
						£394,990

*This estimate for expenses assumes as before that all the benefits granted in the case of the other Funds will be extended to the Widows of this Fund.

VALUATION SUMMARY, 31 *December* 1903.

Name of Fund	Liabilities	Assets	Surplus	Deficiency	Cost of additional allowance of one shilling weekly to incumbent Widows
	£	£	£	£	£
Ashantee War Fund	1,809	1,757	...	52	...
"Atalanta" Relief Fund	4,952	6,700	1,748	...	306
"Captain" ,, ,,	17,033	17,413	380	...	1,100
"Eurydice" ,, ,,	8,746	9,504	758	...	576
Indian Mutiny ,, ,,	25,644	20,994	...	4,650	...
Light Brigade (Balaclava) ,, ,,	1,644	1,900	256	...	...
Patriotic (Russian War, 1854–6) Fund	119,695	130,357	10,662	...	...
Patriotic (General) Fund	88,152	179,236	91,084	...	...
Rodriguez Fund	10,742	16,450	5,708	...	...
Royal Naval Relief Fund	5,154	7,223	2,069	...	...
Soldiers' Effects Fund	115,966	159,277	43,311	...	...
"Victoria" Relief Fund	43,685	54,583	10,898	...	...
Zulu War Fund	17,622	18,600	978	...	1,261
Transvaal War Fund	308,333	394,990	86,657	...	40,685
	£769,177	£1,018,984	£254,509	£4,702	£43,928

The First Report of the Royal Patriotic Fund Corporation, for the year 1904, has been recently published as a Parliamentary Paper, and includes interesting details as to the origin and objects of the several Funds, and as to their administration; also Capital and Income Accounts in respect of each Fund, to 31 December 1904. Reference is made in the Report to the fact that "the Council of the Institute of Actuaries kindly agreed "to furnish re-valuations of the Funds as at 31 December 1903, "gratuitously, in continuance of similar services rendered to the "Patriotic Fund Commissioners"; and in Appendix III is set out a copy of the Report made on behalf of the Council on 17 June 1904 (*see* above), with accompanying Schedules and Valuation Balance Sheets, as at 31 December 1903.

EXAMINATIONS OF THE INSTITUTE, April 1905.

Examination for Admission to the Class of Associate (Part I).

Examiner—Prof. S. L. Loney, M.A.

Supervisors—Messrs. A. D. Besant, B.A., and J. H. Barnes.

First Paper.

1. Given that a foot is equal to 30·48 centimetres, and that a gallon of water weighs 10 lbs. and contains $277\frac{1}{4}$ cubic inches, find, correct to three places of decimals of a pound, the weight of a cubic metre of water.

2. Find to the nearest penny the net return per-cent per annum on money invested in a 5 per-cent stock at $110\frac{1}{2}$, brokerage and other expenses of purchase being £1 per £100 stock, and income tax at the rate of 1*s.* 3*d.* in the £1.

The income tax being reduced to 11*d.* in the £1, to what price should the stock rise to give the same net return as before?

3. Solve the equations

$$\text{(i)}\quad \left.\begin{aligned} x+y&=(a+b)xy,\\ a^2x-b^2y&=(a^3-b^3)xy \end{aligned}\right\};$$

and

$$\text{(ii)}\quad \left.\begin{aligned} x^2-yz&=1,\\ y^2-zx&=2,\\ z^2-xy&=3. \end{aligned}\right\}$$

4. Explain what is meant by saying that one quantity varies as, or varies inversely as, another quantity.

The square of the time of revolution of a planet about the Sun varies as the cube of its distance from the Sun. If the distances of the Earth and Venus from the Sun are as 25 to 18, find, in days, the time of revolution of Venus, the time of revolution of the Earth being taken to be 365 days.

5. Insert three harmonic means between two given quantities a and b.

The product of the arithmetic and harmonic means of two numbers is 576, and the difference of their arithmetic and geometric means is 1; find the numbers.

6. How many different games at lawn-tennis, each side consisting of a man and a lady, may be arranged at a party consisting of 8 ladies and 6 men?

If two of the ladies are the wives of two of the men, and no man may have his wife as a partner, how many games may similarly be arranged?

*7. Prove that $\log a^m = m \log a$.

If the increase in the population of a country in any year be always proportional to the population at the beginning of that year, and if the population be doubled in 40 years, find in how many years it will be trebled.

* "A Short Collection of Actuarial Tables" will be supplied for use in answering this question.

8. State and prove the Exponential Theorem.

Show that the sum to infinity of the series whose nth term is

$$\frac{n+1}{\underline{n}} \cdot 2^n \text{ is } 3e^2-1.$$

9. Show how to resolve an algebraic fraction, whose denominator can be broken into factors, into a series of partial fractions.

Break into partial fractions the quantity

$$\frac{x^2+5x-9}{x^3-2x^2-5x+6},$$

and write down the general term when it is expanded in a series of powers of x.

10. A has a dice-box containing 5 dice and B has one containing 4 dice. It is known that 12 has been thrown by one of them and *a priori* it is equally likely to have been either. Find the chance that it was thrown by A.

11. Prove the formula

$$\Delta^n u_0 = u_n - n u_{n-1} + \frac{n(n-1)}{1\,.\,2} u_{n-2} - \frac{n(n-1)(n-2)}{1\,.\,2\,.\,3} u_{n-3} + \quad . \quad . \quad . \quad .$$

$$\text{Given} \quad \left.\begin{array}{r} u_0 = 98023, \\ u_{10} = 97651, \\ u_{20} = 97246, \\ \text{and } u_{30} = 96802\,; \end{array}\right\} \text{find the value of } u_{15}.$$

12. Define Σu_x, and show that $\Sigma u_x = \Delta^{-1} u_x$.

Prove that $\Sigma a^x x^2$

$$= \frac{a^x}{a-1}\left[x^2 - \frac{2ax}{a-1} + \frac{a^2+a}{(a-1)^2}\right] + C.$$

Second Paper.

13. A bookseller sells books at a discount of three-pence in the shilling, and thereby makes a profit of $15\frac{5}{13}$ per-cent. If he sold at a discount of two-pence in the shilling instead, what percentage of profit would he make?

14. I bought a horse and carriage for £80. I sold the horse at a profit of 20 per-cent, and the carriage at a loss of 4 per-cent, and found that on the whole transaction I had gained 5 per-cent. How much did the horse cost me?

15. Find the highest common factor and the least common multiple of the expressions:

$$a^4 + a^2b^2 + b^4 \text{ and } a^4 - a^2b^2 + 2ab^3 - b^4.$$

Break into factors the expression:

$$2x^3 + x^2 - 7x - 6.$$

16. Find the sum of the cubes of the roots of the equation:

$$ax^2+bx+c=0.$$

Find the condition that one of the roots of the equation $ax^2+bx+c=0$ may be equal to one of the roots of $a_1x^2+b_1x+c_1=0$ with its sign changed.

17. Find the sum of the cubes of the first n natural numbers.

Find the sum of the first n terms of the series whose rth term is

$$(r-1)(r+1)(r+2).$$

18. Find the number of combinations of n things taken r together; and show that it is equal to the number of combinations taken $n-r$ together.

Find the number of combinations of 4 letters taken 5 at a time when each letter may be repeated any number of times.

19. Obtain in its simplest form the general term in the expansion of $(1-2x)^{\frac{5}{2}}$, where $x<\frac{1}{2}$.

Find the coefficient of x^r in the expansion of

$$[1+2x+3x^2+\ \ldots\ ad\ inf.]^n, \text{ where } x<1.$$

*20. Making use of the tables, find the value of:

(1) $(4{\cdot}627)^{1{\cdot}39}$;

(2) x, where $(3{\cdot}79)^x=4{\cdot}93$.

21. State the expansion of $\log_e(1+x)$ in a series of powers of x, and show how it may be used to calculate the value of logarithms.

If x be less than $\frac{1}{2}$, prove that

$$\log_e\frac{2+x}{1+2x}=\log_e 2-\frac{3}{2}x+\frac{15}{8}x^2-\frac{21}{8}x^3+\ \ldots\ ad\ inf.$$

What is the coefficient of x^n in this series?

22. In a game of whist the dealer found, on turning up the last card, that he had the ace, king, queen, knave, ten, and three other trumps in his hand; find the chance that this would occur.

23. An event is known to have occurred from one of several causes whose probabilities are known; and when any of these causes exists the chance of the event following from it is known; on any occasion when the event happens find the chance of any given cause.

There are 4 bags which are known to contain 2 white and 4 black, 4 white and 3 black, 3 white and 3 black, and 2 black balls respectively. A ball is drawn at random and found to be black. Find the chance that it was drawn from the bag containing the most black balls.

24. With the usual notation prove that

$$f(x)=f(0)+\frac{x^{(1)}}{\underline{|1}}\Delta f(0)+\frac{x^{(2)}}{\underline{|2}}\Delta^2 f(0)+\frac{x^{(3)}}{\underline{|3}}\Delta^3 f(0)+\ldots$$

Show that $\Delta^4 0^5=240$.

* "A Short Collection of Actuarial Tables" will be supplied for use in answering this question.

Examination for Admission to the Class of Associate (Part II).

Examiners—Messrs. A. G. Allen, F. T. M. Byers, E. C. Thomas, and T. E. Young, B.A.

First Paper.

1. Given a table showing the present values at the effective rate i of annuities certain payable annually, how would you construct a table showing the present values at the same effective rate, of annuities payable p times a year?

2. What data would be required for the construction of tables of mortality for—

(*a*) The general population.
(*b*) Assured lives?

Explain the terms "select", "ultimate" and "aggregate" when applied to tables of mortality for assured lives.

3. A father, aged 35 next birthday, has a child aged 1 next birthday. An assurance of £150 is to be paid on the child attaining the age of 21, provided the father be then alive. In the event of the father's death an annuity of £5 is to be paid annually until the child attains age 21, with a payment of £100 at that date. The premiums are to cease on the father's death and to be returned on that of the child before reaching age 21.

Deduce in commutation symbols the formula for the annual premium.

4. Express in symbols the value of a complete annuity on a life aged (x), payable half-yearly, with the provision that if death occur within m years the annuitant and his representative shall together receive not less than the purchase money.

5. On the assumption of a uniform distribution of deaths, prove algebraically or by general reasoning that

$$a_x^{(m)} = \frac{1}{j_{(m)}} - \mathrm{A}_x \frac{i}{j_{(m)}}\left(\frac{1}{j_{(m)}} + \frac{1}{m}\right)$$

where $j_{(m)}$ is the nominal rate of interest convertible m times a year corresponding to the effective rate i.

Give the corresponding formula for a continuous annuity.

6. If in the formula $\mu_x = \mathrm{A} + \mathrm{B}c^x$ the quantity c be unity, what effect is produced upon (1) the force of mortality, (2) the annual premium for a whole-life assurance, and (3) the policy value?

7. State Lubbock's formula of summation as far as the term involving second differences, and apply the same to deduce the value of $\mathring{e}_x$ without assumption as to distribution of deaths.

8. Describe the method you would employ in the construction of a table of annual premiums for contingent assurances based upon select mortality tables—$\mathrm{P}^{\;1}_{[x].[y]}$.

Second Paper.

9. Show the identity of the two forms of sinking fund—the investment of the sinking fund instalments in securities independent of the given loan, and the repayment of increasing portions of the loan. If after such a loan had been completed the Income Tax were raised to 20*s.* in the pound, what effect would be produced in each case upon the redemption of the original capital?

10. A loan of £42,000 has been made with the following condition as to repayment: Annual instalments of principal for 20 years, the first being £4,000, the second £3,800, the third £3,600, and so on, Interest at the rate of 4 per-cent being paid annually on the outstanding amounts. Immediately after payment of the fifth instalment, it is arranged to repay the balance of the advance with a premium, the lenders being able to re-invest at only 3 per-cent. Show how the premium should be computed.

11. A Railway Staff in a stationary condition is recruited annually by 500 entrants at age 20, who are required to contribute to a Pension Fund. At age 60 they have the option of retiring on pension, and retirement is compulsory at age 65. Assuming that there are no secessions other than by death, and that one-half of those who reach age 60 retire at that age, the others remaining until age 65, give expressions for—

(i) The total number of present contributors;
(ii) The total number of present pensioners;
(iii) The total number of future years' service with which existing contributors will be credited.

12. If the value of l_{x+t} for all values of t from $-\frac{1}{2}$ to $+\frac{1}{2}$ can be expressed in the form $A-Bt$ where A and B are independent of t, prove algebraically that $\mu_x = m_{x-\frac{1}{2}}$.

13. Investigate a formula for the value of $\mathring{\hat{a}}^{(m)}_{y/x}$.

14. Deduce the Single and Annual premiums for an assurance for ten years, payable as to one-half at the first death, and as to the other half at the second death of three lives (x), (y) and (z). Is there any practical objection to making the quotation, and, if so, how would you propose to meet it?

15. Give verbal interpretations of the expressions

$$ {}_nV_x = A_{x+n}\left(1-\frac{P_x}{P_{x+n}}\right) = 1-\frac{1+a_{x+n}}{1+a_x}, $$

and show that they are identical.

16. Given a complete table of temporary annuities for all ages and durations, describe fully the method you would adopt to obtain and verify a table of policy values for endowment assurances.

Third Paper.

17. What does $Aa_{\overline{n}|}+B\dfrac{a_{\overline{n}|}-nv^n}{i}$ represent? Prove your answer and give a verbal explanation of the formula. What does the expression become when $n=\infty$?

18. A debenture of £100, redeemable at £110 on the 1 July 1915, and bearing interest at the rate of $4\frac{1}{2}$ per-cent per annum, payable half-yearly on the 1 January and 1 July in each year, is purchased on the 1 April 1905 for £109. How would you calculate the yield to the purchaser?

19. Prove that $Q^1_{xy}=\dfrac{1}{l_x}(l_{x-\frac{1}{2}}\cdot\mathring{e}_{x-\frac{1}{2}\,y}-l_{x+\frac{1}{2}}\cdot\mathring{e}_{x+\frac{1}{2}\,y})$ very nearly.

20. What would be the average age at death in a community which would be stationary, but that for the last 100 years there had been immigration at age 20 to the extent of $\frac{1}{10}$th of the number living at that age?

21. Prove that $P_x+d\,.\,{}_nV_x=P_{x+n}(1-{}_nV_x)$ and explain the expression verbally. Show what the formula becomes in the case of a whole-life policy subject to a limited number of premiums.

22. Give the formulæ for—

(i) The annual premium for a deferred annuity on a life aged 40 next birthday to commence at age 60 (the first payment becoming due one year later), with the refund of all the premiums paid in the event of death between ages 50 and 60.

(ii) The single premium for a reversionary annuity to (y) after the death of (x), the latter being permanently resident in an unhealthy climate and therefore subject to a different table of mortality.

23. Show in detail how to obtain a table of annual premiums for whole-life assurances from the values of q_x without constructing the life table. Assuming a rate of mortality represented by a constant addition of ·01 to q_x according to a standard table, explain how the required premiums could be approximately obtained without special tables.

24. Assuming Maclaurin's theorem, show that

$$\int_0^{2n} u_x dx=\frac{n}{3}(u_0+4u_n+u_{2n}).$$

How would you apply this formula to calculate a whole-life benefit?

Examination for Admission to the Class of Fellow (Part III).

Examiners—Messrs. T. G. Ackland, H. J. Baker, H. Foot, B.A., and J. Spencer.

First Paper.

1. Upon what principles were duplicate policies eliminated in the British Offices Life Tables, 1893, in the construction of (1) Aggregate Tables, (2) Select Tables? What are the essential points of difference between the data from which the $O^{M(5)}$ Table, and the $O^{[M]}$ Table, excluding the first 10 years of assurance, were respectively obtained?

2. Given, in the case of a table following Makeham's law, the equation

$$\log_{10} l_{x+t} = \log_{10} k - (x+t)\alpha - \frac{\beta}{c-1} c^{x+t}$$

where $\alpha = -\log_{10} s$ and $\beta = -(c-1)\log_{10} g$,

show what modifications may be introduced in order that the equation may apply to select tables, without loss of the property of uniform seniority. Deduce, from the value of $\log_{10} l_{[x]+t}$ thus obtained, expressions for $\mu_{[x]+t}$ and $\operatorname{colog}_{10}(p_{[x]+t})$; and show the relations between the ultimate and select values $\log_{10} l_{x+t}$, $\log_{10} l_{[x]+t}$; μ_{x+t}, $\mu_{[x]+t}$; $\operatorname{colog}_{10}(p_{x+t})$, $\operatorname{colog}_{10}(p_{[x]+t})$.

3. In the case of a Society whose business consists entirely of deferred and immediate annuities,—the premiums in respect of the deferred annuities being returnable without interest in the event of death before the annuities are entered upon,—how would you trace to its various sources the surplus disclosed at a quinquennial valuation, and how would you distribute it amongst the policyholders?

4. The mortality experience of an office closely approximates to that of the O^M Table, while its valuations are made on the basis of the H^M Table. Discuss the effect on the future profit (1) from mortality, and (2) from interest, of a change in the valuation rate of interest from 3 per-cent to $2\frac{1}{2}$ per-cent.

5. Describe Mr. Sheppard Homans' method of distributing surplus, and discuss its advantages and disadvantages. What are, in your view, the criteria of a satisfactory method of distribution?

6. You are asked to prepare a scale of weekly contributions, securing (1) full sick-pay during 26 weeks; (2) half-pay during a further 26 weeks; (3) quarter-pay during the remainder of sickness; (4) payment on death of a member; (5) payment on death of a member's wife. State upon what lines you would proceed, giving special attention to (*a*) the tables of mortality and sickness employed for the several benefits, (*b*) the question of prolonged sickness, and old age, and (*c*) provision for expenses of management.

*7. Find approximately the net annual premium, on the basis of the Carlisle Table, at 3 per cent. interest, for an assurance payable in the event of A, aged 30, predeceasing B, aged 40, where (1) A only is, and (2) both A and B are, resident in a tropical country. It may be assumed that the extra risk is represented, in the case of a single life, by a constant addition of ·01 to the force of mortality at all ages. Given μ_{30} (Carlisle) $= \cdot 0101$.

How would you treat the second case in practice?

8. On the death of the last survivor of C, aged 75, and D, aged 70, the income arising from a fund now consisting of £50,000 Consols is to be divided equally between A, aged 35, and his brother B, aged 40, so long as they are both alive. The survivor of A and B is entitled to the corpus of the fund, contingently upon his surviving both C and D. How would you estimate the market value of A's interest?

Second Paper.

9. Explain the method adopted in the case of the British Offices Life Tables of estimating the fractional exposure of the withdrawals. Why was the Nearest Duration Method found to be unsuitable?

10. Obtain the graduation formula which may be represented in the form

$$\frac{S_{5:5:3}}{100}\left\{\frac{4}{3}u_0 + (\Delta u_{-2} - \Delta u_1)\right\}$$

and show how it may be conveniently applied in practice. Find what second-difference error the formula involves.

11. A policy for a term of n years is granted at an annual premium to a life aged x, with the option of continuance at the end of the term as an endowment assurance, maturing at age $(x+n+t)$, on payment of the normal yearly premium at age $(x+n)$. Find expressions for the value of the policy on a select basis, (1) at the end of the $(n-1)$th year, and (2) at the end of the $(n+1)$th year assuming the option to have been exercised.

12. Having available the results of a net premium valuation of the whole-life business of an office on an H^M $2\frac{1}{2}$ per-cent basis, how would you determine the approximate difference in the aggregate reserves on an O^M $2\frac{1}{2}$ per-cent basis, the O^M 3 per-cent net premiums being valued? State briefly the special points calling for consideration.

13. Indicate fully what considerations would guide you in determining the basis of valuation of a life office (1) as a test of solvency, and (2) as an investigation to ascertain divisible surplus.

14. State what mortality table you consider most suitable as the basis of premiums for temporary assurances, and justify your choice. Explain on what bases as to loading, rate of interest, &c., you would proceed to compute a table of office annual premiums, and mention any special safeguards which you would think it desirable to provide.

* "A Short Collection of Actuarial Tables" will be supplied for use in answering this question.

15. A Life Assurance Company allows, after the payment of two years' premiums, a surrender-value, in the case of its whole-life policies and endowment assurances, both with and without profits, equivalent to the H^M $4\frac{1}{2}$ per-cent net premium reserve, in respect of both sum assured and reversionary bonuses. Criticize this method.

16. A man aged 35 is entitled to a reversionary life interest in

£10,000 Consols.

£15,000 National War Loan ($2\frac{3}{4}$ per-cent, repayable April 1910).

£300 East Indian Railway Company Annuity, Class A, ceasing 1953.

A leasehold house of the net annual value of £250 with 25 years unexpired.

The present life tenant is a male aged 67. Show fully how you would proceed to estimate the market value of the reversionary life interest.

Third Paper.

*17. A, born 2 September 1865, is entitled, contingently on his surviving a lady born 1 January 1830, and at her death, to one-fifth share of a fund now represented by the undermentioned securities, subject to estate duty at 4 per-cent. He has already received £750 absolutely on account of his expectant share, which will have to be brought into hotchpot. What is the market value of his share?

£5,000 $5\frac{1}{2}$ per-cent Preference Shares of a Hotel Company, now valued at £5,500.

£4,000 $4\frac{1}{2}$ per-cent Debenture Stock of a Furnishing Company, now valued at £3,600.

*18. What is the market value of the absolute reversion to one-third share of a fund now represented by the undermentioned securities, expectant on the decease or re-marriage of a widow born 3 October 1840, the whole being subject to an annuity of £156 to a female born 2 February 1875? State also the maximum amount you would advise to be lent on its security.

£10,000 Great Northern Railway 3 per-cent Debenture Stock.

£5,000 Midland Railway $2\frac{1}{2}$ per-cent Debenture Stock.

£5,000 Great Western Railway 5 per-cent Consolidated Guaranteed Stock.

19. Give a short account of the Manchester Unity Sickness and Mortality Experience, 1893–97, explaining the methods followed in classifying the sickness and mortality data, and pointing out the main features of the Experience, as compared with other Friendly Society Tables.

20. Discuss the relative advantages of adopting the Graphic method, and a summation formula, in the graduation of Sickness Tables, bearing in mind the necessity (owing to conditions of practice) of tabulating sickness claims according to period of attack. How would you determine whether the graduated tables were satisfactory?

* "A Short Collection of Actuarial Tables" will be supplied for use in answering these questions. If the values selected by the Candidate are not those which he would in practice adopt, he should state what bases he considers more appropriate.

21. What are the several methods of determining the ages to be used in making a net valuation of whole-life assurances and immediate annuities, and what are their theoretical justifications and effects?

22. Having given that interest at rate i' will be realised, that i is the valuation rate of interest, and that the mortality experienced will be that assumed in the valuation, deduce an expression for the present value at rate i' of the future profit from surplus interest, in the case of a whole-life policy issued at a uniform annual premium.

23. What tables of mortality would you employ in calculating premiums to be charged for (1) whole-life assurances with profits; (2) whole-life assurances without profits; (3) endowment assurances; (4) joint whole-life assurances; (5) children's endowments; (6) contingent assurances; (7) deferred annuities; (8) reversionary annuities? Give reasons justifying your selections.

24. A is the present holder of a certain title of nobility, B, his wife, C is their eldest son, the heir, and D the next younger son. It is desired to effect an assurance payable on the death of B, provided that during her lifetime D shall have succeeded to the title. Give a formula for the net single premium for the assurance, disregarding the possibility that C will leave issue. What loading would you suggest in a case of this kind, and which of the lives should undergo medical examination?

Examination for Admission to the Class of Fellow (Part IV).

Examiners—Messrs. H. W. Andras, J. Blakey, O. Kentish, and E. A. Rusher.

First Paper.

1. Define and contrast:
   (*a*) Fee simple and fee tail.
   (*b*) Reversion and remainder.
   (*c*) Vested remainder, contingent remainder, and executory interest: state the rules governing the creation of the last two.

2. Describe briefly the following:

Estate Duty, Settlement Estate Duty, Legacy Duty, Succession Duty; and state which of them is payable in respect of each death in the following cases:

   (*a*) On A's death in 1904, in respect of a policy effected on his life by B, a creditor, who has paid all the premiums and owns the policy.
   (*b*) On a policy on the life of C bequeathed by his will upon trust to invest the amount thereunder and pay income to his son for life, then to his son's widow for life, then to divide the capital between his son's children equally. C died in 1900; his son died in 1902, leaving a widow and two infant children, one of whom died in infancy in 1903; the widow died in 1904, leaving the other child surviving.

3. In a proposal for Life Assurance, which proposal is to be treated as the basis of the contract, A, the Life Assured, warrants and agrees that he will not commit suicide within one year from the date of the contract. What claim, if any, will there be on the assuring Office if A commits suicide within the year?

(*a*) If the policy is his own property.
(*b*) If it has been effected by and belongs to his creditors.
(*c*) If he committed suicide whilst temporarily insane.

Give reasons for your answer.

4. What objects would you have in view in preparing for a Life Assurance company a set of account books for recording the transactions of any financial year? Give the names, and state briefly the use, and relation to each other, of the various books of account you would recommend. What accounts would you open in the general ledger?

5. What do you understand to be the main objects aimed at by the Friendly Societies Act, 1896? In what respect do you consider such objects have been attained?

State any measures by means of which you think the Act might be improved.

6. What are the usual regulations of the London Joint-Stock Banks in respect of deposits bearing interest? Why does not the Bank of England accept these deposits?

7. "It is a very dangerous thing to deal with equities of redemption." Comment on this expression, defining the term equity of redemption.

In a purchase by a mortgagee of the equity of redemption in the mortgage, what precautions should be observed?

8. Give the characteristics of the three standard tables employed at the present time for Friendly Society valuations. How would you ascertain whether a given table is suitable in the case of a particular society?

Second Paper.

9. A society, now comprising several thousand members, has been established for a native Christian Community in a certain part of India. It has been in existence 50 years, and the following benefits are given:

(*a*) An annuity to the member on attainment of age 60.
(*b*) A payment to the widow at the member's death.
(*c*) An annuity until attainment of age 14 to such children as are under that age at the member's death.

The members are divided into three classes in decennial ages at entry 20 to 30, 30 to 40, and 40 to 50, the monthly contribution varying with each class.

What particulars would you call for to enable you to value the fund, bearing in mind that no previous valuation has been made?

10. What is your opinion as to the desirability of an office supplying printed forms of assignment? Do you consider that any alteration should be made in the law relating to the assignment of policies, or that an office should be empowered to issue a certificate of title?

11. State, with reasons, what construction is given to the following devises of real estate:—

(*a*) To A (who survives the testator).
(*b*) To A and the heirs of his body (A having pre-deceased the testator).
(*c*) To A, a child of the testator pre-deceasing him.

Wherein does the construction of deeds and wills differ?

12. Describe how a general depression of trade in this country affects Stock Exchange securities and the Money market.

13. Write a brief report to the Directors on the various methods which have been adopted for valuing the Stock Exchange securities in the balance sheet of a life office at the close of a valuation period, stating the advantages or disadvantages of each method. Which do you prefer, and for what reasons?

14. Explain clearly the process by which a high bank rate is used as a means of attracting gold from abroad, noting any influences which may operate to counteract the effects of a high rate.

15. Give a brief account of the terms upon which the Water Board has recently taken over the undertakings of the various Metropolitan Water Companies, and of the method in which it has issued stock for that purpose.

16. Draft the headings of a book or books suitable for recording the new premiums received during the year, and the new sums assured and premium income at the end of the year, in respect of the new life assurance business of any financial year in an office receiving premiums annually, or half-yearly or quarterly on the instalment principle.

Third Paper.

17. What reasons can be assigned to account for the difficulty which some municipalities experience in raising loans by the issue of stock to the public, and what are the salient points for consideration in an application by a local authority for an advance from a life office?

18. Describe the present method of assessment of a life office for income-tax under Schedule D.

(1) In the case of a proprietary life assurance company transacting life assurance and annuity business only.
(2) In the case of a proprietary life and fire insurance company.

State your views as to the principles involved.

19. Discuss the following securities as investments for the funds of a Life Assurance company:

(1) Cape of Good Hope $3\frac{1}{2}$ per-cent Consolidated Inscribed Stock (1929–1949).
(2) Johannesburg Municipal 4 per-cent Inscribed Stock.
(3) Pennsylvania Railroad $3\frac{1}{2}$ per-cent Consolidated Mortgage Sterling Bonds.
(4) Buenos Ayres and Rosario Railway $3\frac{1}{2}$ per-cent Centra Debenture Stock.
(5) Great Indian Peninsula Railway Annuities.
(6) Madras Railway Capital Stocks.

20. What considerations would guide you in dealing with an application for payment of a claim under a Life Policy?

(*a*) To the personal representatives under a probate granted in a foreign country.
(*b*) To the donee of a power of attorney.
(*c*) To the legatee under a specific bequest of the policy.

21. Do you consider Mr. Sutton's Welsh Tables suitable for the valuation of a friendly society whose members consist mainly of miners? If not, what basis as to sickness and mortality would you adopt? Give reasons for your answer.

22. Application is made to a life office for an advance of £20,000 on the security of leasehold property consisting of offices and warehouses in a large city. State precisely the information which you would require before putting the matter before the Board, and draft a letter to the proposed borrowers agreeing to the loan.

23. What are the rates of remuneration paid to the Bank of England for the management of the funded and unfunded debt? Why is the National Debt managed by the Bank of England and not by a Government department, and what advantages does the present system possess?

24. In the last twenty years the proportion of the assets of Life Assurance Companies of the United Kingdom invested in mortgages on property has substantially decreased, and the proportion invested in Stock Exchange or other convertible securities has largely increased. How do you account for this, and what effect has such a tendency had or, if continued, is it likely to have on the finance of Life Offices?

PROCEEDINGS OF THE INSTITUTE.—Session 1904–1905.

First Ordinary Meeting, 28 *November* 1904.

The first ordinary meeting of the Session 1904-1905 was held at the Hall of the Institute, on the 28th day of November 1904.

The President (Mr. Henry Cockburn) in the Chair.

The President delivered an Inaugural Address.

Second Ordinary Meeting, 19 *December* 1904.

The President (Mr. HENRY COCKBURN) in the Chair.

Messrs. Lawrence MacLagan Cathles, F.F.A., John Hamilton Imrie, M.A., F.F.A., Frederick William Robertson, F.F.A., James Leask Robertson, F.F.A., and John Walter Thomson, F.F.A., were duly elected Associates of the Institute.

A paper entitled "On the Retrospective Method of Valuation", by Mr. F. Bell, was read in abstract by the author.

The following gentlemen took part in the discussion:—Messrs. W. P. Phelps, G. King, W. P. Elderton, T. J. Searle, and R. P. Hardy.

Third Ordinary Meeting, 30 *January* 1905.

The President (Mr. HENRY COCKBURN) in the Chair.

A paper entitled "On Staff Pension Funds", by Mr. G. King, was read in abstract by the author.

The following gentlemen took part in the discussion:—Messrs. G. J. Lidstone, A. Hewat, T. G. Ackland, E. C. Thomas, H. W. Manly, and R. P. Hardy.

Fourth Ordinary Meeting, 27 *February* 1905.

The President (Mr. HENRY COCKBURN) in the Chair.

A paper entitled "Changes in Pure Premium Policy-Values consequent upon variations in the Rate of Interest or the Rate of Mortality, or upon the introduction of the Rate of Discontinuance", by Mr. G. J. Lidstone, was read in abstract by the author.

The following gentlemen took part in the discussion:—Messrs. J. E. Faulks, T. E. Young, G. King, T. G. Ackland, and R. P. Hardy.

Fifth Ordinary Meeting, 27 *March* 1905.

The President (Mr. HENRY COCKBURN) in the Chair.

Mr. Robert Raynal Brodie, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "Bonuses in Model Office Valuations and their relations to Reserves", by Dr. J. Buchanan, was read in abstract by the author.

The following gentlemen discussed the paper:—Messrs. H. J. Baker and G. King.

Sixth Ordinary Meeting, 17 *April* 1905.

The President (Mr. HENRY COCKBURN) in the Chair.

Mr. Charles William Steele Jamieson, F.F.A., was duly elected an Associate of the Institute.

Papers entitled "On the Importance and Practicability of a Standard Classification of Impaired Lives", by Mr. S. W. Carruthers, M.D., and "Social Conditions as affecting Widows' and Orphans' Pension Funds", by Mr. S. J. H. W. Allin, were read by their respective authors.

The following gentlemen took part in the discussions:—Messrs. E. A. Rusher, Dr. Light (a visitor), and Mr. S. G. Warner; Messrs. E. C. Thomas, G. King, V. Marr (communicated through the Editor of the *Journal*), H. W. Manly, A. W. Watson, and H. W. Andras.

The Fifty-eighth Annual General Meeting, 5 *June* 1905.

The President (Mr. Henry Cockburn) in the Chair.

The proceedings at the Annual General Meeting will be found on page 408.

REPORT, 1904–1905.

The Council have the pleasure to report to the members upon the progress of the Institute during the session of 1904–1905, the fifty-seventh year of its existence.

There has been an *increase* of 25 in the number of members, as compared with the previous year. 138 candidates have been admitted as Probationers, and 65 as Students conditionally on their passing Part I of the Examination. At the end of the official year in which the Institute was incorporated by the Royal Charter the number of members was 434, while ten years later, at 31 March 1895, it was 775. Since that time the numbers have been as follows:

On 31 March	1896, 788,	On 31 March	1901, 818,
,,	1897, 826,	,,	1902, 842,
,,	1898, 860,	,,	1903, 828,
,,	1899, 834,	,,	1904, 856,
,,	1900, 822,	,,	1905, 881.

The following schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last:

Schedule of Membership, 31 *March* 1905.

	Honorary Members	Fellows	Associates	Students	Corresponding Members	Total
i. Number of Members in each class on 31 March 1904 .	1	226	271	335	23	856
ii. Withdrawals by						
(1) Death . . .	...	6	1	...	1	
(2) Resignation . .	...	2	4	16	...	53
(3) Default in payment of Subscriptions . .	...	1	1	21	...	
	1	217	265	298	22	803
iii. Additions to Membership						
(1) By Election . .	...	...	6	...	...	
(2) By Order of Council	...	...	...	62	...	78
(3) By Re-instatement	...	...	2	8	...	
	1	217	273	368	22	881
iv. Transfers						
(1) By Examination:						
from Associates	...	...	8	...	...	...
to Fellows .	...	8	...	...	...	...
	1	225	265	368	22	881
(2) By Examination:						
from Students	..	...	...	1	...	...
to Fellows .	...	1	...	...	...	...
	1	226	265	367	22	881
(3) By Examination:						
from Students	...	...	...	17	...	...
to Associates .	...	...	17	...	...	...
v. Number of Members in each class on 31 March 1905 .	1	226	282	350	22	881

The Council have, with great regret, to report the loss by death, since the last Annual Meeting, of three Fellows, Mr. Archibald Day, Mr. David Deuchar, and Mr. Frank McGedy; two Associates, Mr. D. Y. Mills and Mr. W. B. Paterson; and two Corresponding Members, Mons. C. L. Landré and Dr. H. Grosse.

By the death of Mr. Archibald Day, whose membership dated from 1849, and who was an Hon. Secretary 1866–1870, and President 1886–1887, the Institute has lost a prominent and highly-esteemed member, who took great interest in its affairs, and whose presence will be missed by a large number of his colleagues.

The Annual Subscriptions, together with admission and other fees, amounted to £2,050. 7*s*., as compared with £2,050. 6*s*. received in the previous year. The total Income for the year was £2,656. 1*s*. 10*d*., and the total Expenditure £2,441. 3*s*. 8*d*. The Revenue Account and Balance Sheet are given herewith (pp. 404-5).

The stock in hand of the Institute publications on 31 March was as follows:

No. of Copies	Description of Work
12,403	Parts of *Journal*.
504	Index to Vols. 1 to 10.
987	,, to Vols. 21 to 30.
989	*Text-Book*, Part I (New Edition).
1,370	,, Part II (Second Edition).
688	Government Joint-Life Annuity Tables.
767	Select Life Tables.
414	A Short Collection of Actuarial Tables.
233	Messenger Prize Essay (Friendly Societies).
27 *in cloth* / 2,922 *in paper*	Lectures on Finance and Law (Clare and Wood Hill).
1,657	Lectures on the Companies Acts (A. C. Clauson).
1,615	Lectures on the Law of Mortgage (W. G. Hayter).
825	Lectures on the Measurement of Groups and Series (A. L. Bowley).
703	Transactions of the Second International Congress of Actuaries.
2,463	Syllabus and Examination Questions.

The following papers were submitted at the sessional meetings of the Institute, namely:

28 *November* 1904.—An Inaugural Address by the President.—Mr. Henry Cockburn.

19 *December* 1904.—"On the Retrospective Method of Valuation."—Mr. F. Bell.

30 *January* 1905.—"On Staff Pension Funds."—Mr. George King.

27 *February* 1905.—"Changes in Pure Premium Policy-Values consequent upon Variations in the Rate of Interest or the Rate of Mortality, or upon the introduction of the Rate of Discontinuance."—Mr. G. J. Lidstone.

27 *March* 1905.—"Bonuses in Model Office Valuations and their Relations to Reserves."—Dr. J. Buchanan.

17 *April* 1905.—"On the Importance and Practicability of a Standard Classification of Impaired Lives."—Dr. S. W. Carruthers.

"Social Conditions as affecting Widows' and Orphans' Pension Funds."—Mr. S. J. H. W. Allin.

[*Continued on page* 406.]

Dr. *Revenue Account for the year*

	£	s.	d.	£	s.	d.
Amount of Funds at the beginning of the year—						
General Fund	7,796	3	10			
Messenger Legacy Fund	369	15	2			
Brown Prize Fund	254	4	9			
	8,420	3	9			
Examination Fees for year 1904	254	2	0			
				8,674	5	
Subscriptions—						
Fellows	647	17	0			
Associates	543	18	0			
Students	369	12	0			
Probationers	75	12	0			
	1,636	19	0			
One Annual Subscription Compounded for	25	4	0			
Fines for Re-instatement	1	16	0			
				1,663	19	
Application Fees—						
Associates	12	12	0			
Students	57	4	6			
Probationers	44	12	6			
				114	9	
Examination Fees for year 1904				22	1	
Class Fees				186	18	
Sales of Publications—						
Journal	314	11	7			
Text-Book, Part I	65	19	0			
Text-Book, Part II	175	14	8			
Government Annuity Tables	4	7	7			
Select Life Tables	7	1	7			
Short Collection of Actuarial Tables	5	16	5			
Hardy's Friendly Societies	1	17	10			
Legal, Financial, and Statistical Lectures	5	16	5			
Transactions of the Second International Congress	16	0	0			
Syllabus and Examination Questions	1	11	1			
				598	16	
Dividends and Interest (*less* Tax)—						
General Fund	239	1	0			
Messenger Legacy Fund	10	10	9			
Brown Prize Fund	7	4	11			
				256	16	
				£11,517	5	

Balance Sheet,

LIABILITIES.

	£	s.	d.	£	s.	d.	£	s.
General Fund				8,247	8	4		
Messenger Legacy Fund	233	9	2					
Accumulated Dividends	146	16	9					
				380	5	11		
Brown Prize Fund	200	0	0					
Accumulated Dividends	61	9	8					
				261	9	8		
							8,889	3
Examination Fees for year 1905							249	18
Sales of British Offices Life Tables							39	19
Contributions of Companies towards cost of proposed further Tables based on the British Offices Experience							312	4
Sundry unpaid accounts							74	8
							£9,565	13

ending 31 *March* 1905. Cr.

	£ s. d.	£ s. d.
Journal—		
Printing of Nos. 216, 217, 218, and 219	648 16 10	
Clerical assistance	40 0 0	
		688 16 10
Library—		
Binding, Purchases, and Index Cards		26 8 6
Publications' Account—Binding		25 13 8
Meetings		75 8 8
Examination charges		81 1 10
Tutors for classes in Parts I and II		239 8 0
Expenditure on account of Mr. G. F. Hardy's Lectures		91 4 9
Syllabus and Examination Questions—Printing		41 6 6
Legal charges		9 0 0
Office Expenditure—		
Rent	600 0 0	
Salaries and Pension	332 2 0	
House expenses	88 3 11	
Corporation Duty	11 8 4	
Fire Insurance	26 16 0	
Stationery and Printing	124 17 11	
Postage and Telegrams	38 7 11	
Furniture and Fittings	16 17 1	
Sundries	6 2 3	
		1,244 15 5
Expenditure on Valuations for the Royal Patriotic Fund Corporation		104 17 6
Amount of Funds at the end of the year, as per Balance Sheet		8,889 3 11
		£11,517 5 7

Examined and found correct, 2 May 1905.

BERNARD WOODS,
W. M. MONILAWS, } *Auditors.*
GEO. A. BROWN,

31 *March* 1905.

ASSETS.

	£ s. d.
Natal 3 per-cent Inscribed Stock (£3,000), cost	2,846 6 0
Metropolitan Railway $3\frac{1}{2}$ per-cent Debenture Stock, converted from £1,050 4 per-cent Debenture Stock, 1 Oct., 1904 (£1,200), cost	1,185 11 3
Great Eastern Railway 4 per-cent Debenture Stock (£1,200), cost	1,546 6 4
Great Northern Railway Preferred Ordinary Stock (£1,000), cost	1,142 11 9
Great Western Railway $4\frac{1}{4}$ per-cent Debenture Stock (£1,000), cost	1,524 5 7
Cash on Deposit Account	500 0 0
Cash on Current Account	820 12 5
	£9,565 13 4

[The Institute also possesses certain copyrights and stocks of publications (see p. 403).]

Examined and found correct, 2 May 1905.

BERNARD WOODS,
W. M. MONILAWS, } *Auditors.*
GEO. A. BROWN,

For the Examinations held in the United Kingdom and the Colonies on 14, 15, 17, and 18 April last, 285 entries were received, namely:

124	for Part	I.
98	,, ,,	II.
44	,, ,,	III.
19	,, ,,	IV.

The results of the Examinations will be duly announced.* The Council warmly acknowledge the valuable services of the Honorary Examiners and Supervisors.

Slight modifications have been made in the wording and arrangement of the Examination Syllabus, in order to define more clearly the allocation of the subjects in each Part, and the Examination Time Table has been altered so as to allow three periods of three hours each (instead of two periods of four hours) for the Examinations in Parts II, III, and IV.

For the further convenience of Students a pamphlet has been published containing the Syllabus, and the questions set during the past three years, and it is intended to issue this pamphlet annually.

A series of six lectures on "The Theory of the Construction of Tables of Mortality, and of similar Statistical Tables in use by the Actuary", has been delivered by Mr. G. F. Hardy. The lectures, which worthily maintained Mr. Hardy's reputation in the branch of actuarial science referred to, were followed with close attention by a large number of the members.

The long and arduous labours of the Joint Committee of the Institute of Actuaries and the Faculty of Actuaries on Mortality Investigation have been finally brought to a close, and the Committee has been dissolved. The thanks of the Institute, and of all interested in vital statistics, are due to the members of that Committee for the patience and skill with which the immense aggregate of crude facts has been sifted, and sorted, and reduced to an ordered and intelligible collection of tables which are of the highest interest, and of the greatest value as tools of precision in the hands of the practising actuary.

Mr. George King has felt compelled to resign the position of Editor of the *Journal*, which he has filled, with such marked ability, since October, 1896; and the Council are pleased to report that Mr. T. G. Ackland has been appointed to succeed him.

The Council have much pleasure in announcing that Mr. King has accepted an invitation to deliver an annual course of lectures on the actuarial subjects comprised in the Syllabus for the Examinations in Parts III and IV of the Examinations. The first series of lectures will commence in October next, and full particulars will shortly be communicated to the members.†

EXAMINATIONS, 1905.

Examinations were held on the 14th, 15th, 17th, and 18th of April, 1905, in the United Kingdom and the Colonies, at London, Edinburgh, Dublin, Adelaide, Melbourne, Sydney, Montreal, Toronto, Ottawa, and Winnipeg, with the following results, the names in each class being arranged in alphabetical order:—

Part I.

One hundred and twenty-four candidates sent in their names, of whom one hundred and eighteen presented themselves, and seventy-one passed, namely:—

* These results, for the United Kingdom and Colonies combined, are given on pp. 407-8.

† *See* p. 413.

Class I:

Alder, M. C.
Allen, J.
Bromby, W.
Burrows, G. E.
Burrows, V. A.
Clarke, H. G.
Dark, T. A.
Derrick, V. P. A.
Homan, R. C.
Laing, J. M.
McKechnie, J. H.
Sinclair, C. C.
Stone, M.
Thompson, J. S.
Walker, D. A.
Wellington, F.
White, R. C. S.

Class II:

Adam, C. C.
Atkins, F. C.
Beamish, E. C.
Clemens, F. B.
Edwards, H. A.
Edwards, H. H.
Flynn, B. D.
Forbes, J.
Goodall, E. V.
Hamley, E. F.
Harrington, E. W.
Johns, A. H.
Keevil, N.
Kenchington, F.
King, A. E.
Lafford, H. G.
Langstaff, M. P.
Osborne, W. A.
Pickup, J. R.
Reeve, G. M.
Rowland, S. J.
Schooling, T. H.
Sloan, J. J. E.
Welsh, W.
White, O. D.
Woodward, J. H.

Class III:

Agutter, W. J.
Blackadar, E. G.
Bradshaw, F. L.
Brown, J.
Cogar, W. E.
Coutts, K. V.
Cox, S. N.
Dore, H. W.
Dulley, J. F.
Eastcott, W. M.
Hammond, H. P.
Harnack, F. W.
Harrison, L.
Hill, C. D.
Jennings, R. W.
Kidd, H. D.
Marshall, A. W.
Marshall, J. E.
Perry, S. J.
Phillips, T. A.
Pollard, E. C.
Sharp, H. G.
Stuart, C. J. S.
Thompson, J. H. R.
Thompson, J. W.
Tutill, H. L.
Williams, H. C.
Wisdom, S. H.

PART II.

Ninety-eight candidates sent in their names, of whom ninety presented themselves, and forty-five passed, namely:—

Class I:

Langstaff, J. M.
Melville, H. E.

Class II:

Barford, F. W.
Cooper, J. J.
Downes, E. G.
Falk, O. T.
File, L. K.
Hallman, M. S.
Hancock, E. J.
Jefferson, J. A.
Leigh, S. G.
Macfarlane, J. A.
McKechnie, J. B.
McPhail, F. C.
Maltby, C. H.
Monkhouse, C. C.
Moore, G. L.
Raynes, H. E.
Savery, R. S. B.
Story, C. L. W. S.
Stuckey, R. R.
Townley, E. W.
Tully, A. P. T.
Wansbrough, T. P.
Winstanley, C. W.

Class III:

Bain, W. A.
Chubb, W.
Crump, P. C.
Elderton, R. L.
Ellis, R. G. G.
Farmer, E. C.
Gould, W. H.
Hammond, R.
Hitchins, W. R.
Latham, B.
Makepeace, F. L.
Monilaws, W. B.
Moore, G. E.
Newnham, E. W.
Paton, A. G.
Shute, O. B.
Somerville, W. H.
Strong, A. W.
Sturt, H. R.
Touzel, P. D.

Part III.

Forty-four candidates sent in their names, of whom thirty-nine presented themselves, and eight passed, namely:—

Class I:

None.

Class II:

None.

Class III:

Humphreys, H. T.
Kelham, C. S.
Mackenzie, M. A.
Neill, S. B.
Robertson, F. W.
Spurgeon, E. F.
Stewart, L. W.
†Wandless, J. R.

Part IV.

Nineteen candidates sent in their names, all of whom presented themselves, and twelve passed, namely:—

Class I:

None.

Class II:

†Chandler, T. R.
†Penman, W., Jr.
†Thompson, T. P.

Class III:

Court, A. G. D.
Denmead, J. C.
†Gemmill, W.
†Green, G.
†Milligan, C. L.
†Oakley, H. J. P.
†Robinson, H. T. K.
†Sherriff, F. H.
†Wilson, J. S.

Those marked (†) have now completed the Examination for the Class of Fellow.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The fifty-eighth Annual General Meeting of the members was held at Staple Inn Hall, on Monday, June 5, the President, Mr. Henry Cockburn, in the chair.

The Report of the Council (given on p. 402) having been taken as read,

The President, in moving that the Report and Accounts, as presented, be adopted, said that he need not trouble the meeting with many remarks in moving the resolution. It would be seen that the number of members had increased to 881, that figure being the highest ever reached in the

history of the Institute. It was also of interest to notice that since the Royal Charter was obtained, 20 years ago, the numbers had rather more than doubled. The increase during the past year had taken place in the Associates and Students; curiously enough the number of Fellows remained exactly the same as in the previous year. The Institute had, as usual, to regret the loss by death of various members—during the present year to a less extent, happily, than in the previous year; but, nevertheless, there had been some serious deprivations. Firstly, they had to regret the death of their old friend and former President, Mr. Archibald Day. There never was a more faithful supporter of the Institute than Mr. Day, who was the friend both of the older and of the junior members. His kindly presence would, he was sure, be long missed in the Hall and elsewhere. The Institute had also lost an old friend in Mr. Frank McGedy, who had been out of office for some time, and whom they had therefore not seen very recently. Mr. David Deuchar's death came very suddenly while he was in full vigour and work, and the Council deeply regretted his loss. Mr. Deuchar was associated with some present in the Joint Mortality Investigation, and they all had a great regard for him. The Institute had also suffered loss in the death of two Associates, Mr. Mills and Mr. Paterson, and two corresponding members, M. Landré and Dr. Grosse, the last two gentlemen not having been members for a very long period.

With reference to the funds, the balance-sheet, again, as last year, showed an increase of about £200. That increase was accounted for almost altogether by one item, *i.e.*, the sales of the *Journal* and of the *Text-Book*, Parts I and II. The increase during the past year was nearly £200 in those items alone. He did not think there was anything else calling for special attention in connection with the accounts, but he would be happy to answer any question which might be put. During the past session a number of interesting and practical papers had been given, some from old contributors, and some, he was happy to say, from new ones. This showed that what might be called old subjects were deserving of fresh treatment and consideration from time to time, and received fresh impetus from the new thoughts brought to bear upon them by active-minded members. He thought also that they need have no fear that new topics would fail to arise.

No fewer than 285 entries were made for the examinations. This was 10 more than in the previous year, and appeared to be a record number of entries. As the members had heard more than once recently, the proportion of passes was not quite so large in the different examinations as the Council could wish. The Council had had the matter under consideration during the past year, and it would be seen that slight modifications had been made in the syllabus and in the time allowed for the examinations, while a pamphlet had been printed containing the questions set in each examination during the last three years. In that way the Council had endeavoured, not to modify the standard, but to afford all reasonable assistance to men fairly well prepared for the examination.

The members had also had the pleasure during the past session of listening to a series of lectures from Mr. George Hardy. Those lectures, as was to be expected, were very well attended, and listened to with great interest throughout. The Council hoped, ere long, to publish the lectures, which were at present under revision by the author.

The Joint Mortality Committee had at last been dissolved, their sittings having extended over a period of nearly eight years; and a final account had been taken of the receipts and expenditure. In that connection, no doubt the members would have noticed in the accounts that a sum of £312. 4*s*. appeared in the balance-sheet as contributions of companies towards the cost of proposed further tables, based on the British Offices' Experience. The meaning of that item was, that a number of companies in London were good enough to return to the Institute the surplus which was sent to them on the final account, with the idea that the Institute should bring out at all events one more volume of a practical kind for every-day use—perhaps something on the lines of Mr. Ralph Hardy's Valuation Tables, as was suggested at the annual meeting last year by Mr. Browne. The Council had appointed a small committee to deal with the matter, and he hoped some useful result would accrue before long.

The Institute had lost Mr. King as editor of the *Journal*. Mr. King had filled that position since the year 1896, and he need not tell the members that the Council received Mr. King's resignation with very great regret, and accorded him their warmest thanks. He was particularly pleased to be able to make an announcement in connection with that gentleman of a more pleasing nature, namely, that the Council had persuaded Mr. King to deliver an annual course of lectures on the actuarial subjects comprised in the Syllabus of Parts III and IV of the Examinations. He could hardly recollect a time when Mr. King had not been doing something for the Institute, and there was no one, he was sure, who would undertake the proposed duty more ably than Mr. King. He would throw his whole experience and great powers into it, and he had no doubt that many of the members—whether students or others—would be very glad to come and listen to him. The particulars of the lectures would be announced, he hoped, very shortly. They would commence in October, and continue thereafter.

About a year ago a Committee was appointed by the House of Commons on the subject of income tax, and it was hoped by the Council that the terms of the reference might permit of the subject of income tax in its relation to life offices being discussed. He was, therefore, asked to approach the authorities, and he volunteered to give evidence. It was found, however, that the scope of the reference did not permit of that; the only matter upon which he was permitted to give some evidence was in relation to the taxation of terminable annuities. He attended before the Committee on behalf of the Institute, and his friend Mr. Ryan attended also at the same time on behalf of the Life Offices' Association, and they stated their views in regard to taxing the whole of the annuity, whether it was terminable or for life. He confessed that he did not think they made much practical headway with the Committee on that occasion, not because the Committee did not see the point which was brought before them, but because they apparently saw great difficulty in giving practical effect to the suggestions which were made. As President of the Institute, he had just been appointed by the Board of Trade to serve on the Committee dealing with a certain class of investment companies, and he hoped that that Committee might prove of some use.

The President concluded by moving "That the Report and Accounts, as presented, be adopted."

Mr. George King said that in the absence of Mr. Browne it devolved on him to second the motion for the adoption of the Report, and there were one or two points to which he would like to refer. All the members deeply appreciated the lectures that Mr. G. F. Hardy had delivered. Such lectures, however, were not easy to follow without notes being in the hands of the audience; and they all, therefore, looked forward to the time when the lectures would be printed and circulated. He felt perfectly certain that, just as they had learned so much from the other works of Mr. G. F. Hardy, they would be under a further obligation to him through the publication of the recent lectures. The Report showed that the Council and Officers did a great deal of work for the Institute, but there was one direction in which some of the junior members might assist in the work; he referred to the prize essays which the Council hoped would be sent in.* It would be very useful to the Institute if a number of good prize essays were sent in, and it would also be very useful to those who wrote them, even if they should not be fortunate in winning prizes. The members were also very pleased to know that further tables were to be published, based on the British Offices' Experience. Were it not for such subsidiary tables the main tables would be of comparatively little use. As an illustration of this he might point to the fact that, because there were few or no deduced tables based upon the 17 Offices' Experience, those tables had practically never been used in this country; and it was curious that, although there had been various English Life Tables issued since No. III, they had never come into use: mainly because there were so many tables based on No. III. It was therefore of importance that an abundance of tables should be issued, based on the British Offices' Experience, and the Institute had to thank the companies for their generosity in placing money at their disposal, enabling them to do work in that direction. He had nothing further to add, except in reply to the President's kind reference to himself, to say that he would do his best to make the lectures useful to all who cared to attend. It was not an easy task, and a great responsibility would rest upon him; but having undertaken it, he would not shrink from it.

The resolution for the adoption of the Report was then put and carried unanimously.

Election of Officers.

Messrs. J. H. Barnes and A. W. Tarn were appointed scrutineers of the ballot for the election of officers.

The ballot was then taken, and the President subsequently announced that the following list of Fellows, recommended by the Council for election, had been unanimously approved:—

President.

Henry Cockburn.

Vice-Presidents.

George King.
Ernest Woods.
Frederick Schooling.
Thomas Gans Ackland.

* See pages 127, 128.

Council.

Thomas Gans Ackland.	George King.
*Henry Walsingham Andras.	George James Lidstone.
Henry James Baker.	Henry William Manly.
James Blakey.	*Geoffrey Marks.
Henry Cockburn.	Arthur Ernest Molyneux.
*Francis Ernest Colenso, M.A.	William Peyton Phelps, M.A.
Gordon Douglas.	Gerald Hemmington Ryan.
Joseph Ernest Faulks, B.A.	Frederick Schooling.
Duncan Cumming Fraser, M.A.	George Todd, M.A.
Niel Ballingal Gunn.	Samuel George Warner.
Ralph Price Hardy.	Alfred William Watson.
*Arthur George Hemming.	James Douglas Watson.
Charles Daniel Higham.	Ernest Woods.
*Lewis Frederick Hovil.	Frank Bertrand Wyatt.
*William Hutton.	Thomas Emley Young, B.A.

Treasurer.

Gerald Hemmington Ryan.

Honorary Secretaries.

George Todd, M.A. | Samuel George Warner.

* Not Members of the previous Council.

On the motion of Mr. J. S. Hazell, seconded by Mr. W. C. Laing, Messrs. W. M. Monilaws and George A. Brown were re-elected auditors, and Mr. H. Lugton was elected an auditor, for the ensuing year.

Mr. H. E. Nightingale, in moving a vote of thanks to the President, Vice-Presidents, Council, Officers, Examiners, and Supervisors, for their services during the year, thought the success and progress of the Institute were largely due to the continued good administration of the President and Council. The fruits of their wise and liberal policy were shown by the steady maintenance of the membership, the high character of the general contributions, and by the strong position which the Institute held in the profession and in the estimation of the public. The Institute was also in an excellent financial condition, and no mark of appreciation was too high for the onerous and valued services rendered by the Examiners.

Mr. James Chatham, in seconding the motion, said the work of the officers was a labour of love, and he therefore thought the vote of thanks should be all the more hearty on that account.

The resolution having been carried with acclamation,

The President, on behalf of the Vice-Presidents, Council, Officers, Examiners, and himself, thanked the members very sincerely for the kind vote which had just been accorded to them. He was sure that all who served with him round the table, or who served as examiners, were actuated by the spirit of loyalty to the Institute and the desire to see it flourish.

Mr. E. A. Rusher, in moving that a vote of thanks be given to the auditors, Messrs. Bernard Woods, W. M. Monilaws and G. A. Brown, for their services during the past year, said that the accounts seemed to grow greater in volume, and he therefore thought that as each year passed the

members owed to the auditors an increasing debt of gratitude for seeing that the statements were drawn up in proper order.

Mr. HEMMING seconded the motion, which was carried with acclamation.

Mr. BERNARD WOODS, on behalf of Messrs. Monilaws, Brown and himself, thanked the members for so kind a recognition of their services. It was certainly true that the volume of business was larger, but at the same time, owing to the excellent way in which Mr. Jarvis prepared the accounts, the auditors' labours were very light.

The PRESIDENT then adjourned the meeting to Monday, 27 November 1905.

LECTURES

ON THE ACTUARIAL SUBJECTS OF PARTS III AND IV OF THE EXAMINATIONS.

The Council have arranged for a series of twenty-four Lectures, to be delivered by Mr. George King during the Session 1905-6, commencing on Thursday, 5 October, and continuing weekly (subject to the usual vacation at Christmas) until 5 April 1906. These Lectures, whilst mainly intended for Students in Parts III and IV, may also be regarded as partaking of the nature of a post-graduate course, and will be available for all members of the Institute possessing a knowledge of the subjects of Part II of the Examinations. The Syllabus of the first twelve Lectures, to be delivered before Christmas, 1905, is as follows:—

History of Life Assurance.
History of Mortality Tables.
Construction of Mortality Tables from Population Statistics, by Milne's Method.
Construction of Mortality Tables from Population Statistics, by the Analytical Method.
Construction of Mortality Tables from the Records of Life Offices.
Law of Mortality.
Graduation of Mortality Tables.

Further particulars as to fees, time of meeting, &c., have been communicated to all members of the Institute.

Additions to the Library.

The following works have been added to the Library since the publication of the *Journal* for October 1904:

By whom presented (when not purchased).

Accountants, Institute of Chartered, in England and Wales.
List of Members, 1905. — *The Institute.*

Actuarial Society of America.
Transactions, 1904–5. — *The Society.*
Containing *inter alia*—
"On the Principles which should determine the Maximum Single Risk, and the Acceptance of exceptional Classes of Risks", by W. S. Nichols.
"Notes on a Factor, hitherto overlooked, of the Rate of Interest", by C. T. Lewis.
"An Experiment with the Specialized Investigation", by R. W. Weeks.
"The Decline in the Birth Rate in New South Wales", by R. Teece.
"Methods of Intervaluation Ascertainment of Reserves", by M. M. Dawson.
"Effect of Total Abstinence on the Death Rate", by J. G. Van Cise.
"A Graduation of the Specialized Mortality Experience", by A. Hunter.
"On the Repayment of Loans by Contingent Instalments", by H. W. Robertson.

Actuaries, Faculty of
Transactions, 1904–5. — *The Faculty.*
Containing *inter alia*—
"Life Offices and Trust Funds", by W. B. Paterson.
"Life Assurance without Medical Examination", by J. Nicoll.
"The Investments of Australasian Life Offices", by J. B. Gillison.
"The Use of Select Tables for the Calculation of Survivorship Benefits", by Dr. J. Buchanan.
"The Influence of Expenses and Selection on Reserves for recently-effected Policies", by Dr. J. Buchanan.
"Office Premiums", by H. Moir.
"The English Land Registry", by J. R. Hart.
"Widows' Funds: The Valuation of a Widow's Annuity", by V. Marr.
"Titles to Life Assurance Policies", by A. E. Sprague.
"Notes on Local Loans", by T. Kyd.

Alpe (E. N.).
The Law of Stamp Duties on Deeds and other Instruments. Revised and amplified by A. B. Cane, B.A. Tenth Edition, 1905. — *Purchased.*

Altenburger (J.).
Beiträge zum Problem der Ausgleichung von Sterblichkeitstafeln. Budapest, 1905. — *The Author.*

American Mathematical Society.
Transactions, 1904–5. — *The Society.*

American Statistical Association.
Transactions, 1904–5. — *The Association.*

	By whom presented (when not purchased).
Andrae (A.).	
Die Sterblichkeit in den Berufen die sich mit der Herstellung und dem Verkauf geistiger Getränke befassen. Berlin, 1905.	*The Author.*
Australian Mutual Provident Society.	
Fifty-sixth Annual Report.	*The Society.*
Austria.	
Die privaten Versicherungsunternehmungen in den im Reichsrathe vertretenen Königreichen und Ländern im Jahre, 1902.	*The Austrian Government.*
Baker (H. J.), and Raisin (A. H.).	
British Offices Life Tables, 1893. Select Tables deduced from the Graduated Experience of Whole-Life Participating and Non-Participating Assurances on Male Lives, at $2\frac{1}{2}$, $2\frac{3}{4}$, 3, $3\frac{1}{4}$, $3\frac{1}{2}$, and 4 per-cent interest. 1904.	*The Authors.*
Another copy.	*The Publishers.*
A further copy.	*Purchased.*
Bankers.	
Journal of the Institute of Bankers. List of Members, 1905.	*The Institute.*
Belgium.	
Bulletin de l'Association des Actuaires Belges.	*The Association.*
Bulletin du Comité permanent des Congrès internationaux d'Actuaires. 1904.	*The Permanent Committee.*
Compte Rendu des Opérations et de la Situation de la Caisse Générale d'Épargne et de Retraite. 1904.	*The Belgian Government.*
Bertelsen (N. P.), and Steffensen (J.).	
A Table for determining the Rate of Interest in an Annuity-Certain.	*The Authors.*
Bertillon (Dr. Jacques).	
Nomenclatures des Maladies. (Statistique de Morbidité—Statistique des causes de décés) arrêtées par la Commission internationale chargée de reviser les nomenclatures nosologiques, 1900. Avec notices et annexes. Montévrain, 1903.	*The Author.*
"Biometrika."	
Vol. III, part IV, and Vol. IV, parts I and II, containing, *inter alia*— "Das Fehlergesetz und seine Verallgemeinerungen durch Fechner und Pearson." A Rejoinder, being an Historical Account of Skew Variation, by Karl Pearson.	*Purchased.*
Brabrook (Sir E. W.).	
Institutions for Thrift. 1905	*Purchased.*
Bunyon (C. J.).	
The Law of Life Assurance: Edited by J. Vesey Fitzgerald. Fourth Edition, 1904.	*Purchased.*
Cantelli (Dr. F. P.).	
Sull' adattamento delle curve ad una serie di misure o di osservazioni. Rome, 1905.	*The Author.*
Davenport, (C. B.)	
Statistical Methods with Special Reference to Biological Variation. New York, 1904.	*Purchased.*

	By whom presented (when not purchased).
Dawson (M. M.).	
Various derived Tables. One Life. American Experience Tables of Mortality. Second Edition. With Logarithmic Tables and Select and Ultimate Values. New York, 1905.	*The Author.*
Depas (L.).	
Le Régime des Sociétés Françaises et etrangères d'Assurances sur la vie en France. Paris, 1903.	*Purchased.*
Dizler (C.).	
In Arithmetischen Reihen steigende Renten. 1904.	*The Author.*
d'Ocagne (M.).	
Le Calcul Simplifié par les Procédés Mécaniques et Graphiques. Second Edition. Paris, 1905.	*The Author.*
Eedy (A. M.).	
Industrial Life Assurance in Australasia. Sydney, N.S.W. 1903.	*The Author.*
Elsner (Dr. A. F.).	
Assecuranz Almanach, 1905.	*The Publishers.*
Engineers, Institution of Civil	
List of Members, 1905.	*The Institution.*
France.	
Bulletin de l'Institut des Actuaires Français.	*The Institute.*
Revue des Assurances sur la vie: Doctrine—Instructions Pratiques—Jurisprudence—Questions Médicales—Statistique—&c. 9 vols. Paris, 1874–82.	*R. H. Curtis.*
Germany.	
Deutscher Verein für Versicherungs-Wissenschaft. List of Members. 1905.	*The Society.*
Veröffentlichungen des Deutschen Vereins für Versicherungs-Wissenschaft. 1904–5.	*The Editor.*
Zeitschrift des Königlich Preussischen Statistischen Bureaus. 1904–5.	*The German Government.*
Zeitschrift für die gesamte Versicherungs-Wissenschaft. 1904–5.	*The Society.*
Gunn (N. B.).	
Stock Exchange Securities and how to select them. Edinburgh, 1904.	*The Author.*
Henderson (R.).	
Frequency Curves and Moments. 1904.	*The Author.*
Holland.	
Archief voor de Verzekerings-Wetenschap. 1904–5.	*The Society.*
Jaarboekje van de Vereeniging voor Levensverzekering. 1905.	*The "Algemeene Maatschappij."*
Mededeelingen der Vereeniging voor Levensverzekering. 1904–5.	*The Society.*
Infantile Mortality.	
A Collection of Cuttings from the *Times*. 1904.	*Purchased.*
Ingall (G. D.), and Withers (G.).	
The Stock Exchange. 1904.	*Purchased.*
Insurance Blue Book and Guide.	
1904–5.	*The Publishers.*
"Insurance Register." By William White.	
1905.	*The Publishers*

By whom presented (when not purchased).

Ireland, Insurance Institute of.
Journal, 1898–1905, containing *inter alia*— *The Institute.*
"On the Construction of Life Assurance Premiums", by C. E. Howell.
"The Elements of the Theory of Probability", by W. A. Martin.
"Some Aspects of the Workmen's Compensation Act", by M. Good.
"Death Rates in Former and Recent Times", by Sir C. A. Cameron, C.B., M.D.
"Insurance and Banking Finance: A Contrast and a Forecast", by G. P. Blizard.
"Suggested Amendments in the Workmen's Compensation Acts", by H. Hanna, LL.B.

Joint Committee of the Institute of Actuaries and the Faculty of Actuaries in Scotland on Mortality Investigation.
Transactions, 1893–1905. Six copies, for reference only. *The Joint Committee.*

"Journal of the Federation of Insurance Institutes."
Vol. VII, containing *inter alia*— *Purchased.*
"A Life Office Prospectus", by J. Stirling.
"Notes on Investments", by J. Robb.
"Observations on Medical Examination in connection with Life Assurance", by B. J. Baron, M.B.
"Old Age Pensions", by D. C. Fraser.
"The Main Principles to be followed in the Selection of Lives by a Life Assurance Company", by J. B. Hall.
"The Principles of Organization in an Insurance Office", by T. E. Young, B.A.
"The Value of New Business to a Life Office", by D. Y. Mills.
"Weights and Measures", by C. E. Galwey.

Lepreux (O.).
De la Réparation des Accidents du Travail conformément à la loi du 24 Décembre 1903: Critique d'un projet de Caisse Commune des industries du Bassin de Liége. Brussels, 1905. *The Author.*

London Mathematical Society.
Proceedings, 1904–5. *The Society.*

McKellar (J. A.).
Note on Lubbock's Formula. 1904. *Geoffrey Marks.*

Maluquer y Salvador (J.).
Caja Nacional de Seguro Popular. Ponencia, 1904.
Cajas Nacionales de Seguro Popular. Informaciòn Legislativa. 1904.
Problemas Internacionales del Seguro. 1905. *The Author.*

Maluquer y Salvador (J.), and others.
Proyecto de Ley de Instituto Nacional de Previsiòn. 1905. *J. Maluquer y Salvador.*

Manchester, Insurance Institute of
Report and Proceedings, 1903–4. *The Institute.*
Containing *inter alia*—
"Income Tax as affecting Insurance Companies", by R. N. Carter.

	By whom presented (when not purchased).
Moir (H.).	
Life Assurance Primer: A Text-Book dealing with the practice and Mathematics of Life Assurance. New York, 1904. Two copies.	*The Author.*
Molt (C. G.).	
Die Kredit-Versicherung. Berlin, 1905.	*The Author.*
New Zealand, Insurance Institute of	
Report and Proceedings, 1904, containing *inter alia*—"The Life Insurance Policies' Act, 1884", by M. Chapman.	*The Institute.*
Ontario Medical Association.	
Symposium on Life Assurance: Being original Papers contributed at the Annual Convention, June, 1904. Toronto, 1904.	*The Northern Assurance Co. of Canada.*
Parliamentary Papers.	
Acts of Parliament. Finance Act, ch. 7. 1904. Friendly Societies. 1896. Public Works Loans. 1904. Aged Pensioners' Bill. Report and Special Report from the Select Committee, Proceedings, Minutes of Evidence, &c. 1903. Census, 1901, England and Wales. General Report, with Appendices. 1904.	*Purchased.*
Colonies.	
Australia and New Zealand. Statistical Account, 1903–4. Statistics, 1861–1903.	*The Government of N.S.W.*
Canada.	
Census, 1901. Vol. I. Population.	*M. D. Grant.*
Report of the Superintendent of Insurance for the year 1903.	*The Government Insurance Department.*
Statistical Year Book, 1902 and 1903.	*M. D. Grant.*
New South Wales. Annual Report of the Government Statistician on Births, Deaths, and Marriages in 1904. Annual Report of the Government Statistician on the Vital Statistics of the Metropolis, 1904. Results of Census, taken 31 March 1901. Life Assurance, 1904. Eight parts in one volume. Statistical Register, 1903, 1904. Vital Statistics. Report for 1903 and previous years.	*The Government of N.S.W.*
New Zealand. Official Year Book, 1904. Statistics of the Colony for the year 1903.	*The Government of N.Z.*
Victoria. Friendly Societies. Twenty-sixth Annual Report of the Actuary, 1903. Fourteenth Report of the Registrar, 1903. Statistical Register, 1904.	*The Government of Victoria.*
Western Australia. Statistical Register for 1903 and previous years. Year Book for 1900–3. Vol. II.	*The Government of W. Australia.*

By whom presented (when not purchased).

Parliamentary Papers—*(continued).*

Friendly Societies.

Building Societies.

Ninth Annual Report, Parts I and II.

Chief Registrar's Report for 1903. Parts A, B and C.

Guide Book of the Registry Office, 1905. — *Purchased.*

Government Insurance and Annuities.

Account, 1903. — *Purchased.*

Life Assurance Companies.

Returns, 1904. — *The Board of Trade.*

National Debt.

Annuities. Account to 5 January 1904.

Return showing Liabilities and Assets at the close of each financial year, from 1835–6 to 1903–4, both inclusive.

Savings Banks and Friendly Societies. Account to 20 November 1904.

Sinking Funds. Account showing amount received from the Exchequer during the year ended 31 March 1904. — *Purchased.*

Registrar-General. England.

Sixty-fifth and Sixty-sixth Annual Reports on Births, Deaths and Marriages. 1902, 1903.

Registrar-General. Ireland.

Fortieth detailed Annual Report on Births, Deaths and Marriages. 1903.

Supplement to the Thirty-seventh detailed Annual Report for 1900. Decennial Summaries, 1891–1900.

Registrar-General. Scotland.

Forty-eighth detailed Annual Report on Births, Deaths and Marriages. (Abstracts of 1902.)

Forty-ninth Annual Report, 1902; and Thirty-ninth Annual Report on Vaccination. 1904.

Workmen's Compensation.

Memorandum on Foreign and Colonial Laws relating to Compensation for Injuries to Workmen. By Sir Kenelm E. Digby.

Report of the Departmental Committee appointed to enquire into the Law relating to Compensation for Injuries to Workmen. Minutes of Evidence, &c. 1904.

Statistics of Proceedings under the Acts of 1897 and 1900, and the Employers' Liability Act, 1880, during the year 1903. — *Purchased.*

Periodicals.

Accountants' Magazine. — *Purchased.*

Insurance Record. — *The Editor.*

Post Magazine. — *The Editor.*

Poort (W. A.).

Mededeelingen van de Nederlandsche Vereeniging voor Gemeente-belangen, 1903. Part I.

Verslag betreffende de grondslagen voor Pensioneering van weduwen en weezen van de ambtenaren der Gemeente 's-Gravenhage. 1904.

Verslag betreffende de Pensioneering der ambtenaren van den Provincialen Waterstaat in Friesland en van hunne weduwen en weezen. 2 parts. 1903–4. — *The Author.*

Poussin (R.).

Sur l'application des procédés Graphiques aux Calculs d'Assurances. Paris, N.D. — *Purchased.*

	By whom presented (when not purchased).
Powell (Sir R. D.), M.D. The Prevention of Consumption. 1904.	*W. J. H. Whittall.*
Rebstein (J.), and Schaertlin (G.). Befinden über die Pensions und Hülfskasse der Schweizerischen Bundesbahnen, vom 30 November 1904.	*The Authors.*
Samwer (K.). Die Selbstentleibungen bei der Gothaer Lebensversicherungsbank in den Jahren 1829 bis 1903.	*The Gotha Life Office.*
Scientific and Learned Societies of Great Britain and Ireland. Official Year-Book. 1904.	*Purchased.*
South Australian School of Mines and Industries. Annual Report, 1904.	*The Public Actuary of S. Australia.*
Statistical Society (Royal). Journal, 1904–5.	*The Society.*
Stillman (C. F.), M.D. The Life Insurance Examiner: A Practical Treatise upon Medical Examinations for Life Insurance. New York, 1888.	*I. S. Lidbury.*
Strahan (J. A.). A General View of the Law of Property. Third edition, 1901.	*Purchased.*
Switzerland. Rapport du Bureau Fédéral des Assurances sur les Enterprises Privées en Matière d'Assurances en Suisse, 1903.	*The Swiss Government.*
Toronto, Insurance Institute of Report and Proceedings, 1903–4, containing *inter alia*— "A Method of Ascertaining the Cost of an Agency", by A. J. Hughes. "The Assurance of Under-Average Lives", by J. Burn. "Some Points on Tuberculosis", by J. L. Davison, B.A., M.D.	*The Institute.*
Trefzer (F.). Versicherungstechnische Beilage zum Statutenentwurf der Hülfskasse für das Personal der eidgenössischen Verwaltungen. 1904.	*The Author.*
Typaldo-Bassia (A.). Les Assurances sur la vie au point de vue Théorique et Pratique. Paris, 1892.	*The Author.*
Yale Insurance Lectures. Life, Fire and Miscellaneous. 2 Vols. 1903–4.	*The Yale Alumni Weekly.*
Yorkshire, Insurance Institute of Report and Proceedings, 1903–4.	*The Institute.*

The following additional copies of works already in the Library have also been added:—

Curtis (F. A.). Scheme of Graphic Notation applicable to Life Assurance Computations. 1869.	*R. H. Curtis.*
Journal of the Institute of Actuaries. Vols. ix to xxii inclusive.	*The Pharmaceutical Society.*

INDEX TO VOL. XXXIX.

END OF VOL. XXXIX.

Printed by C. & E. Layton, Farringdon Street.